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FREGE *on* LANGUAGE, LOGIC, & PSYCHOLOGY

selected essays

EVA PICARDI

edited with an introduction by
Annalisa Coliva

Frege on Language, Logic, and Psychology

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OXFORD
UNIVERSITY PRESS

OXFORD
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Great Clarendon Street, Oxford, OX2 6DP,
United Kingdom

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Introduction © Annalisa Coliva 2022

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First Edition published in 2022

Impression: 1

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Published in the United States of America by Oxford University Press
198 Madison Avenue, New York, NY 10016, United States of America

British Library Cataloguing in Publication Data
Data available

Library of Congress Control Number: 2021951876

ISBN 978-0-19-886279-6

DOI: 10.1093/oso/9780198862796.001.0001

Printed and bound by
CPI Group (UK) Ltd, Croydon, CR0 4YY

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Eva Picardi 1948–2017

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Acknowledgements

This volume originates in conversations I had with Eva Picardi over the last few years of her life. She wanted to collect her papers on Frege, and was hoping to be able to do it before dealing with the dire consequences of her long-lasting illness. She asked me to help her put the volume together and I gladly accepted. However, we never really got started on the volume, for we both had more urgent projects at hand. Moreover, our personal lives put many demands on our time and energy. When, all of the sudden, things precipitated, I felt I had to honor the promise I had made. I am sure Eva would have been critical of my choices, but I also think that she would have been pleased to see that I had kept my promise. My hope is that having several of her major papers collected in one single volume will enhance the significance of each and will allow readers to appreciate the overall interpretation of Frege's philosophy that emerges from them. Another volume with Eva's writings on Wittgenstein, Quine, Davidson, and American neo-pragmatists, such as Rorty and Brandom, has in the meanwhile appeared in print (*The Selected Writings of Eva Picardi: From Wittgenstein to American Neo-Pragmatism*, A. Coliva (ed.), Bloomsbury, 2020). The two volumes complement each other and will allow scholars to appreciate the width and depth of Eva's scholarship.

This volume would not have been possible without the precious help of several people: Paolo Leonardi, who helped me reconstruct the corpus of Eva's work; Alessia Pasquali, a former student of Eva's and a former doctoral student of mine, who helped me collect all of her papers, even when quite inaccessible; Antonio Ferro, a former student of Eva's, who translated one of the papers that appear here in English for the first time;¹ and, finally, Edward (Ted) Mark, a graduate student of mine, who helped me prepare the manuscript and obtain permissions to reprint the material contained in this volume.

My sincere gratitude goes also to the Humanities Research Center, the hub for research in the School of Humanities at the University of California, Irvine, and to Judy Tzu-Chun Wu in particular, for a generous award that has made work on this book possible. Last but not least, my deepest thanks to Peter Momtchiloff at Oxford University Press, who has generously provided much needed editorial help in re-keying several papers of which only a PDF file of the printed version was at my disposal.

¹ Chapter 2 of the present volume. Chapters 5, 6, and 11, originally in Italian, have been translated by me.

Credits

Chapter 1: ‘The Logic of Frege’s Contemporaries, or “Der verderbliche Einbruch der Psychologie in die Logik”’, in D. Buzzetti and M. Ferriani (eds.), *Speculative Grammar, Universal Grammar and Philosophical Analysis of Language*, Amsterdam, Benjamins, 1987, 173–204. Italian translation reprinted in E. Picardi, *La chimica dei concetti: linguaggio, logica, psicologia 1879–1927*, Bologna, Il Mulino, 1994, chapter 1.

Chapter 2: ‘Kerry und Frege über Begriff und Gegenstand’, *History and Philosophy of Logic*, 15, 1994, pp. 9–32. Italian translation reprinted in E. Picardi, *La chimica dei concetti: linguaggio, logica, psicologia 1879–1927*, Bologna, Il Mulino, 1994, chapter 2.

Chapter 3: ‘Sigwart, Husserl, and Frege on Truth and Logic, or Is Psychologism Still a Threat?’, *European Journal of Philosophy*, 5, 2, 1997, pp. 162–82.

Chapter 4: ‘Frege’s Anti-Psychologism’, in M. Schirn (ed.), *Frege: Importance and Legacy*, Berlin, De Gruyter, 1998, pp. 307–29. Reprinted in M. Beaney (ed.), *Gottlob Frege, Critical Assessments of Leading Philosophers*, Routledge, 4 vols., vol. 1, *Frege in Context*, 2006, pp. 340–58.

Chapter 5: ‘Frege, Peano e Russell sulle idee primitive della logica’, in N. Vassallo (ed.), *La filosofia di Gottlob Frege*, Milan, Angeli, 2003, pp. 181–210.

Chapter 6: ‘Über Sinn und Bedeutung: un’esposizione elementare (Parte I)’, *Lingua e Stile*, 24, 3, 1989, pp. 331–64; ‘Über Sinn und Bedeutung: un’esposizione elementare (Parte II)’, *Lingua e Stile*, 25, 2, 1990, pp. 159–99. Reprinted in E. Picardi, *La chimica dei concetti: linguaggio, logica, psicologia 1879–1927*, Bologna, Il Mulino, 1994, chapter 3.

Chapter 7: ‘The Chemistry of Concepts’, in B. Naumann, F. Plank, and G. Hofbauer (eds.), *Language and Earth*, Studies in the History of the Language Sciences, 66, Amsterdam, Benjamins, 1992, pp. 125–46. Italian translation reprinted in E. Picardi, *La chimica dei concetti: linguaggio, logica, psicologia 1879–1927*, Bologna, Il Mulino, 1994, chapter 4.

Chapter 8: ‘Assertion and Assertion Sign’, in G. Corsi, C. Mangione, and M. Mugnai (eds.), *Teorie delle modalità, Atti del convegno internazionale di storia della logica di San Gimignano 5–10 dicembre 1987*, 1989, pp. 139–54. Italian translation reprinted in E. Picardi, *La chimica dei concetti: linguaggio, logica, psicologia 1879–1927*, Bologna, Il Mulino, 1994, chapter 5.

Chapter 9: ‘A Note on Dummett and Frege on Sense Identity’, *European Journal of Philosophy*, 1, 1, 1993, pp. 69–80.

Chapter 10: 'Michael Dummett's Interpretation of Frege's Context Principle: Some Reflections', in M. Frauchinger (ed.), *Justification, Understanding, Truth, and Reality: Themes from Dummett*, Berlin, De Gruyter, 2018, pp. 29–62.

Chapter 11: 'Carnap interprete di Frege', in A. Pasquinelli (ed.), *L'eredità di Carnap*, Bologna, CLUEB, 1995, pp. 357–72.

Chapter 12: 'Frege and Davidson on Predication', in M. C. Amoretti and N. Vassallo (eds.), *Knowledge, Language and Interpretation: On the Philosophy of Donald Davidson*, Frankfurt, Ontos Verlag, 2008, pp. 49–79.

Chapter 13: 'Davidson and Frege on the Unity of the Proposition: Some Remarks', *Dianoia*, 14, 2009, pp. 185–209.

Chapter 14: 'Was Frege a Proto-Inferentialist?', in J. J. Acero and P. Leonardi (eds.), *Facets of Concepts*, Padua, Il Poligrafo, 2005, pp. 35–48.

Complete List of Publications by Eva Picardi

The list was compiled by Picardi herself. The papers in this volume are marked in bold.

1973

'Organismo vivente e organismo linguistico', in L. Heilmann (ed.), *Wilhelm von Humboldt nella cultura contemporanea*, Bologna, Il Mulino, 1976, 61–82.

Review of K. R. Popper, *Congetture e confutazioni*, *Rivista di filosofia*, LIV, 1, 68–70.

1976

'Note sulla logica di Port-Royal', *Lingua e Stile*, XI, 3, 337–91.

Review of W. V. Quine, *I modi del paradosso*, *Lingua e Stile*, XI, 4, 614–15.

Review of M. Dummett, *Frege: Philosophy of Language*, *Lingua e Stile*, XI, 4, 597–600.

1977

'Some Problems of Classification in Linguistics and Biology: 1800–1830', *Historiographia Linguistica*, IV, 31–57.

'Gottlob Frege: opere postume e lettere', *Lingua e Stile*, XII, 4, 547–81.

1978

Review of S. Haack, *Deviant Logic*, *Lingua e Stile*, XIII, 4, 573–7.

Translation of B. F. McGuinness, 'Il così detto realismo del Tractatus di Wittgenstein', *Lingua e Stile*, XII, 2, 161–74.

1979

'Asserzioni con nomi vuoti', *Lingua e Stile*, XIV, 2/3, 217–32.

1980

'Significato, traduzione e forma logica', *Lingua e Stile*, XV, 3, 511–24.

1981

Assertibility and Truth: A Study of Fregean Themes, Bologna, CLUEB.

Review of M. Dummett, *Truth and Other Enigmas*, *Lingua e Stile*, XVI, 4, 613–17.

1982

'Interpretazione radicale e teorie della verità', *Lingua e Stile*, XVIII, 1, 51–67.

'Alcune osservazioni sulla corrispondenza Frege-Russell', *Atti del Convegno Nazionale di Logica, 1–5 ottobre Montecatini 1979*, Naples, Bibliopolis, 717–35.

Review of W. L. Harper and R. Stalnaker (eds.), *Ifs: Conditionals, Belief, Decision, Chance, and Time*, *Lingua e Stile*, XVIII, 1, 575–8.

Italian translation of L. Wittgenstein, *Lezioni sui fondamenti della matematica*, Cambridge 1939, C. Diamond (ed.), Torino, Boringhieri.

1983

‘On Frege’s Notion of Inhalt’, in *Atti del Convegno internazionale di Storia della logica*, San Gimignano, 4–8 dicembre 1982, Bologna, CLUEB, 307–12.

‘Wittgenstein e la teoria del significato’, in A. Gargani (ed.), *Ludwig Wittgenstein e la cultura contemporanea*, Ravenna, Longo Editore, 101–7.

Review of L. Wittgenstein, *Il libro blu e il libro marrone*, *Lingua e Stile*, XVIII, 639–40.

Review of H. Scholz, *Storia della logica*, *Lingua e Stile*, XVIII, 640.

Review of H. D. Levin, *Categorical Grammar*, *Lingua e Stile*, XVIII, 4, 639.

1984

Asserting, D.Phil. dissertation, Oxford, unpublished.

Review of J. Barwise and J. Perry, *Situations and Attitudes*, *Lingua e Stile*, XIX, 4, 625–8.

Review of U. Eco, *Semiotica e filosofia del linguaggio*, *Lingua e Stile*, XIX, 4, 690.

Review of M. Dummett, *Filosofia del linguaggio: Saggio su Frege*, *Lingua e Stile*, XIX, 4, 689–90.

1986

‘Osservazioni sulla nozione di composizionalità’, *Lingua e Stile*, XXI, 2/3, 187–205.

‘Peano’, in *Encyclopedic Dictionary of Semiotics*, T. Sebeok (ed.), vol. II, 672–3, Berlin, Mouton–De Gruyter.

1987

Critical edition and Italian translation of G. Frege, *Scritti postumi*, Naples, Bibliopolis, 9–66.

‘A proposito della Centenar Ausgabe delle *Grundlagen der Arithmetik* di Gottlob Frege’, *Rivista di filosofia*, LXXVIII, 1, 131–5.

‘The Logics of Frege’s Contemporaries or “der verderbliche Einbruch der Psychologie in die Logik”’, in D. Buzzetti and M. Ferriani (eds.), *Speculative Grammar, Universal Grammar and Philosophical Analysis of Language*, Amsterdam, Benjamins, 173–204.

‘Ramsey fra Wittgenstein e Russell’, in R. Simili (ed.), *L’epistemologia di Cambridge 1850–1950*, Bologna, Il Mulino, 329–55.

Review of Gabbay e Guenther (eds.), *Handbook of Philosophical Logic*, vol. III, *Lingua e Stile*, XXII, 4, 543–6.

Review of M. Di Francesco, *Parlare di oggetti*, *Lingua e Stile*, XXII, 4, 561–3.

Review of J. L. Austin, *Come fare cose con le parole*, *Lingua e Stile*, XXII, 4, 621–2.

1988

'Meaning and Rules', in C. Nyíri, B. Smith (eds.), *Practical Knowledge: Outlines of a Theory of Traditions and Skills*, London, New York, and Sydney, Croom Helm, 90–121.

'Frege on Definition and Logical Proof', in *Temi e prospettive della logica e dalla filosofia della scienza contemporanee, Atti del congresso della S.I.L.F.S., Cesena 7–10 gennaio 1987*, Bologna, CLUEB, vol. I, C. Cellucci and G. Sambin (eds.), 227–30.

Review of R. M. Martin, *Metaphysical Foundations, Mereology, Logic, Lingua e Stile*, XXIII, 4, 538–41.

Review of N. Tennant, *Anti-Realism and Truth, Lingua e Stile*, XXIII, 4, 607–8.

Review of P. Simons, *Parts, Lingua e Stile*, XXIII, 4, 6.

1989

'Über Sinn und Bedeutung: un'esposizione elementare (Parte I)', *Lingua e Stile*, XXIV, 3, 331–64.

'Assertion and Assertion Sign', in *Teorie delle Modalità, Atti del convegno internazionale di Storia della logica di San Gimignano 5–10 dicembre 1987*, G. Corsi, C. Mangione, M. Mugnai (eds.), 139–54.

'Davidson on Assertion, Convention and Belief', in J. Brandl and W. L. Gombocz (eds.), *The Mind of Donald Davidson*, Amsterdam, Rodopi, 97–108.

Review of E. Lepore (ed.), *New Directions in Semantics, Lingua e Stile*, XXIV, 4.

Review of G. Frege, *Die Grundlagen der Arithmetik, Centenarerausgabe, History and Philosophy of Logic*, X, 98–100.

1990

German edition, translation, and introduction, with J. Schulte, *Die Wahrheit der Interpretation, Beiträge zur Philosophie Donald Davidsons*, Frankfurt, Suhrkamp, 7–54.

'Über Sinn und Bedeutung: un'esposizione elementare (Parte II)', *Lingua e Stile*, XXV, 2, 159–99.

'La chimica dei concetti', *Lingua e Stile*, XXV, 3, 363–81.

Italian translation of M. Dummett, *Alle origini della filosofia analitica*, Bologna, Il Mulino.

1991

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Review of W. V. Quine, *Pursuit of Truth, Lingua e Stile*, XXVI, 4, 574–7.

1992

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'Alfred North Whitehead/Bertrand Russell: Principia Mathematica', in *Interpretationen: Hauptwerke der Philosophie. 20. Jahrhundert*, Stuttgart, Reclam, 7–42.

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'Formalizzazione linguistica', in M. Cortellazzo and A. Mioni (eds.), *La linguistica italiana degli anni 1976–1986*, Rome, Bulzoni, 395–408.

1993

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'A Note on Dummett and Frege on Sense-Identity', *European Journal of Philosophy*, I, 1, 69–80.

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'Dummett on Analysis and Cognitive Synonymy', in J. Czermak (ed.), *Philosophy of Mathematics, Proceedings of the 15th International Wittgenstein Symposium, Part I*, Vienna, Hölder Pichler, 77–85.

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Review of D. Davidson, I. Hacking, M. Dummett, *Linguaggio e interpretazione*, L. Perissinotto (ed.), *Lingua e Stile*, XXVIII, 4, 622–3.

1994

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'Kerry und Frege über Begriff und Gegenstand', *History and Philosophy of Logic*, XV, 9–32.

'Davidson and Quine on Observation Sentences', in G. Preyer (ed.), *Language, Mind and Epistemology: On Donald Davidson's Philosophy*, Kluwer, Synthese Library, 97–116.

'Convention and Assertion', in B. McGuinness, G. Oliveri (eds.), *The Philosophy of Michael Dummett*, Kluwer, Synthese Library, 59–78.

'Frege on Anti-Psychologism and Objectivity', in C. Cellucci, M. C. Di Maio, G. Roncaglia (eds.), *Logica e filosofia della scienza: problemi e prospettive, Atti del convegno di logica e filosofia della scienza*, Lucca, 7–10 gennaio 1993, Pisa, Edizioni ETS, 93–102.

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'Comments on Norman Malcolm (1989)', in R. Egidi (ed.), *Wittgenstein: Mind and Language*, Kluwer, Synthese Library, 207-10.

Review of S. Read, *Thinking about Logic*, *Lingua e Stile*, XXX, 4, 701-4.

Review of H. L. Kretzenbacher, H. Weinrich (eds.), *Linguistik der Wissenschaftssprache*, *Lingua e Stile*, XXX, 4, 717-19.

Review of L. Formigari, *La sémiotique empiriste face au kantisme*, *Lingua e Stile*, XXX, 4, 771-2.

1996

Italian edition and translation of M. Dummett, *La base logica della metafisica*, Bologna, Il Mulino.

Review of B. Smith, *Austrian Philosophy: The Legacy of Franz Brentano*, *Erkenntnis*, XLV, 123-7.

Review of M. Kusch, *Psychologism: A Case Study in the Sociology of Philosophical Knowledge*, *Lingua e Stile*, XXXI, 4, 620-4.

1997

'Sigwart, Husserl and Frege on Truth and Logic, or Is Psychologism Still a Threat?', *European Journal of Philosophy*, V, 2, 162-82.

'Varietà di naturalismo', *Lingua e Stile*, XXXII, 1, 102-25.

'Is Language a Natural Object?', in M. Sainsbury (ed.), *Thought and Ontology*, Milan, Angeli, 107-23.

Review of H.-J. Glock, R. Arrington (eds.), *Wittgenstein and Quine* (Routledge 1996), *Lingua e Stile*, XXXII, 4, 588-92.

1998

'Semantica naturalizzata?', in E. Agazzi, N. Vassallo (eds.), *Introduzione al naturalismo filosofico contemporaneo*, Milan, Angeli, 240-65.

'Frege e Peano sul segno d'asserzione', in B. McGuinness (ed.), *Language, Logic and the Formalization of Knowledge*, Gaeta, Bibliotheca, 199-222.

Review of A. Bottani, *Il riferimento imperscrutabile* (1996), *Epistemologia*, XXI, 2, 355-9.

'Frege's Anti-Psychologism', in M. Schirn (ed.), *Frege: Importance and Legacy*, Berlin, De Gruyter, 307-29.

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- Teorie del significato*, Rome and Bari, Laterza.
- 'Ist der kritische Weg noch offen?', *Deutsche Zeitschrift für Philosophie*, XLVII, 3, 511–23.
- 'Reference, Conceptual Scheme and Radical Interpretation', in R. Rossini Favretti, G. Sandri, R. Scazzieri (eds.), *Incommensurability and Translation*, London, Elgar Press, 53–67.
- 'Sensory Evidence and Shared Interests', in M. De Caro (ed.), *Interpretations and Causes: New Perspectives on Donald Davidson's Philosophy*, Dordrecht, Kluwer, 171–85.

2000

- 'Empathy and Charity', in L. Decock, L. Horsten (eds.), *Quine: Naturalized Epistemology, Perceptual Knowledge and Ontology*, Amsterdam, Rodopi, 121–34.
- 'Frege und Peano über Definitionen', in G. Gabriel, U. Dathe (eds.), *Gottlob Frege: Werk und Wirkung*, Paderborn, Mentis Verlag, 171–89.

2001

- Introduction in M. Dummett, *Origini della filosofia analitica*, Torino, Einaudi, vii–xiv.
- Italian translation and translation note of G. Frege, *Senso, funzione e concetto: scritti filosofici*, ed. C. Penco and E. Picardi, Bari, Laterza, xxix–xxxviii.
- Italian translation of M. Dummett, *La natura e il futuro della filosofia*, Genoa, Il Melangolo.
- 'Normativity and Meaning', in *The Dialog, Das Gespräch, Il Dialogo, Yearbook for Philosophical Hermeneutics, Normativity and Legitimation* (Proceedings of the II Meeting Italian/American Philosophy, New York 1999, Riccardo Dottori ed.), 1, Münster, Hamburg, and London, LIT Verlag, 233–50.
- 'Rorty, Sorge and Truth', *International Journal of Philosophical Studies*, IX, 3, 431–9.
- Preface to R. Dionigi, *La fatica di descrivere*, Macerata, Quodlibet, 9–24.
- 'Compositionality', in G. Cosenza (ed.), *Grice's Heritage*, Turnhout, Brepols, 52–72.
- Teorias del significado* (Spanish translation of Picardi 1999, *Teorie del significato*), Madrid, Alianza.

2002

- 'Teoria del significato e olismo: alcune osservazioni sul programma di Michael Dummett', in M. Dell'Utri (ed.), *Olismo*, Macerata, Quodlibet, 91–112.
- 'Il principio del contesto in Frege e Wittgenstein', in C. Penco (ed.), *La svolta contestuale*, Milan, McGraw Hill, 1–23.

2003

‘Frege, Peano e Russell sulle idee primitive della logica’, in N. Vassallo (ed.), *La filosofia di Gottlob Frege*, Milan, Angeli, 181–210.

Italian edition of H. Putnam, *Mente, corpo, mondo*, Bologna, Il Mulino.

2004

‘La verità nell’interpretazione: alcune osservazioni su Gadamer e Davidson’, in M. Gardini and G. Matteucci (eds.), *Gadamer: bilanci e prospettive*, Macerata, Quodlibet, 275–86.

Edition and introduction, with Annalisa Coliva, *Wittgenstein Today*, Padua, Il Poligrafo, 15–21.

2005

‘Semantica: afferrare pensieri’, in C. Bianchi, N. Vassallo (eds.), *Filosofia della comunicazione*, Bari and Rome, Laterza, 17–41.

Reprint of ‘Frege’s Anti-Psychologism’, in M. Beaney (ed.), *Gottlob Frege, Critical Assessments of Leading Philosophers*, London/ New York, Routledge, 4 vols., vol. I, *Frege in Context*, 340–58.

‘Was Frege a Proto-Inferentialist?’, in J. J. Acero, P. Leonardi (eds.), *Facets of Concepts*, Padua, Il Poligrafo, 35–48.

2006

‘Linguistics and Logic I: The Influence of Frege and Russell on Semantic Theory’, in S. Aurox, K. Koerner, H.-J. Niederehe, K. Versteegh (eds.), *History of the Language Sciences*, vol. 3, Berlin and New York, De Gruyter, 2600–12.

‘Concept and Inference’, *Preprints*, XXVIII, 123–41.

‘Individualismo semantico e significato letterale’, in R. Calcaterra (ed.), *Le ragioni del conoscere e dell’agire: scritti in onore di Rosaria Egidi*, Milan, Angeli, 392–402.

‘Significato letterale, traduzione e interpretazione’, in R. Pititto, S. Ventura (eds.), *Tradurre e comprendere, Atti del XII congresso della società di filosofia del linguaggio, Piano di Sorrento, 29 settembre–1 ottobre 2005*, Rome, Aracne Editrice, 433–54.

‘Colouring, Multiple Propositions and Assertoric Content’, in M. Carrara, E. Sacchi (eds.), *Propositions: Semantic and Ontological Issues*, Grazer Philosophische Studien, LXXII, 21–43.

2007

‘Pragmatismo e teorie del significato: Rorty, Davidson, Brandom’, in R. Calcaterra (ed.), *Pragmatismo e filosofia analitica*, Macerata, Quodlibet, 139–61.

‘On Sense, Tone, and Accompanying Thoughts’, in R. B. Auxier, L. E. Hahn (eds.), *The Philosophy of Michael Dummett*, The Library of Living Philosophers, Chicago and LaSalle, Open Court, 491–520.

2008

- Italian edition and introduction, M. Dummett, *Pensiero e realtà*, Bologna, Il Mulino, vii–xvii.
- ‘Frege and Davidson on Predication’, in M. C. Amoretti, N. Vassallo (eds.), *Knowledge, Language and Interpretation: On the Philosophy of Donald Davidson*, Frankfurt, Ontos Verlag, 49–79.
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Introduction

A Biographical Sketch

Eva Picardi, one of the leading Italian analytic philosophers of her generation, was born in Reggio Calabria on January 16, 1948. Soon after, her family moved to Bologna. At Bologna, shortly after 1970, she graduated in Philosophy. In 1984 she received a DPhil in Philosophy at Oxford, presenting a dissertation on *Asserting*, written under Sir Michael Dummett's supervision. Asserting was also the topic of her first book, *Assertibility and Truth: A Study of Fregean Themes* (1981). Picardi taught philosophy of language at the University of Bologna from 1976 until 2016. She died in Bologna on April 23, 2017.

Picardi was one of the promoters of the European Society for Analytic Philosophy (ESAP); she was on the editorial board of the *European Journal of Philosophy* and of the *Journal for the History of Analytic Philosophy*. She was also a member of the advisory board of the Palgrave Macmillan series on the History of Analytic Philosophy, edited by Michael Beaney, and a member of the advisory board of the group coordinated by Crispin Wright for the English translation of Frege's *Grundgesetze der Arithmetik*. During her long career, she held visiting positions in several German universities and at Oxford.

Her research was on themes and authors of analytic philosophy of language. She was an internationally renowned expert on Gottlob Frege, whose work she related to, and compared with that of Giuseppe Peano, Bertrand Russell, Ludwig Wittgenstein, Frank Ramsey, Rudolf Carnap, and Donald Davidson. Some of her essays on these topics were collected in the volume *La chimica dei concetti: linguaggio, logica, psicologia 1879–1927* (The Chemistry of Concepts: Language, Logic, Psychology 1879–1927) (1994). After 1990, Picardi began intense research on American neo-pragmatists—W. V. O. Quine, Donald Davidson, Hilary Putnam, Richard Rorty, and Robert Brandom—often comparing their work with Michael Dummett's anti-realist program. Later, she became interested in the contextualist debate, tracing its origin back to Frege's distinction between sense and tone, his context principle, later radicalized by Wittgenstein and by Quine's and Davidson's holism—of which Dummett proposed a milder, molecularist variant.

During her career, Picardi was also very active in translating analytic philosophers—Frege, Wittgenstein, Davidson, Putnam, Dummett—into Italian. Together with Carlo Penco, she worked on a new Italian edition of Frege's works until her very last days. These translations were instrumental in developing

analytic philosophy in Italy because they made it possible to study it at the undergraduate level, where, for a long time, only texts in Italian could be included in the syllabi. Her contribution to the development of Italian analytic philosophy is much greater, though. She was the editor of the philosophical section of *Lingua e stile*, from 1992 until 2000, a journal that soon became the most important one for Italian philosophers of language. Besides, she was one of the ten founders of the Italian Society for Analytic Philosophy (SIFA), and its president from 2000 until 2002. Together, we organized in 2001 the conference Wittgenstein Today, which assembled in Bologna the best Wittgenstein scholars of the time. From the meeting an anthology by the same title originated. She was the director of Cogito Research Center in Philosophy, at Bologna University, and a member of the Academy of Sciences of the Alma Mater.

A very dedicated teacher, Picardi motivated and directed her students, devoting a lot of time and energy to them. Many of them have become researchers and professors in Italy, in other European countries (United Kingdom, Finland, Portugal, Germany), in the United States, and New Zealand. Her engagement with teaching led her to write one of the most complete introductions to the philosophy of language, *Linguaggio e analisi filosofica: elementi di filosofia del linguaggio* (1992) and later a more agile one, *Teorie del significato* (1999), translated into Spanish in 2001.

Her stand on meaning was moderately literalist. Picardi was aware of the role that pragmatic aspects and context play in understanding, but she kept to a normative view of meaning. Normativism made her critical of naturalist programs, such as Chomsky's and Fodor's, which she carefully examined. A Fregean, she never became a supporter of direct reference, her admiration for Saul Kripke, Hilary Putnam, and Tyler Burge notwithstanding. Philosophy of language was, for her, "first philosophy"—that is, the *via maestra* for investigating the main if not all philosophical topics. She largely endorsed Dummett's view as presented in his *Origins of Analytic Philosophy*—a series of lectures Dummett first delivered in Bologna at Picardi's invitation in the 1980s.

Picardi had style—philosophical and personal. She mastered her field and had knowledge beyond it. Besides her mother tongue, she was fluent in English and German, she had knowledge of French, Spanish, Latin, and Ancient Greek. She was no skeptic and had firm philosophical convictions. In discussion, she was precise and insightful. At the same time, she would often not argue the last steps. She was convinced that matters can be seen in more than one way, and that this is what rewards us in a vast knowledge of the literature.

She was deeply respected by her colleagues, both in Italy and beyond. During her lifetime, Dummett and Davidson replied to some of her papers. A further testament to that are the essays comprised in the volume in her memory, *Eva Picardi on Language, Analysis and History*, edited by A. Coliva, P. Leonardi, and S. Moruzzi (2018). She did not live to see it in print, even though I was able to

present it to her in draft form at the last conference of the Italian Society of Analytic Philosophy we both attended in Lucca in 2016.

During her long illness, she neither hid her condition nor turned it into a problem to participate, going on as if there were no deadlines, undertaking various new projects. Elegant and beautiful, intelligent and cultivated, fearless as she was.¹

About the Volume: Themes and Background

The present volume collects most of Picardi's papers on Frege's views about logic, language, and psychology, which she developed by, on the one hand, looking closely at the milieu within which Frege operated and, on the other, by bringing out and insisting on their relevance for contemporary debates, particularly in the philosophy of language.

This is all the more significant nowadays, as these two dimensions of philosophy—the historical and the theoretical—are typically perceived as separate, if not even in competition with one another, particularly within analytic philosophy itself. Reading Picardi's work, in contrast, cannot but show that they complement and enrich one another. Such a methodological lesson is, in my opinion, the most original and impactful element in Picardi's legacy and something deeply dear to her heart and constitutive of her identity as a scholar. I still vividly remember that when, for entirely practical reasons, we decided to divide the activities at Cogito Research Center (at Bologna University) between history of analytic philosophy and philosophy of language, and had her lead the former, she resented it, at some level. For, literally, she could not conceive of the two as separate. Nor could she conceive of the analytic tradition, which she saw as originating in the writings of Frege and Wittgenstein, whose German-speaking milieu she knew in depth, as separate from the rest of philosophy (call it "continental" if you so wish). This may surprise some readers, particularly those born and raised within the analytic tradition. Yet, it is to be kept firmly in mind that Picardi had actually been born and raised in a different milieu. Like many well-educated Italians, she had attended *liceo classico*, and had started studying philosophy from a very early age (sixteen). Her father was a philosophy professor at *liceo classico* too, and her mother had also graduated in philosophy before becoming a psychoanalyst, as was common in those days. That meant, simply, that for Eva Picardi the study of philosophy had been, from very early on, also the study of the history of the entire

¹ A glimpse of Picardi's serious irony can be had here: <https://www.youtube.com/watch?v=kiWVa4llyU4>. One of her lectures, in Italian, on Frege's reflections on identity, can be found here: <http://www.cattedrarosmini.org/site/view/view.php?cmd=view&id=213&menu1=m2&menu2=m37&menu3=m410&videoid=935>.

Western tradition. Furthermore, like most European intellectuals, she was deeply aware of the relevance of that tradition for the development and critique of the most important ideologies of the twentieth century. In that sense, I think, it is not oxymoronic to say that she was a continental intellectual doing analytic philosophy; or, as one might put it, the most analytic among continental philosophers and the most continental among analytic philosophers.²

The organization of the papers in this volume broadly—though not strictly—reflects the development of Picardi's ideas, rather than a stringent historical order. One strand in her work on Frege concerns understanding and contextualizing Frege's anti-psychologism. Picardi's contention is that it is much more motivated by semantic considerations than by adherence to Kantian transcendentalism. Furthermore, her deep knowledge of German and her being a native speaker of Italian put her in a privileged position to reconstruct the intricacies of Frege's relationship with other logicians of his time, both in Germany, such as Kerry and Sigwart, and in Italy, such as Peano and his school.

Furthermore, for Picardi, the core theme of analytic philosophy of language, since Frege's *Über Sinn und Bedeutung* is the reflection on meaning. Indeed, it is fair to say that, in her view, the subsequent development of the philosophy of language within analytic philosophy may be seen as a series of reactions against, or developments of, the Fregean distinction between sense and reference (or Meaning, with a capital 'M', as she preferred to translate the German term *Bedeutung* as used by Frege), with the attendant distinction between sense and tone, and the relationship between meaning, truth, and context. Virtually all subsequent debates about proper names, definite descriptions, propositional attitudes, concepts, synonymy, the semantic/pragmatic divide, and the role of context in the determination of the thought expressed by a sentence containing indexicals or demonstratives, can, according to Picardi, be traced back to Frege's seminal paper. It is thus apparent that Picardi's Frege is continuous with Dummett's, given the insistence on the semantic aspects and the relevance of his work for the philosophy of language.

A further testament to that are Picardi's detailed analyses of the influence of Frege on key philosophers of language, such as Carnap and Davidson, but also, as carried out in the final section of Picardi (2020), Wittgenstein, Grice, and for contemporary debates between representationalists and inferentialists about meaning. By so doing, Picardi provides a commanding picture of Frege's originality with respect to other thinkers of his time and his enduring significance for twentieth- and twenty-first-century analytic philosophy of language.

² A testament to that are chapters 5, 6, and 7 in the other volume of Picardi's writings, *The Selected Writings of Eva Picardi: From Wittgenstein to American Neo-Pragmatism*, London, Bloomsbury, 2020.

About the Volume: Structure and Contents

The volume is divided into three parts. The first part—"Frege in Context: Logic and Psychology"—contains five papers that address, on the one hand, the relationship between Frege's logic and the logics developed by some of his contemporaries, such as Benno Kerry and Christoph Sigwart, whom Frege accuses of permitting "a pernicious intrusion of psychology into logic". Adherence to psychologism, for Frege, blurs important distinctions such as the one between sense (*Sinn*) and representation (*Vorstellung*), or between justification and subjective conviction. Sigwart's *Logik* was also the target of Edmund Husserl's, and later of Martin Heidegger's, criticisms, and Picardi perceptively reconstructs this complex intellectual milieu. Still, Kerry's work was also significant in that it raised two important issues, which were also central to Frege's reflections. That is, (i) the distinction between the content and the object of a concept; and (ii) the logical roles of the article "the". According to Picardi, not only did Frege fail to offer a convincing solution to Kerry's puzzle concerning "the concept horse", but he also overlooked the criticisms leveled by Kerry against the notion of an (indefinite) extension on which his own definition of number was based. Yet, Frege's anti-psychologism, according to Picardi, does not depend on appealing to a form of Kantian transcendentalism, contrary to what Hans Sluga and Philip Kitcher have maintained. Rather, it is driven by semantic considerations through and through. In particular, it is predicated on the sharp distinction between sense and representation, and by conceiving of the former first as public meaning, given in a language and, later, in *Der Gedanke*, as an abstract entity.

On the other hand, by looking at the correspondence between Frege, Giuseppe Peano, and Bertrand Russell between 1890 and 1914, Picardi develops a close comparison between their different conceptions of logic, with special attention to their respective accounts of the primitive notions of logic and the role of definitions. Frege's conception is deemed superior to those of the other two logicians and intimately intertwined with Ludwig Wittgenstein's, especially with respect to the role of elucidations in logic.

Part II—"Frege's Philosophy of Language"—opens with Picardi's detailed reading of Frege's *Über Sinn und Bedeutung*. The subtitle, "An Elementary Exposition," is deceptive, for this is far from merely a presentation of Frege's main claims in that seminal paper. Rather, it is a thorough examination of all its themes with illuminating forays into how they became the object of heated debates in subsequent philosophy of language. Chief amongst them are Frege's treatment of identity and identity criteria, the objectivity of sense, his views about *oratio obliqua* and the doctrine of indirect senses, synonymy and its relationship with Benson Mates's paradox and Alonzo Church's synonymous isomorphism, as well as the recognition of the role that context may play in the determination of the thought expressed by a sentence in which indexicals and demonstratives occur.

The part continues with an examination of the origins and value of the comparison, drawn by Frege and Peirce, between chemical molecules and the composition of concepts within a proposition. Special relevance is given to Frege's notion of "unsaturadness" and to his way of connecting this theme with the possibility of deriving quantified sentences from their atomic instantiations and vice versa.

A characteristic element of Frege's ideography is the assertion sign. Its role and its connections with Frege's conception of assertion, truth, and judgment is explored in the third paper of this part. Picardi points out how Frege's views about the assertion sign changed over the years and how deeply they influenced the early Wittgenstein's and Frank Ramsey's positions.

The final two papers in this part are Picardi's contribution to a conversation between her and her former mentor, Michael Dummett, on Frege's ideas on sense identity and the role of the context principle. According to Picardi, the former issue played a crucial role in the justification of Frege's foundational program. In her view, two features of this program made it desirable to have stringent criteria for sense identity. The first was the claim that deductive reasoning and pure conceptual thought unaided by intuition can enlarge our knowledge; the second was the thesis that arithmetical propositions are analytic, if they can be shown to be derivable from purely logical axioms by means of definitions of arithmetical concepts and rules of a purely logical nature. According to Picardi, Frege conjectured that arithmetical propositions are contained in logical axioms as plants are contained in a seed and not as bricks are contained in a house. What then needed explaining was (a) the mechanism of deductive inference—that is, the peculiar mode of containment of the conclusion in the premises of a deductive argument, in such a way as to account for its validity and fruitfulness; and (b) the status of definitions.

Picardi then turns to a detailed examination of the role that the context principle plays in Frege's philosophy, in the light of Dummett's distinction between a thin and a robust notion of reference and between an idle and an operative notion of reference. According to Picardi, by adopting the compositional reading of the context principle suggested by Dummett in chapter 16 of *Frege: Philosophy of Mathematics*, it is possible to put the notion of semantic value into sharper focus and disclose an operative construal of reference which is not hostage to the thin vs. robust divide.³

³ The conversation, both in person and in print, between Picardi and Dummett has been intense over the years. As mentioned in "A Biographical Sketch," Dummett's *Origins of Analytic Philosophy* were first delivered at Bologna University, at Picardi's invitation. Dummett was a regular visitor in the Department of Philosophy there, and Picardi was likewise a regular visitor in Oxford over many summers, and, after Dummett's death, as a Visiting Fellow at Trinity College. Regarding their exchanges in print, see also chapters 13–15 in *The Selected Writings of Eva Picardi: From Wittgenstein to American Neo-Pragmatism*, London, Bloomsbury, 2020.

In the third and final part, “Frege’s Legacy,”⁴ Picardi’s papers explore the influence of Frege’s ideas on various subsequent philosophers of language. Rudolf Carnap’s distinction between the intension and the extension of predicates, called “Begriffswörter” by Frege and “predicators” by Carnap, is traced back to Frege’s lectures in 1910, 1913, and 1914 that Carnap attended in Jena. The paper also shows how these notions were differently developed by Carnap in the *Logische Aufbau der Welt* and in *Meaning and Necessity*.

The second and third paper in this part examine Frege’s theories of predication and of the unity of proposition and Davidson’s criticisms of them, which Picardi finds wanting. In her opinion, Frege’s theories are superior to Davidson’s and still viable.

Finally, Picardi turns to the relevance of Frege’s ideas for current debates between inferentialists and representationalists about meaning, with special attention to the dispute between Paul Boghossian and Tim Williamson on the meaning of logical constants. Picardi shows how Williamson’s ideas oppose suggestions first put forward by Dummett, and later developed by Robert Brandom and Boghossian, concerning the meaning of pejoratives. According to her, Williamson’s account draws substantially on Frege’s remarks on tone and coloring, thus indirectly joining forces with Jerry Fodor in the anti-inferentialist camp. Picardi then ponders why Frege’s views can lend themselves to such different interpretations and uses it as a foil to highlight the different conceptions that inferentialists and representationalists have of analyticity, synonymy, and definition.

⁴ For other papers by Picardi that explore the significance of Frege’s ideas for subsequent debates in the philosophy of language, and for philosophers such as Wittgenstein, Grice, Dummett, and Charles Travis, see the five chapters in part 4—“The Semantic/Pragmatic Divide”—of *The Selected Writings of Eva Picardi: From Wittgenstein to American Neo-Pragmatism*, London, Bloomsbury, 2020.

PART I

FREGE IN CONTEXT: LOGIC
AND PSYCHOLOGY

1

The Logic of Frege's Contemporaries

I

The German mathematician and philosopher Gottlob Frege (1848–1925) is, according to the received view, both the father of analytic philosophy and the founder of mathematical logic.* However, he almost certainly deserves a third award for having established anti-psychologism, if not as a doctrine, at least as a body of arguments and of instructions for detecting the symptoms of psychologism, instructions which we are well advised to follow if we are to avoid the many pitfalls of a dangerous doctrine. For psychologism wears various guises: it appears in wolf's clothing, e.g. in Sigwart (1873–8), Husserl (1891), Erdmann (1892), Lipps (1893), in sheep's clothing, e.g. in Wundt (1880–3), Lotze (1852, 1880), and as a slip of the pen in the writings of many mathematicians, even of those who, like Georg Cantor (1845–1918) and Richard Dedekind (1831–1916), are otherwise immune from committing the 'genetic' fallacy of confusing the meaning and import of mathematical propositions with the real processes whereby we apprehend them. The favoured aim of psychologism is the blurring of distinctions—between laws of thinking and laws of thought, between the sense of linguistic expressions and that which they can be used to talk about ('*die Vorstellung*' and '*das Vorgestellte*' in the terminology of some psychologistic authors), between sense and subjective representation (sometimes distinguished in terms of *Vorstellung* (representation) vs. *Bild* (image) but more often conflated under the same label), between that which is overtly said and that which is merely suggested (*angedeutet*), implied (*mitgemeint*), or presupposed (*vorausgesetzt*). An especially apt characterization of psychologism is given by Frege in his posthumously published *Logik* of 1897:

Bei der psychologischen Auffassung der Logik fällt der Unterschied zwischen den Gründen, die eine Überzeugung rechtfertigen, und den Ursachen, die sie wirklich hervorbringen, weg. Eine eigentliche Rechtfertigung ist dann nicht möglich; an

* I wish to thank Dr. Barry Smith for his helpful comments and advice.

ihre Stelle wird die Erzählung treten, wie die Überzeugung gewonnen wurde, aus der zu entnehmen ist, dass alles seine psychologischen Ursachen gehabt hat.
(Frege 1983 [1897]: 159)¹

Nowadays anti-psychologism is not in vogue. In part its interest has simply faded away: logic has become an established discipline, and if it runs any risks, psychologism is not among them. Besides, the antidote prescribed by Frege himself may land us straight in a barren 'third realm' of thoughts (1967 [1918]: 360–1) in comparison with which even the jungle of mental representations, mental acts, mental states, mental capacities, etc. of the psychologistic logicians might look appealing. Moreover, it is by no means obvious that Frege's own doctrines of sense and (assertoric) force are unstained by psychologism, if 'psychologism' is construed in austere Fregean terms. But the most serious objection to Frege's views comes from certain quarters within the philosophy of language, a discipline in which Frege's ideas have gained currency and importance thanks to the influential writings of Bertrand Russell (1872–1970), Ludwig Wittgenstein (1889–1951), and Rudolf Carnap (1891–1970). This objection amounts to the following claim. A classical theory of meaning for a natural language and the model-theoretic semantics which goes with it cannot coexist with a psychologically realistic model of the linguistic competence of an ideal speaker–hearer. Because of their exclusive concentration on the theory of reference analytic philosophers have lost sight of the other face of semantics, namely the theory of sense, or, to use a more congenial terminology, they have failed to give an account of the mental representations of propositional contents. For too long, the objection continues, philosophy of language has borrowed its key notions from logic (i.e. notions such as logical form, truth, entailment); now it is time to return to, or rather advance towards, a psychologically relevant treatment of that in which understanding language really consists.

Plainly, a discussion of these and kindred issues would exceed the scope of the present paper. I have mentioned them in order to hint at some of the reasons why even the non-historically minded reader might profit from rehearsing Frege's rebukes to the psychologistic logicians of his day. In this way such a reader may even be encouraged to have a look at those ponderous books to the actual neglect of which among philosophers (but see Poggi 1977) Frege no doubt contributed a good deal. I am not competent to judge whether in the cognitive sciences these books have had a better fate. After all, the authors under consideration are Wilhelm Wundt (1832–1920), the founder of modern experimental psychology,

¹ 'With the psychological conception of logic we lose the distinction between the grounds that justify a conviction and the causes that actually produce it. This means that a justification in the proper sense is not possible; what we have in its place is an account of how the conviction was arrived at, from which it is to be inferred that everything has been caused by psychological factors.' (Engl. transl. 1979: 147.)

Hermann Lotze (1817–81), whose *Medizinische Psychologie* (1852) surely deserves a place in the history of psychology, Theodor Lipps (1851–1914), Benno Erdmann (1851–1912), Theodor Ziehen (1862–1950), who all came to logic from the medical sciences. A different approach to logic is to be found in the works of the early Edmund Husserl (1859–1938), Alexius Meinong (1853–1920), and Alois Höfler (1853–1922), whose inspiration came from the Austrian school of ‘descriptive psychology’ inaugurated by Franz Brentano’s (1838–1917) *Psychologie vom empirischen Standpunkt* (1874).

The landscape of German (and Austrian) philosophy in the second half of the nineteenth century is extremely rich and varied. What makes those times particularly interesting from the point of view of the present paper is the way in which three disciplines—psychology, logic, and linguistics—that came of age at almost the same time claimed privileged access to the phenomenon of linguistic meaning. Many treatises which were called *Logic* or contained the word ‘logic’ in their titles would, by modern standards, at most count as essays in the philosophy of logic. They dealt with assorted topics, such as language and perception, the acquisition of language, the formation of concepts and their linguistic representation, judgement and other mental activities, the status of the laws of thought, the phenomena of attention, memory, perceptual stimulation, and their relations to the contents of sentences, etc. Linguists, philosophers, psychologists, and mathematicians all wrote logic books; there were ‘pure’ (that is, impure) logics, ‘formal’ logics, and ‘empirical’ logics. The latter heading can be taken to comprise all those logic books which followed the model of John Stuart Mill’s (1806–73) *System of Logic* (1843). The German translation by J. Schiel of its first edition appeared in 1849 (this is the translation from which both Frege and Wundt quote); its eighth and last edition of 1872 was translated by Theodor Gomperz and appeared in his edition of Mill’s *Gesammelte Werke* in twelve volumes (1869–81). The influence of Mill’s works on both German and Austrian philosophy can hardly be overestimated; for Sigwart (1830–1904), Brentano, and Wundt (to mention only a few) Mill’s work was a source of profound insights; for others, e.g. Frege and Husserl, it was an object of critical confrontation. It was probably through Mill’s writings that the works of James Mill (1773–1836), Alexander Bain (1818–1903), George Boole (1815–64), and William Stanley Jevons (1835–82) reached a large philosophical audience. Among mathematically trained logicians the works of Boole, Venn (1834–1923), and Jevons became known mainly through the writings of Ernst Schröder (1841–1902) and Robert Grassmann (1815–1901).² By far the best critical accounts of Boole’s algebra of logic were given by Wundt (1880) and by Frege

² In his review of Frege (1879) Ernst Schröder (1880) criticizes Frege for not mentioning any contemporary logicians, e.g. Robert Grassmann. The latter’s work is referred to and discussed in Frege’s posthumously published paper ‘Booles rechnende Logik’ (Frege 1983: 38). But, curiously enough, Felix Klein (1849–1925), the editor of *Mathematische Annalen* (to whom Frege had offered

in his posthumously published paper 'Booles rechnende Logik und die Begriffsschrift' (1880–1) (cf. also Alois Riehl (1844–1924) 1877). Frege and Husserl dealt at length with Schröder's work, and in the context of that discussion the contrast between *Logiker des Inhalts* and *Logiker des Umfangs* came to the fore. Frege summed up the whole controversy in terms which seem to me the most satisfactory and penetrating. In this context mention must also be made of the German scientist and philosopher who in that period most luminously incarnated the ideal of interdisciplinary research; this was Hermann von Helmholtz (1821–94). His *Handbuch der physiologischen Optik* (1856–67) was an essential contribution to psychology, and his influential work on non-Euclidean geometries and the philosophy of arithmetic, made accessible to a general audience through his popular essays (Helmholtz 1971), contributed to creating a climate which was not really favourable to the new ideas developed in mathematics by Dedekind, Frege, and, above all, Cantor.³ Cantor and Frege concurred in their opinions of Helmholtz's work on number theory and, presumably, also in their opinions of Helmholtz's friend Kronecker (1823–91) as editor of *Crelles Journal für reine und angewandte Mathematik* (cf. Cantor (1932 [1887]: 383); Frege (1967 [1892]: 166; 1893–1903: §137)).

In a sense German science and philosophy suffered from an embarrassment of riches during the period under consideration; there simply were too many people having brilliant ideas and publishing them at the same time. This richness must be borne in mind by everyone who attempts to arrive at a clear conception of the problems then at issue. To take just one example: in order to understand what Lotze meant by '*Lokalzeichen*' or Helmholtz by '*Zeichen*' we have to look at their discussions of visual and spatial perception. An echo of these discussions lingers on in the nativism vs. empiricism controversy, but if we concentrate only on the purely philosophical statements of these authors we are likely to miss the point and the specific flavour of that debate.

Needless to say, neither now nor then did everyone agree on whether that richness was a positive feature. The Neo-Kantian Wilhelm Windelband (1848–1915) for example took a fairly dim view of that period and thought that

that paper for publication), complains in his letter of rejection (Frege 1976: 134) that neither Robert nor Hermann Grassmann are mentioned. Thus either Klein had failed to notice Frege's reference to Grassmann, or Frege inserted this reference in the course of revising his paper at some later time. That he did revise it can be taken for granted because of the fact that there are references to passages in Wundt's *Logik* to which nothing corresponds in the 1st ed. (1880)—which was the only one available at the time Frege originally wrote his paper—while they *do* occur in the 2nd ed. (1893).

³ On the whole Frege was in agreement with Cantor's 'ontological' claim to having established infinite totalities, cf. Frege (1967 [1892]: 163). Thus it is unclear which contrast Sluga has in mind when he remarks, 'If the philosophy of mathematics and the philosophy of logic have recently been dominated by ontological considerations that is not due to the influence of Frege's thought. Such ontological questions seem to derive from a number of different sources, such as Cantor's examination of infinite totalities' (1976: 29). For an historically accurate account of Cantor's relations with his contemporaries, cf. Dauben (1979), especially ch. 6 'Cantor's Philosophy of the Infinite'.

both 'psychologism' and 'positivism' were offspring of an all too scientific interpretation of Otto Liebmann's (1840–1912) slogan '*Zurück zu Kant*' pronounced in his book *Kant und die Epigonen* (1865). Windelband writes:

Die Erneuerung des Kantianismus hat aber das Geschick erfahren, daß sie zunächst durch die Interessen des naturwissenschaftlichen Denkens auf die Erkenntnistheorie eingeengt wurde: und deren ausgesprochen empiristische Zuspitzung führte auf der einen Seite zu positivistischen Umbildungen, auf der andern Seite zur Auflösung der philosophischen Probleme in psychologische. So breitete sich, wie in der Zeit vor Kant, einige Jahrzehnte lang namentlich auf den deutschen Universitäten eine verderbliche Vorherrschaft des Psychologismus aus. (1980 [1892]: 539)⁴

Even if one does not agree with Windelband's fondness for *Werttheorie* or, rather, for *Werttheorien* (cf. Schnädelbach 1983: 205ff.), one may still see a grain of truth in his assessment of certain aspects of the Kantian heritage. Not so dissimilar is the judgement expressed by the mathematician Cantor, who complains about the dominant 'Kantian' school of scepticism and positivism represented by Helmholtz⁵ and Kronecker. In this context the only type of 'infinite' which could be recognized was the infinite of analysis, which for Cantor was spurious, not a concept of infinity at all. Cantor objects to the Kantian claim that without sensibility no objects are given to us and says that this dogma makes it impossible for the creations of purely conceptual thinking to gain recognition and acceptance (for a similar argument, cf. Frege 1884: §89). Thus Cantor's theory of transfinite order-types clashed with deeply ingrained prejudices of his time. The source of these prejudices was the philosophy of Kant, about which Cantor expressed a very harsh judgement in his criticism of Wundt's misunderstandings of his—Cantor's—theory of order-types:

Die Begriffsschwankungen und die damit zusammenhängende Verwirrung, welche seit ungefähr *hundert* Jahren zuerst vom fernen Osten Deutschlands her in die Philosophie hineingetragen wurden, zeigen sich nirgends deutlicher als in den das *Unendliche* betreffenden Fragen, wie aus unzählig vielen, sei es

⁴ 'It was the fate of the revival of Kantianism that because of the interests of the scientific way of thinking it was at first restricted to the theory of knowledge, whose extreme developments towards a decidedly empiricist position led to positivistic transformations on the one hand and to the dissolution of philosophical problems into psychological ones on the other. Thus, just as during the time before Kant, the disastrous predominance of psychologism became widespread, for a few decades, particularly at German universities.'

⁵ To be exact, the Helmholtz attacked by Cantor is the author of 'Zählen und Messen'; the Helmholtz attacked by Windelband is the author of 'Die Tatsachen in der Wahrnehmung', where Helmholtz criticizes Kant's view that the axioms of geometry are known *a priori*. In fact, as far as the propositions of arithmetic are concerned, Helmholtz holds an extreme form of psychologism.

kritizistisch oder positivistisch, psychologisch oder philologistisch gehaltenen Publikationen unsrer heutigen philosophischen Literatur hervorgeht.

(1932 [1885]: 376)⁶

Clearly it will not be possible here to pursue in detail all the topics mentioned in the sketch given above. What I shall try to do is to convey an impression of some of those issues as they must have appeared to Frege. This will involve a preliminary look at the relevant philosophical works which Frege can be presumed to have read or regarding which he can be shown to have evinced some interest. An account will be given of what Frege meant by 'psychologism', followed by a discussion of what Frege might have considered positive features of certain claims and arguments put forward by psychologistic authors.

II

Whereas Frege forged and perfected in detail an impressive range of arguments against the 'corrupting incursion of psychology into logic' (Frege 1893: xiv), it was Husserl who in his *Prolegomena zur reinen Logik* (1900) drew the first accurate map of the spreading of the psychologistic epidemic in German philosophy during the second half of the nineteenth century. Frege's contribution is not sufficiently acknowledged in the *Prolegomena* (Husserl 1913: 169 n.); but it seems clear that most of Husserl's arguments against psychologism are borrowed from Frege,⁷ the sharpness of whose criticisms Husserl himself had experienced and whose writings he knew well (cf. Gabriel 1976: 91).

From a 'positivistic' point of view a detailed survey of German logical writings of the period covered by Husserl's *Prolegomena* was given by Theodor Ziehen in his *Lehrbuch* (1920). Ziehen taught psychiatry at Frege's University (Jena) from 1892 to 1900. Together with Richard Avenarius (1843–96) and Wilhelm Schuppe (1836–1913) he is referred to by Ernst Mach (1838–1916) as a philosopher with similar interests (Mach 1905: vii). Ziehen coined the word 'Konzinnisten' (1920: 205) to characterize those philosophers who, like Sigwart, Wundt, Erdmann, Lipps, and to some extent himself, aimed at placing logic within the general framework of a theory of knowledge, also by drawing on the results of empirical psychology. In the *Lehrbuch* Frege is often quoted in appropriate contexts; Ziehen

⁶ 'The conceptual oscillations and the confusion connected with them, which, at first coming from the far east of Germany, have for nearly a hundred years been instilled into philosophy, manifest themselves most obviously in questions concerning the infinite, as becomes clear from countless many publications of our present-day philosophical literature, be they criticistic or positivistic, psychologistic or philologistic.'

⁷ For a similar judgement, cf. Føllesdal (1982). A different opinion is expressed by Mohanty (1982, ch. 2).

even endorses Frege's notion of 'argument' (and function) and embellishes this notion by way of adducing psychological considerations (1920: 459ff.).

From Husserl's *Prolegomena* and Ziehen's *Lehrbuch* we can cull a fairly accurate picture of the state of the art in Frege's days and of the philosophical climate Frege complains about in the preface to the first volume of his *Grundgesetze*. Frege's own writings, however, do not contain many clues to the philosophical contemporaries he has in mind. To be sure, in his *Grundlagen* (1884) many philosophers of the past are mentioned, but the only *contemporary* German philosophers named are Benno Erdmann and Kuno Fischer (1824–1907), whose *System der Logik und Metaphysik oder Wissenschaftslehre* (1852) is cited as an example of the 'crudity' of the prevailing philosophical theories that prevented Frege's own work on the foundations of arithmetic from being duly recognized. An equally negative view of Kuno Fischer is expressed in Frege's posthumously published piece 'Über den Begriff der Zahl' (1983: 93). This fact should be taken into account by those who conjecture that Kant's philosophy came to have a lasting influence on Frege through Kuno Fischer's teaching at the university of Jena (cf. Sluga 1984). It is by no means clear why it is supposed that there needed to be any *special* incentive for Frege to read Kant. At any rate, a more likely influence would have been Frege's friend and colleague Otto Liebmann (1840–1912), who taught in Jena from 1882; Liebmann's 'critical realism' (cf. Thiel 1984: 611; Kreiser 1984; Gabriel 1986) may have been congenial to Frege.

In contrast with the paucity of references to contemporary philosophers, the writings of contemporary mathematicians are frequently mentioned in the *Grundlagen* and elsewhere, e.g. the works of Ernst Schröder, Hermann Hankel (1839–73), Robert Grassmann, Hermann Grassmann (1809–77), Karl Theodor Weierstraß (1815–97), Hermann von Helmholtz, Leopold Kronecker, Georg Cantor, H. Eduard Heine (1821–82), Richard Dedekind, Johannes Thomae (1840–1921), David Hilbert (1862–1943), to mention just a few of the better known ones. Frege also wrote reviews of works by the Neo-Kantians Hermann Cohen (1842–1918) and Friedrich Albert Lange (1828–75); while Cohen's work *Das Prinzip der Infinitesimal-Methode und seine Geschichte* (1883) is severely criticized in Frege's review (1885), the article on Lange's *Die geschichtliche Entwicklung des Bewegungsbegriffs* (1886) is altogether positive (1891).⁸

The philosopher who, besides Husserl, was singled out by Frege as one of the real offenders was Benno Erdmann. Erdmann, the first (and last) volume of whose *Logik* appeared in 1892, was also the author of a book on the axioms of geometry

⁸ At the close of this article Frege does not miss the opportunity to stigmatize the confusion between logical and psychological notions, encouraged by the use of the word 'Vorstellung'. This article shows that, as far as physics is concerned, Frege's thinking is strikingly holistic and even conventionalist; cf. his comment that only the whole of the basic laws of dynamics can be compared with experience and be confirmed by it ('Nur das Ganze der dynamischen Grundgesetze kann als Hypothese mit der Erfahrung verglichen und durch sie bestätigt werden' (1967: 116)).

and of a study of Kant. In many respects he was a rather traditional philosopher. The account of predication he endorses comes from William Hamilton (1788–1856), he heavily draws on the Aristotelian doctrine of categories and predicables, and, predictably, he reveals himself to be a *Logiker des Inhalts*. Moreover, he was unsympathetic to formal treatments of logic. Although in his *Logik* he does refer to Frege's *Grundlagen*, he gives the wrong year of publication (1888 instead of 1884); but surely this oversight (Erdmann 1892: 108) cannot have been Frege's reason for his hostility towards Erdmann.

In 1969 and 1976 conjectures and speculations as to what Frege might have read (but probably did not) were given a firmer basis. In 1969 the first edition of the surviving papers from Frege's scientific *Nachlaß* was published, and in 1976 a list of the entire original *Nachlaß* prepared by Heinrich Scholz (1884–1956) appeared in print (Veraart 1976). This list contains a number of interesting clues, which for all I know have been ignored by those who have written on Frege. It is a curious fact that, instead of paying close attention to that list, Frege scholars have speculated about Lotze's possible influence on Frege (cf. Sluga 1980, Dummett 1982) and, more recently, about Frege's indebtedness to Kant. Already in 1966 Günther Patzig had expressed his conviction that much in Frege's philosophy comes from Kant (Patzig 1966: 15). In the absence of a detailed analysis to back this type of claim such attributions of influence are either trivial or highly controversial. To make the reader vividly aware of the conceptual tangle involved it will suffice to mention one fact. In 1879 two masterpieces were published: Frege's *Begriffsschrift* and Helmholtz's rectorial address 'Die Tatsachen in der Wahrnehmung'. Both Frege and Helmholtz set out to undermine two pillars of Kant's doctrine, and they did so from opposite ends. Against Kant's claim that the propositions of arithmetic are synthetic *a priori* Frege wanted to show that they are analytic by pointing out that no appeal to any *a priori* intuition of time was needed to grasp the concept of following in a sequence for example. Helmholtz wanted to demonstrate that Kant had attributed too rich and too specific a content to the intuition of space; rejecting Kant's principles he argued that the *axioms* of geometry are synthetic and can be known only *a posteriori*, even though the pure intuition of space (as Helmholtz was still prepared to concede) might be *a priori*. Helmholtz explicitly conceived of himself as improving on the Kantian doctrine and thus, in a sense, as a 'Kantian'. Frege, on the other hand, saw very clearly that his views on logic and arithmetic were irreconcilable with Kant's theory and, besides, he heartily disagreed with Helmholtz's views on arithmetic and geometry. And I am sure that Frege's lamentably retrograde views on geometry were not due to his alleged adherence to Kantian dogmas.⁹ Now, if either (or both) of these

⁹ Although in the context of discussing Hilbert's views Frege makes some very interesting remarks about the *formalization* of the axioms of geometry and the underlying logic, on the whole his position is unmistakably retrograde: cf. Dummett (1976).

thinkers, who contributed like few others to dismantling Kant's conception of knowledge, are to be called 'Kantians', then surely each was a '*Kantianer eigener Art*' (cf. Weyl 1966, Teil II: §18).

It will be better not to follow the *Prinzip der Nichtunterscheidung des Verschiedenen*, which Frege exposed as one of the mainsprings of confusion (Frege 1967 [1899]: 241 and passim), but rather to try to find out what it is that makes an author himself and not another author. For this reason we shall return to Scholz's list. The grouping and sorting out of the various items in his copious *Nachlaß* had been attended to by Frege himself. What Scholz contributed was some further ordering of those items, their numbering, and the description of some of the pieces. A feature of this list which is interesting from the present point of view is that there are a number of mentions of philosophers from whose writings Frege had taken notes. Part of this material was evidently used for articles and books Frege wrote and in many cases managed to publish; thus it is not surprising to find references to Leibniz, Spinoza, Herbart, Mill, and Erdmann. More interesting are the references to the *Logiken* of Wundt and Schuppe as well as a notebook bearing Frege's title 'Logik von Dr. Christoph Sigwart', which seems to have contained nineteen pages of notes on, or extracts from, that work (Veraart 1976: 103). Another interesting mention is that of Friedrich Ueberweg (1826–71) (Veraart 1976: 96). On the basis of these references one may then speculate whether Frege's reading of Schuppe's *Logik* for instance was occasioned by his reading of Husserl's *Philosophie der Arithmetik*, where the views Frege had put forward in his *Grundlagen* (1884) are discussed and compared with those of Schuppe and Herbart (Husserl 1970: 161ff.), or whether his perusal of Ueberweg was due to a remark in Cantor's review of *Grundlagen*, where Frege's notion of the *Umfang* of a concept is treated as a residue of scholasticism possibly picked up from Ueberweg's *System der Logik* (Cantor 1932: 440).

As far as Sigwart is concerned, things are at the same time less mysterious and more intriguing. They are less mysterious because Sigwart's *Logik* was by far the best known among the 'psychologistic logics' of those days. Ziehen (1920: 206) describes it as 'eines der wichtigsten Werke der gesamten logischen Literatur' ('one of the most important works of the entire logical literature'). Cantor (1932: 206) calls it an 'ausgezeichnetes Werk' ('an excellent work'), and Husserl speaks of Sigwart's 'bedeutendes Werk, das wie kein zweites die logische Bewegung der letzten Jahrzehnte in die Bahn des Psychologismus gebracht hat'¹⁰ (1913: 125). According to Husserl, Sigwart's kind of psychologism is a form of 'anthropologism'. Together with J. S. Mill, Alexander Bain, Wilhelm Wundt, Benno Erdmann, and Theodor Lipps he is declared guilty of what Husserl calls 'skeptischer Relativismus', a hopelessly vague label which seems to fit virtually all empiricists,

¹⁰ 'an important work, which as no other has helped the logical development of the last decades to take the path of psychologism'.

who, in the tendentious words of Windelband approvingly quoted by Husserl (1913: 84 n.), attempt ‘durch eine empirische Theorie dasjenige zu begründen, was selbst die Voraussetzung jeder Theorie bildet’.¹¹ And Heinrich Maier (1867–1933), who wrote the introduction to the fourth edition of Sigwart’s *Logik*, sees the latter’s efforts as part of a Neo-Kantian ‘renaissance’ (1921: ix). Both Husserl’s and Maier’s judgements will have to be qualified because Sigwart severely criticized, not only Mill’s views on the significance of deduction, but also Kant’s distinction between the synthetic and the analytic, his classification of judgements, and his account of the role of intuition in the understanding of numerical statements. Sigwart acknowledges as important influences on himself Mill, Adolf Trendelenburg (1802–72), and Ueberweg (1889: vi). The second edition of his *Logik* (1889–93) was translated into English in 1895 and thus found followers and admirers in thinkers as diverse as William James (1842–1910, cf. Zweig 1969: 441) and William Ernest Johnson (1858–1931) (cf. Johnson 1921: 198ff.).

What is intriguing with respect to Sigwart is that, although there is much in his *Logik* that Frege would have disagreed with, there is a good deal that he might have found interesting. Above all, Sigwart always shows himself to be an acute thinker, and even in his arguments against the principle of excluded middle he does not display the facile psychologism of Erdmann’s discussion of the possibility that different people accept different logical laws, which is then used to draw the conclusion that also the laws of logic may evolve (Erdmann 1892: 385ff.). These arguments and claims of Erdmann’s are explicitly discussed and rejected in Frege’s preface to the first volume of *Grundgesetze* (1893: xiv–xxv). Although this does not really speak in favour of Resnik’s (1980: 50) claim that Frege’s non-specific attacks on psychologism are directed at Sigwart, the fact that Frege did study Sigwart’s work (incidentally a fact which Resnik fails to mention) might support his thesis to a higher degree than the lack of evidence for Frege’s having made a close study of Lotze’s *Logik*, together with the hypothesis that during his student days at Göttingen (1871–3) he did *not* attend Lotze’s lectures on logic but only those on the philosophy of religion, could possibly support Sluga’s claim that Frege was profoundly influenced by Lotze (Sluga 1984, cf. Bynum 1972).¹²

We shall come back to Sigwart’s logic in section IV. However, perhaps the most interesting references to a psychologistic logician in Frege’s *Nachlaß* are those to

¹¹ ‘the attempt by means of an empirical theory to establish that which itself is the presupposition of all theory’.

¹² Surely the fact that in presenting Frege’s essay ‘Der Gedanke’ (1918) to the readers of the *Beiträge zur Philosophie des deutschen Idealismus* the Neo-Kantian Bruno Bauch stressed certain alleged affinities between Lotze and Frege can at best be seen as evidence for Bauch’s way of reading Frege but not—as Sluga (1984: 342) claims—for Lotze’s ‘influence’ on Frege. I do not mean to claim that Frege was not acquainted with Lotze’s *Logik*; actually, there is a piece in Frege’s *Nachlass* which contains a criticism of a certain doctrine held by Lotze. Cf. Dummett (1981), Picardi (1983). I doubt, however, that Lotze had any specific or constructive influence on Frege.

Wundt's *Logik*, both in his long article 'Booles rechnende Logik und die Begriffsschrift' (1880–1)—meant for publication but rejected by the editors of three journals to whom Frege had offered it—and in the fragment 'Ausführungen über Sinn und Bedeutung' (1892–5). As Wundt was one of the major representatives of psychologistic logic, one might expect Frege's references to Wundt to be critical. But this is not at all the case; their tone is rather respectful. Frege sees certain analogies between Wundt's treatment of concept-words and his own account of the unsaturated and predicative character of concepts. Since Frege's notions of concepts and concept-words are far from uncontroversial, those references to Wundt clearly merit some comment. I shall go into this question, too, in section IV.

Naturally Frege's scientific *Nachlaß* and Scholz's list cannot answer all the relevant questions arising from reading Frege's works with our present set of problems in mind. Here I wish to mention just one of the several questions that are bound to go unanswered. One may reasonably wonder whether Frege was acquainted with any of Brentano's works. Plainly he must have heard of Brentano, for not only did he review Husserl's *Philosophie der Arithmetik*, which is dedicated to Brentano, but he also corresponded with Carl Stumpf (1848–1936) and/or Anton Marty (1847–1914), who had both of them been pupils first of Brentano and then of Lotze. Thus one may ask whether Frege's terminology of *Anerkennung* and *Verwerfung*, which he uses from 1891 onward, or the term 'Vorstellungsverbindung' (Frege 1879: 2) is in any way due to Brentano, who uses these very terms to designate the acceptance as true, or the rejection as false, of possible contents of judgements. While for Brentano and Marty acceptance and rejection constitute two different types of *act*, for Frege there is no such difference, as he regards negation as part of the content of a thought (cf. Marty 1884; Angelelli 1967: 152ff.; Gabriel in Frege 1976: 162).

This terminology of *Anerkennung* and *Verwerfung* occurs also in *Windelband*.¹³ Moreover, Gabriel has recently (1984) suggested that Frege's use of the word 'Wahrheitswert' may in some way be seen as reminiscent of *Windelband*, whose philosophical standpoint is classified by Ziehen (1920: 188ff.) as *werttheoretischer Logizismus*. It is difficult to decide whether the possibility of any 'influence' coming from *Windelband* is more or less likely than Frege's being influenced by Brentano. My impression is that the relevant values in Frege are those that a function takes for a given argument and have nothing to do with anything specifically *Windelbandian*. In order to emphasize the functional character of concepts Frege used a technical term directly reminiscent of

¹³ A detailed discussion of Brentano's and *Windelband*'s use of this terminology is to be found in Sigwart (1889: 154 n. ff.); cf. Ziehen (1920: 365 n.).

mathematical talk about functions; and this way of looking at the matter is consonant with Frege's repeated claim of having enlarged the extension of the notion of a function (Frege 1879: §10; 1967 [1891]: 131ff.).

III

In the preface to the *Begriffsschrift* Frege says that what he has accomplished by constructing a 'formula language of pure thought' should be of interest to philosophers, at least in so far as it is recognized as one of the tasks of philosophy to break the power of words over the human mind by freeing thought from that which attaches to it solely because of the nature of our linguistic means of expression (1879: vi–vii).

That there is an intrinsic relation between thought and language was a commonplace both for Frege and his psychologistic contemporaries. Thus Wundt wonders how it could have been possible for a mind to be mature enough to invent language without already possessing it; and he argues that the only psychologically conceivable answer is that thinking and language evolved simultaneously, which means that, even though in a developed language a word is nothing but a *Gedankenmünze*, the psychological process of concept formation must have left traces in our language (1880: 44). In a similar vein Helmholtz points out that, although a child learns the meanings of linguistic expressions only by means of being confronted with actual examples of the employment of those expressions, the representations associated with them will not form part of their meanings; at most, they will leave traces which play a certain role in 'unconscious inferences' (1971: 266f.).

However, it is not the genetic aspect of the relation between thought and language which attracts Frege's attention; his attitude towards natural language is critical. He observes that, among the many purposes language can be used to serve, logical perfection is not paramount. The logical relations between concepts are hidden and it is difficult to grasp the patterns of argument really underlying our transitions from one thought to another. Thus, according to Frege, if we are to uncover logical form we must not follow too closely the outline of the dress which clads our thoughts. This 'dress' is the grammatical (in contrast with the logical) structure of historically grown languages; it is through grammar that psychology exerts its power and influences and distorts our understanding of the thoughts imperfectly expressed by natural language. Even the employment of a given system of signs or an artificial notation will entail a mixing of intuitive and purely conceptual elements. Plainly, if a *logical* critique of language is to be feasible by *linguistic* means or by resorting to an artificial notation, thinking and speaking must not be identified: otherwise there would be no way of pulling ourselves out of that particular predicament by our own bootstraps.

Analysis is possible only if language itself provides us with a means of overcoming the psychological conditioning we are exposed to through the use of linguistic symbols (*Zeichen*). In 1882 Frege says, 'Die Zeichen sind für das Denken von derselben Bedeutung wie für die Schifffahrt die Erfindung, den Wind zu gebrauchen, um gegen den Wind zu segeln' (1882: 49).¹⁴ Without symbols there would be no such thing as conceptual thinking. By using the same symbol to express what several similar things have in common we do not designate individual objects but that which is common to them, viz. a concept. As concepts are not perceivable by our senses, it is only by way of designating them by symbols that they can become accessible to us. Thus we can move by sensible means to the world of what is not sensible (1882: 50). Just as we can refer to individual objects by means of what Frege calls their proper names, so we can designate concepts by another type of words; but natural language tends to blur the distinction between concept-words and singular terms. Thus the expression 'horse' can be used to designate both an individual and a species, as e.g. in the sentence 'The horse is an herbivorous animal'; but it can also be used to designate a concept, e.g. in 'This is a horse'. According to Frege the peculiar nature of concept-words and names of abstract entities had eluded philosophers and logicians; for this reason they had been led to talk about the *representations* standardly or occasionally evoked by the occurrence of certain words while ignoring questions about their meaning and reference. But if the semantic role of words consisted just in their evoking certain representations or images, no account could be given of their different logical functions as components of different sentences.

Much later, after the discovery of the antinomies, Frege reached the conclusion that even an artificial notation will not sufficiently protect us from our tendency to let the linguistic clothing of thought mislead us. This, however, is no reason to despair, Frege claims; but we shall have to admit that thought transcends language and must to some extent be ineffable (1983 [1924–5]: 288f.). Certain fundamental notions, such as 'truth', 'function', 'identity', can only be alluded to, never defined. And yet, we seem to grasp these notions. Without linguistic symbols, however, we should surely never have been in a position to consider them.

To reflect upon thought in abstraction from the linguistic means of articulating and expressing it may pave the way for undesirable conclusions. Consider for example the special twist the metaphor of language as the clothes of thought is given by the arch-psychologist, the early Theodor Lipps (1893: viii):

Lernt man ja doch auch nicht den Bau des menschlichen Körpers aus der Betrachtung des bekleideten Körpers kennen; vielmehr weiß jedermann, daß

¹⁴ 'Symbols have the same importance for thought that discovering how to use the wind to sail against the wind had for navigation.' (Engl. transl. Bynum 1972: 84.)

umgekehrt die Formen und Faltungen der Gewandung erst aus der Kenntnis des Körpers verständlich werden können.¹⁵

From this Lipps, unlike Frege, draws the conclusion that, thinking being a matter of mental acts, an investigation of its forms and laws must be a psychological investigation: ‘...daß die Logik eine Sonderdisziplin der Psychologie ist, unterscheidet beide genügend deutlich voneinander’ (1893: 2).¹⁶ But logic, in contrast with psychology, is a normative science; its task is not only to prescribe how to think soundly but also to indicate how this aim can be reached in practice, given our specific mental equipment. According to Lipps thinking is an ‘objektiv bedingtes Vorstellen’ (1893: 4), and objectivity is that which characterizes logic. Objectivity, however, is regarded as being based on subjective certainty, which gives us access to the ‘necessary order of objects’. Thus for Lipps truth comes to rest on subjective certainty, a view evidently at odds with Frege’s views.

According to Frege’s account, logic has little to do with any *mental* process of thinking; nor can thoughts be gathered from such processes, as rules of grammar can in Frege’s opinion be read off from speech. Consequently it is better to avoid the word ‘*Denkgesetze*’ (Frege 1983 [1897]: 157) and more appropriate to speak of ‘logical truths’ or ‘logical propositions’. Frege did not conceive of his formula-language as a means of characterizing an idealized—let alone an actual—inferential *process*. To be sure, the logical ‘laws’ employed in a deduction may be regarded as standards that we have to conform to if we wish to think soundly, that is, if we wish to infer true propositions from true propositions. But Frege never made any claims about possible uses of his *Begriffsschrift* as a *Kunstlehre* or a technique of reasoning, while among his psychologistic contemporaries there was an animated discussion of the issue of *Normalgesetze* (i.e. ideal laws) vs. psychological laws of thinking.

But here a problem will remain: even if thoughts may be regarded as language-transcendent Platonic entities, the grasping of thoughts, the judging of thoughts, and the presenting of thoughts as holding good are mental *acts* occurring in time and performed by actual people. But the logical relations between concepts constituting thoughts cannot be understood in terms of the psychological laws of association, complication, and agglutination of representations. Frege argues that even for psychological reasons the process of *grasping a thought* cannot be explained by pointing to a mere reshuffling of representations; nor can allusions to representations throw light on the unity of thought or the nature of the building blocks constituting thought. In the *Logik* of 1897 Frege points out that the special

¹⁵ ‘One does not, after all, learn about the structure of the human body by looking at the clothed body; on the contrary, everyone knows that the shapes and folds of the dress can become intelligible only if one knows about the body.’

¹⁶ ‘that logic is a special branch of psychology is sufficient to distinguish these two disciplines from each other’.

problems concerning the notion of grasping a thought are due to the fact that grasping a thought is a process

...der schon an der Grenze des Seelischen liegt und der deshalb vom rein psychologischen Standpunkte aus nicht vollkommen wird verstanden werden Können, weil etwas wesentlich dabei in Betracht kommt, was nicht mehr im eigentlichen Sinne seelisch ist: der Gedanke; und vielleicht ist *dieser Vorgang der geheimnisvollste von allen*. (1983 [1897]: 157, my emphasis)¹⁷

And on the same page Frege adds the following footnote:

Diese Frage ist in ihrer Schwierigkeit wohl noch kaum erfaßt. Meistens begnügt man sich wohl damit, das Denken durch eine Hintertür in das Vorstellen einzuschmuggeln, so daß man selbst nicht weiß, wie es eigentlich hineingekommen ist.¹⁸

Historically this footnote is not entirely true to the facts: Brentano had stressed that judging (a prime example of the act of thinking) is an act *sui generis* and quite different from simple representational acts. Wundt had reinvented the notion of synthetic apperception in order to account for the spontaneity of the act of thinking. And especially William James's chapter on 'The Stream of Thought' in the first volume of his *Principles of Psychology* (1890)—where he spells out the difficulties of telling what the unity of thought resides in and what the components of a thought can be supposed to be—would have been a challenge to Frege, had he known it. Whatever the faults of the psychologistic logicians with respect to this particular set of problems, Frege cannot be said to have done more than point out some of those faults and thereby indicated an important philosophical difficulty; his own notion of a 'third realm' of thoughts, however, surely was not a promising step towards its solution.

There is yet another way of describing the relation between logic and psychology criticized by Frege. This does not concern the examination of psychological processes in order to find out whether they are at the basis of what is accepted as true for reasons of logic; rather it concerns the rules with which our thoughts have to accord in order to achieve objective validity. In Frege's opinion all investigations aimed at establishing whether certain psychological laws of thinking are in harmony with objective laws of thought are doomed to remain sterile or, at best,

¹⁷ '...which takes place on the very confines of the mental and which for that reason cannot be completely understood from a psychological standpoint. For in grasping the law something comes into view whose nature is no longer mental in the proper sense, namely the thought; and this process is perhaps the most mysterious of all.' (Engl. transl. 1979: 145.)

¹⁸ 'I should say that this question is still far from being grasped in all its difficulty. People are usually quite content to smuggle thinking in through a back door in the imagination, so that they don't themselves know how it really got in.' (Engl. transl. 1979: 145.)

misleading (1983 [1897]: 158). The claim Frege is objecting to in this context may be attributed to both Wundt and Sigwart. According to them logic is more akin to *ethics* than to physics; it is a normative science prescribing how to think if one wishes to think correctly. And up to a point Frege agrees with this idea (expressed by Herbart, Sigwart, Wundt, and others) of comparing logic to ethics. 'True' expresses the objective of logic in a way similar to that in which 'good' expresses the aim of ethics (1983 [1879/1891]: 4, 139). The parallel is not perfect, however, because our normal use of the word 'true' can be misleading, as Frege points out, in suggesting that basically it is an adjective, whereas in reality truth and falsity are objects: truth is what all true sentences refer to, falsity is what all false sentences refer to. What really characterizes logic is *assertoric force*, i.e. the expression of our striving towards truth, which has no counterpart in any particular part of speech, let alone in the adjective 'true' (Frege 1983 [1915]: 271).

Still, in spite of using the parallel between logic and ethics of which the psychologistic logicians were so fond, Frege soon parts company with Wundt and Sigwart. Frege thinks that the primary task of logic is that of rendering explicit the laws holding in the realm of thought; whether there can be any justification of these laws in terms of psychological evidence or conditions of the possibility of apprehending them is of merely marginal interest. To mention a simple example: to grasp a sentence containing one or more expressions of negation is, as a matter of psychological fact, more difficult than grasping sentences without negation. *This* fact, however, can cast no doubt on, for instance, the validity of the principle of double negation. Even though the usual textbook treatment of that law may fail to do justice to our normal way of reasoning in accordance with it—e.g. for the reason that in ordinary language negation is rarely used in the sense of *propositional* negation—possible questions as to its validity do not concern our empirical difficulties of applying it. Relevant questions that can be asked in this context concern its independence or its being derivable from, say, the principle of non-contradiction together with the principle of excluded middle. Such questions concerning the dependence of one logical law on others *can* be raised and are recognized by Frege as genuine logical problems. In an important passage of *Grundgesetze* (1893: xvii) Frege writes:

Die Frage nun, warum und mit welchem Rechte wir ein logisches Gesetz als wahr anerkennen, kann die Logik nur dadurch beantworten, daß sie es auf andere logische Gesetze zurückführt. Wo das nicht möglich ist, muß sie die Antwort schuldig bleiben.¹⁹

¹⁹ 'The question why and with which right we acknowledge a law of logic to be true, logic can answer only by reducing it to another law of logic. When this is not possible, logic can give no answer.' (Engl. transl. 1967: 15.)

It is precisely at this point that Frege parts company with Sigwart, Wundt, and the later Husserl: their appeals to a logical feeling of necessity, to acts of evident judgement accompanying the acceptance of logical axioms, can at best be seen as ways of describing the symptoms of fulfilment which accompany subjective conviction and subjective certainty. The fact that we cannot 'imagine' people who do not experience the logical inexorability of the principle of identity does not render this principle any truer or firmer than it actually is; in a sense, we *can* imagine such people, Frege argues, only we could not understand them; it would be like being confronted with a new form of madness (1893: xvi).

As has been said at the beginning of this paper, psychologism is aptly characterized by Frege as residing in the confusion of subjective entitlement to holding a sentence true with an objective justification referring to the grounds on which its truth may rest, no matter whether we shall ever be in a position to grasp the thought expressed by it, let alone recognize it as true. To spell out what Frege meant by 'the objective justification' of a proposition would involve a discussion of his foundational programme for arithmetic, and, more specifically, of the issues of reductionism and the logicist conception of definitions, but that would obviously exceed the scope of the present paper.

One may wonder whether Frege's anti-psychologistic standpoint has any relevance or application outside the limited province of logic. In my opinion the answer is yes; but here I can only mention, not argue for, Frege's *Grundgedanke* which, as it were, licenses the transition from anti-psychologism in logic to anti-psychologism in the theory of meaning. According to Frege, mathematical propositions, like ordinary declarative sentences, have sense, they express thoughts. In order to give an account of what 'grasping the sense' of a sentence consists in, we have to clear the ground of all those ingredients of meaning which are of merely psychological or conversational significance and isolate those features which are relevant to the thought *qua* truth-bearer. In order to render explicit the import and significance of a sentence we need not allow for our way of certifying it as true or the representations that happen to cross our minds when we utter a sentence with the force of an assertion; what we have to take into account is that which makes the sentence true. Thus the notion of truth becomes of central importance to an account of meaning in general and provides a link between logical and linguistic considerations.

IV

Readers of Frege's *Grundlagen* may have puzzled over the origins of the notion of *Wiedererkennungsurteil* to which he alludes in §§62–8; such judgements are said to be judgements whose sense has to be elucidated—and if possible sharpened by means of a definition—if our use of number words as names of abstract objects is

to count as legitimate. Such judgements of recognition are preparatory to statements of identity or their denials. According to Frege identity is a decidable relation in which only objects can stand; judgements of recognition on the other hand make sense also with reference to concepts—cf. the example given in Frege (1882), where ‘This is a horse’ serves to ‘present’ a concept—and even with reference to thoughts (cf. Veraart 1970: 100, ‘Wiedererkennung des Gedankens’).

In *Grundlagen* Frege deals with problems concerning the sense of statements expressing judgments of recognition regarding abstract objects such as numbers, shapes, colours, directions. Of course, we cannot enter into the details of Frege’s account of contextual and explicit definitions as ways of fixing the reference of number words; what is of interest from the point of view of the present discussion is that in this context too Frege was fighting his anti-psychologistic battle: his slogan ‘Only in the context of a sentence does a word have a meaning’ was primarily coined in order to defend the claim that from the absence of a concrete object of reference for number words we must not jump to the conclusion that we are dealing with representations or mental images.

Frege’s celebrated distinction between *Sinn* and *Bedeutung* was part of his answer to the question where the cognitive value (*Erkenntniswert*) of identity statements resides and this is the context where the notion of the sense of a proper name is introduced. Now, it is in order to clarify what Frege meant by the sense of a proper name that we need to refer to the discussion of judgements of recognition. From this discussion we learn that a full grasp of the sense of a proper name, e.g. ‘3’ or ‘Venus’, entails, among other things, possession of a criterion (*Kennzeichen*) by means of which the truth-value of identity statements containing such proper names can in principle be decided (cf. Dummett 1973).

It is not to be excluded that as regards his understanding of those notions Frege was indebted to Sigwart for his account of simple judgements and, in particular, his characterization of *erzählende Benennungsurteile*, or narrative judgements of naming. Here it is important to distinguish subjective and perhaps idiosyncratic criteria which happen to enable us to identify, e.g. persons, melodies, and places from objective criteria of the identity of persons, melodies, and places. The latter, difficult as they are to spell out, form part of our concepts of a person, a place, a melody. Now, the accounts of the mechanism whereby we recognize a face, a tune, a *Gestalt*, a shade of colour, etc. were at that time becoming an object of intense study on the part of physiologists and psychologists. Here it will suffice to mention the work of Helmholtz, Stumpf, Meinong, and the Austrian school of *Gestalt* psychology. Their studies were read by psychologistic logicians like Erdmann, Ziehen, and, of course, Wundt. However, the first sustained philosophical discussion of issues related to these scientific questions is to be found in Sigwart’s *Logik*.

According to Sigwart examples of judgements of recognition are ‘This is Socrates’, ‘This is snow’, and perhaps also utterances such as ‘Fire’, which are expressions of the immediate recognition of physical events. The same form of

words, e.g. 'This is snow' or 'This is the *Kölner Dom*', can be used *both* in situations of ostensive teaching *and* on occasions where we express our recognition of an object as the bearer of a proper name which is, so to speak, already in our repertoire. Although in both situations we are presented with perceptually given objects, only the second type of judgement deserves the name 'recognition judgement'; the first type is more appropriately called '*Benennungsurteil*'. These judgements of recognition and naming belong to Sigwart's *simple judgements*, which according to him are the starting point of all our knowledge (1889: 61).

Simple judgements are divided into narrative and explanatory judgements; the first always imply a reference to an existing individual picked out by means of a demonstrative pronoun possibly accompanied by an ostensive gesture; the latter serve to elucidate the meanings of words and thus say nothing about individual objects. Sigwart explains his choice of terminology thus:

Ich wähle diesen Ausdruck, um eine gemeinschaftliche Bezeichnung für die Aussagen zu haben, welche sonst teils als Subsumtionsurteile (wo das Prädikat eine allgemeinere Vorstellung ist) teils als Identitätsurteile (wo das Prädikat dem Subjekt vollkommen kongruent ist) aufgeführt werden. Zwischen beiden besteht in den einfachsten Fällen keine bestimmte Grenze; und der Vorgang, das Bewußtsein der Einheit des Gegebenen als Ganzen mit einer von früher bekannten Vorstellung, ist in beiden Fällen im wesentlichen derselbe. (1889: 69 n.)²⁰

Of course, Frege would not have liked the psychologistic wording chosen by Sigwart. But Sigwart is calling attention to sentences of a type which express judgements not readily classifiable by the traditional scheme of subsumption of an individual under a concept or subordination of one concept under another. More to the point is the fact that these judgements of recognition are related to judgements of identity. After 1892 Frege refrains from using terms like '*Identitätsurteil*' or '*Wiedererkennungsurteil*' because this terminology seems to suggest a difference between types of acts of judgement whereas, from a logical point of view, all these distinctions concern the contents of judgements; in this respect Frege is in agreement with Sigwart: '*Urteilsformen*' is a misleading expression; the relevant distinction is that between the act of judging and the content of such an act.

Sigwart, although himself pressing this point (and thus anticipating Frege's disregard for the distinction between assertoric, apodictic, and problematic

²⁰ 'I choose this expression in order to have a common designation for those statements which otherwise would be called either judgements of subsumption (where the predicate is congruent with a more general representation) or judgements of identity (where the predicate is completely congruent with a subject). In the most simple type of case there is no clear boundary between those two; and a process—the awareness of agreement between that which is given as a whole and a previously known representation—is essentially the same in both cases.'

judgements (Frege 1879: §4; cf. Schmit 1985)) and suggesting an account of the quantity of judgements according to which words like ‘all’ and ‘some’ do not belong to the subject-term but really are predicates, still urges that the *synthesis* accomplished in forming such judgements is of relevance to understanding their contents. Like other psychologistic logicians, Sigwart is particularly fond of genetic explanations and often in his examples he refers to the stages a child goes through in learning his mother tongue, starting from perceptually relevant circumstances. Such a genetic account, Sigwart thinks, could help to show that the analytic–synthetic distinction is merely a relative one. Like other psychologistic logicians, he is interested in the problem of how from simple judgements of perception other judgements can be culled, not by analysis of concepts and inferences therefrom, but through mechanisms of tacit ‘inference’, memory, selective attention, and noticing the salience of certain features, which render those transitions possible (e.g. from the sight of an apple to the thought of its edibility).

Through these considerations psychologistic logicians became involved in the immensely difficult issue of the relation between *sense perception* and the language in which we talk about perception and in the equally formidable problem of the relation between perceptual information and the *semantic* information obtained from linguistic means of expression. Thus in Erdmann for instance we find the remarkable distinction between sentences expressing *direct* judgements and sentences expressing *communicated* judgements (Erdmann 1892: §36). According to that distinction the information that we can extract from, say, ‘The child is running after the dog’ as a perceptual judgement differs significantly from that which we can gather from that sentence taken as a communicated judgement. Or, if we take the trite Kantian examples ‘A body is heavy’ and ‘A body is extended’ as experiential judgements, the alleged greater informativeness of the first (synthetic) judgement as compared with the second will disappear.

This difference—a Fregean might object—has nothing to do with meaning proper but is connected with differences in the observable circumstances under which such sentences are uttered. But even according to Frege we have to be endowed with the relevant recognitional abilities in order to be able to utter a sentence with assertoric force in the relevant circumstances. Therefore, even though we may concede that the *semantically* relevant meaning is that conveyed by the ‘communicated judgement’, still the question remains how understanding that meaning is related to the judgements of recognition we utter in the relevant circumstances.

The way Sigwart tackles these problems is reminiscent of Quine’s sophisticated analysis of the ontogenesis of reference. I think it is not at all far-fetched to regard the examples of simple judgments discussed by Sigwart as corresponding to Quine’s descriptions of situations with perceptually salient features from which stimulus-meaning is somehow distilled (Quine 1960: §§8, 19). Another example of this approach is Sigwart’s account of how the notions of falsity and truth fight

their way through the jungle of simple representations and get entangled with the linguistic notions of denial and negation on the one hand and of assertion on the other (1889: 159–60 n.).

One further reason why Sigwart's account of simple judgements could have been of interest for Frege is to be found in the latter's doctrine of predicate words as unsaturated expressions (in contradistinction to proper names as saturated expressions). The starting point of Frege's analysis are singular sentences, that is, sentences formed by means of a predicate word and one or more proper names (e.g. 'Plato is a philosopher', 'Plato taught Aristotle', etc.). Such sentences had been disregarded by philosophers and logicians because of their exclusive concern with the logic of terms; but it is precisely by concentrating on the logical structure of those sentences that we can bridge the gulf between the logic of terms and the logic of propositions. Now, according to Frege the linguistic expressions for saturated and unsaturated entities (objects or concepts, respectively) have to be themselves saturated or unsaturated; moreover, also the *senses* of proper names must somehow be 'saturated' (in contrast with the senses of predicate words):

... von den Teilen eines Gedankens dürfen nicht alle abgeschlossen sein, sondern mindestens einer muß irgendwie ungesättigt oder prädikativ sein, sonst würden sie nicht aneinander haften. So haftet z. B. der Sinn der Wortverbindung "die Zahl 2" nicht an dem des Ausdrucks "der Begriff *Primzahl*" ohne ein Bindemittel. Ein solches wenden wir an in dem Satze "die Zahl 2 fällt unter den Begriff *Primzahl*". Es ist enthalten in den Worten "fällt unter", die in doppelter Weise einer Ergänzung bedürfen... und nur durch diese Ungesättigtheit ihres Sinnes sind sie fähig, als Bindemittel zu dienen. (Frege 1967 [1892]: 178)²¹

Thus the sense of a proper name cannot possibly be an individual concept, for otherwise it could not 'stick on' the sense of a predicate expression. The same holds good for the levels of reference and syntax. (In a letter to Russell Frege (1976: 224) calls it a 'logische Urtatsache' that 'Der Zerlegung des Satzes entspricht eine Zerlegung des Gedankens und dieser wieder etwas im Gebiete der Bedeutungen'.)²² Now, on any account on which the sense of a proper name does not collapse into that of a definite description an appeal to ostensive acts and related procedures by means of which we bestow names on things will have to be

²¹ 'For not all the parts of thought can be complete; at least one must be "unsaturated" or predicative; otherwise they would not hold together. For example, the sense of the phrase "the number 2" does not hold together with that of the expression "the concept *prime number*" without a link. We apply such a link in the sentence "the number 2 falls under the concept prime number"; it is contained in the words "falls under" which need to be completed in two ways... only because their sense is thus "unsaturated" are they capable of serving as a link.' (Engl. transl. 1984: 193.)

²² 'The analysis of the proposition corresponds to an analysis of the thought, and in turn to something in the realm of meanings, and I should like to call this a primitive logical fact.' (Engl. transl. 1980: 142.)

essential. It is precisely in this context that Sigwart's discussion of judgements of naming and recognition indicates ways of usefully supplementing Frege's account.

But also the notion of an unsaturated expression poses certain problems. Frege does not want to construe predication as a relation which connects two pre-existing entities, for to do so would presuppose—as he explains in a letter to Marty (Frege 1976: 164)—that concepts have an independent subsistence. It is a pillar of Frege's theory that concept-words can be extracted from the complete sentences in which they occur as components. No wonder that the part of Wundt's theory in which the 'predicative' character of concepts is brought to the fore finds Frege's approval. According to Wundt one of the most important features of concepts is their being capable of entering logical relations with other concepts (1893: 96); and these relations are expressed in judgements. However, it is not as if in an act of judgement we combined two already given representations; rather, it is the other way around: we start from judgements and by analysing them we get to their components. The dispute as to whether logically and epistemologically judgement or concept 'comes first' is on the verge of futility, unless it is made clear what the point of such a claim of priority may be. To elucidate this point Wundt employs a metaphor similar to that used by Frege according to which a concept is like an atom: it can leave one compound only to join another one. Neither concepts nor concept-words occur in isolation; the former occur instantiated in the objects whose properties they are, the latter in the sentences whose predicates they are. In the second edition of Wundt's *Logik*—but not in the first edition—we find the metaphor Frege alludes to:

Die Tatsache, daß die logischen Begriffe nicht ursprünglich selbständig gegebene Denkinhalte, sondern Zerlegungsprodukte der Urteile sind, hat manche Logiker veranlaßt, der Untersuchung des Begriffs die des Urteils voranzustellen.... Andererseits ist es aber ebenso unleugbar, daß sich die logische Analyse des Urteils auf die Untersuchung der Eigenschaften seiner Begriffselemente stützen muß. Die Logik ist also hier offenbar in der nämlichen Lage wie andere Gebiete, die zu analogen Abstraktionen genötigt werden. Aus denselben Gründen, aus denen der Grammatiker, obgleich das Wort so wenig wie der Begriff isoliert vorkommt, doch der Satzbildung die Wortbildung, oder aus denen der Chemiker, obgleich die chemischen Elemente meist in Verbindungen vorkommen, den Eigenschaften der Verbindungen die der Elemente voranstellt, wird auch für den Logiker diese Ordnung der Gegenstände die zweckmäßigere sein.
(1893: 97–8)²³

²³ 'The fact that originally logical concepts are not independently given contents of thought but products of the analysis of judgements has induced some logicians to let the investigation of judgement precede that of concepts... On the other hand, it is equally undeniable that logical analysis of judgement must base itself on an examination of the properties of their conceptual elements. Thus it

A similar argument occurs in Frege's article on Boole, where he stresses that a concept is obtained by extracting it from the sentences in which it occurs and adds:

Allerdings muß der Ausdruck des beurteilbaren Inhaltes, um so zerfallen zu können, schon in sich gegliedert sein. Man kann daraus schließen, daß mindestens die nicht weiter zerlegbaren Eigenschaften und Beziehungen eigne einfache Bezeichnungen haben müssen. Daraus folgt aber nicht, daß losgelöst von den Dingen die Vorstellungen dieser Eigenschaften und Beziehungen gebildet werden; sondern sie entstehen zugleich mit dem ersten Urteile, durch das sie Dingen zugeschrieben werden. (1983 [1880–1]: 18–19)²⁴

In a later piece, written after 1892 (that is, after the distinction between *Sinn* and *Bedeutung* had been drawn), Frege acknowledges that the terms 'unsaturated' and 'predicative' fit the sense of an expression better than its reference; and yet, he adds, 'es muß dem ["ungesättigt" und "prädikativ"] doch auch etwas bei der Bedeutung entsprechen; und ich weiß keine besseren Wörter. Vgl. Wundts Logik' (1983 [1892–5]: 129).²⁵

As has already been pointed out, according to Wundt a judgement is a unitary act whose components can be distinguished only consequently. Wundt invokes a law of *Zweigliederung*, i.e. of binary articulation, to show that all judgements are amenable to being rendered in binary form, which is somewhat reminiscent of Frege's distinction between function and argument. According to Wundt the three main logical operations are negation, addition, and determination. He goes back to Leibniz's theory of determinables, of which he sets out to give an outline in the form of an elaborate calculus burdened however with a redundant and cumbersome notation. One of the main objections Wundt levels against Boole is that in his logical calculus the relation of *determinans* and *determinandum* is not expressed. Characteristically, Boolean conjunction is commutative, whereas the relation of determination, which is more akin to that of *genus* and *species*, is not. Therefore in general it does not make sense to exchange the order of *determinans*

is plain that here logic finds itself in the same position as other disciplines which are constrained to perform analogous abstractions. For the same reasons, for which the grammarian gives word-formation precedence over sentence-formation, although words as well as concepts do not occur in isolation, and for which the chemist gives the properties of the elements precedence over the properties of combinations, although chemical elements mostly occur in combination, the same order will be the most suitable one for the logician.'

²⁴ 'Of course, if the expression of the content of a possible judgement is to be analysable in this way, it must be already itself articulated. We may infer from this that at least the properties and relations which are not further analysable must have their own simple designations. But it doesn't follow from this that the ideas of these properties and relations are formed apart from objects; on the contrary, they arise simultaneously with the judgement in which they are ascribed to things.' (Engl. transl. 1979: 17.)

²⁵ 'The words "unsaturated" and "predicative" seem more suited to the sense than the meaning; still there must be something on the part of the meaning which corresponds to this, and I know of no better words. Cf. Wundt's *Logik*.' (Engl. transl. 1979: 119 n.)

and *determinandum*. Frege considers this distinction of Wundt's of little logical relevance and argues for the superiority of his own notation over those invented by Boole, Schröder, and Wundt (1983 [1880–1]: 37). In spite of their different views on the issue of what notions are worth singling out and representing by the symbolic notation of a logical calculus, Frege considered Wundt's account of the predicative nature of concept-words similar to his own conception of the unsaturatedness of concept-words and functional expressions. The implications of Frege's theory of concept-words are deeper and reach further than those of Wundt's account. According to Frege, unsaturatedness at the linguistic level is a mere hint at the functional character of concepts; and appreciating their functional character is only a first step towards understanding the important notion of a complex predicate. The latter notion plays an essential role in Frege's account of the meaning and truth-conditions of sentences containing expressions of multiple generality and is thus part and parcel of his most celebrated discovery, i.e. quantification theory. Generally, it is appropriate to say that many aspects of Frege's philosophy are corollaries of his theory of concepts and concept-words.

A first step in the development of this theory is Frege's distinction between assertion and predication and a clear statement of the logical relations in which objects and concepts can stand. He points out that confusion of assertion and predication and confusion of subsumption (of an object under a concept) and subordination (of a concept under another concept) are due to a deep-rooted misunderstanding of the role of the copula. For this reason he criticizes the doctrine of the double role of the copula (as a syncategorematic sign ambiguous between membership, class inclusion, and identity as well as a sign carrying assertoric force, signifying affirmation). To separate the assertoric force from the predicate is regarded by Frege as a first step towards a new conception of what counts as a predicate expression, applicable not only to simple predicates, such as '*x* walks', but also to complex predicates containing expressions of generality, such as 'being the only *x* anybody wishes to be a friend of everybody else'. This way of constructing new concept-words had been neglected by traditional philosophers. Terms were supposed to represent concepts, and a concept was conceived as a sum of marks; the import of a predication was usually rendered either in the intensional idiom of the concept expressed by the predicate being contained in the concept expressed by the subject or in the extensional idiom of class inclusion ('the extension of the subject term in the extension of the predicate term'). According to this view, when a judgement is put forward it is the task of the copula to mark either of these relations and to present it as holding.

In Frege's opinion, the traditional doctrine of the copula does not do justice to the most important features of concept-words and concepts, viz. their unsaturatedness and incompleteness, for it presupposes the formation of concept-words and the determination of their sense and reference as already accomplished, in advance of the judgements in which they occur. But, as both Wundt and Frege

clearly saw, to stress the priority of the act of judging over that of forming a concept is not of itself of great significance; also Kant had emphasized this point, and yet his insight had not issued in a new conception of predication. What had escaped the attention of philosophers and logicians was the peculiar role that singular (atomic) sentences play as a connecting link between the logic of propositions and the logic of predicates (of which the logic of terms constitutes the monadic part). Frege often stresses the heuristic importance of starting from atomic sentences for gaining an understanding of how more complex propositions can be obtained from atomic propositions, which can be viewed as their generating instances (Frege 1983 [1880–1]: 16, 38ff.). If, on the other hand, we follow Boole and Schröder and start from the calculus of concepts, there is little chance to recover the truth-functional mode of composition characteristic of the propositional calculus. To understand Frege's point we must remember that in constructing a formal system it was his aim to find the most appropriate way of representing a content, not to build a *calculus ratiocinator*. Frege's notion of a formal system is inseparable from his insight into the recursive properties of language and his identification of the linguistic devices employed in building atomic, molecular, and quantified propositions out of their components. In order to understand this process of composition we have to start from singular propositions expressing judgeable contents.

How had it been possible for singular propositions to baffle philosophers and logicians for such a long time? A reason may be that in the case of sentences like 'Caesar crossed the Rubicon', 'Aristotle talks and walks', 'Plato taught Aristotle' neither the intensional nor the extensional account of predication has a natural application, while sentences expressing class inclusion—or the part-whole relation, which is its intuitive counterpart—seem quite amenable to the logic of terms. 'Horses are animals' can be rendered as 'The extension of "horse" is included as a proper part in the extension of "animal"'; but surely we do not want to say that Aristotle is a part (a piece) of the extension of 'thing that walks and talks', possibly conceived as a scattered universal denoted by a mass term. An obvious way out of this difficulty is to appeal to the distinction between membership and class inclusion. And of course Frege would agree that we need some such distinction. But he is wary of the notion of a set, because he thinks it suggests the picture of a heap, of an aggregate; therefore he does not want to avail himself of membership as a primitive notion. Frege's 'classes' are extensions of concepts; he emphatically denies that within these (logical) objects one could discern parts, i.e. the individual component members (cf. Frege 1893–1903: §150). Besides, also in the case of class inclusion, Frege urges, a more natural paraphrase of 'Horses are animals' is 'If x is a horse, then x is an animal', where there is no allusion to extensions but only to concepts. According to Frege, the key notion of logic is that of an object falling under a concept, not the idea of set and membership. Therefore he considers the notion of the *application* of a function to an argument to yield a unique value as

comprising concepts and relations. Thus a concept (the *Bedeutung* of a concept-word) is a mapping from objects into other objects (the two classical truth-values, truth and falsity).

It is not as if Fregean concepts had less being than tables and chairs, as if they were, so to speak, attenuated entities. The correct way of putting the question about their reality is not to ask what a concept-word stands for, but whether it singles out a function. A sober philosopher may object that at this stage talk of functions and mappings is quite unnecessary: all we have to do is replace the question as to the reference of a concept-word with a question about the objects of which a given concept is *true* (Quine 1970). It is conceivable that Frege would have agreed with this objection; but since he had no qualms about quantifying over functions and properties, the edge of Occam's razor showing itself in the objection would have left him unimpressed.

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Kerry and Frege on Concept and Object

Benno Kerry is not a household name to most contemporary historians of logic.¹ His name is hardly found in any reference work. Not a single doctoral dissertation—at least none that I am aware of—has ever been devoted to him. His name is likely to have struck the expert audience in the context of Frege’s article ‘On Concept and Object’. Yet, despite the wealth of scholarly literature devoted to the article, it appears that Kerry’s views have not been accurately portrayed in any of these contributions.

I

So, who was Benno Kerry? Here I will confine myself to some quick remarks about Kerry’s philosophical pedigree and the response that his works have received. The most significant—and, apparently, only—source of information about Kerry’s philosophical lineage is a review, which appeared in the *Vierteljahrsschrift für wissenschaftliche Philosophie* in 1892. In his review, Alois Höfler discusses a book by Kerry edited by Gustav Kohn, *System einer Theorie der Grenzbegriffe: Ein Beitrag zur Erkenntnislehre* (1890).

To begin with, it should be pointed out that Alois Höfler is mentioned in the first of a series of articles published by Kerry in the *Vierteljahrsschrift für wissenschaftliche Philosophie*, which will be addressed in the following. In that context Kerry states that the expression ‘psychic work’ (*psychische Arbeit*), indicating the psychic activities required by the subject in order to process what is intuitively given (1885, I, 427 n.), is borrowed from Höfler. In his review (p. 231), Höfler alludes to this remark himself. Höfler prefaces his review with a biographical sketch, from which one can gather that Kerry first took an interest in historical and legal subjects, and only later on devoted himself to philosophical and mathematical topics. As Höfler writes in his short biography (pp. 230f.):

¹ I would like to thank Volker Peckhaus, Joachim Schulte, and Christian Thiel for their helpful comments on an earlier draft of this paper, which appeared in the proceedings of the Erlanger *Logikhistorisches Kolloquium* with the title ‘Einige Bemerkungen zur Auseinandersetzung zwischen Kerry und Frege’ (vol. 4, November 1991). I am also grateful to the Alexander von Humboldt Stiftung for its financial support during the academic year 1989/90, which I spent in Erlangen, where the first draft was written.

Between 1881 and 1885 we spent many hours—often several half-afternoons a week—discussing philosophical issues. At first, I thought he pursued interests and methods that considerably deviated from what I took to be scientific philosophy. For, as Kerry told me, he took his first steps into philosophy not through Kant, as was most common back then, but rather through Schelling. Yet, even around that time it was the aversion to the psychological way of looking at philosophical problems—which was traced back to “neo-Kantian” influences—that first caused a falling out with respect to our theoretical commitments and then benefited us in that it prompted us to overcome our conflict. I should like to mention that the most important fact about Kerry’s philosophical development is that in his own works he began to use that very psychological method to which I had grown accustomed in philosophy particularly through the example of Brentano and Meinong. He did so with increasing conviction and in an ever more persuasive fashion.

Höfler seems to have played a significant role in Kerry’s conversion to psychologism. The application of the ‘psychological method’, of which Höfler wholeheartedly approves, is the very aspect that earned him rebuke from Cantor and Frege. As for the book *System einer Theorie der Grenzbegriffe*, it may be interesting to recall that it is mentioned in the first volume of Rudolf Eisler’s *Wörterbuch der philosophischen Begriffe* under the entry ‘Grenzbegriff’. According to Eisler, the book appeared in 1910, which is obviously a sheer mistake.

In 1887 Kerry also wrote a brief author’s report, in which he presents a volume he himself edited, *Ernst Laas’ literarischer Nachlaß*. The work on this volume is clearly connected with Kerry’s interests in the history of philosophy. Ernst Laas’s best-known work was *Positivismus und Idealismus* (three volumes, 1879–84), where he argues against epistemological idealism (construed as ‘rationalism’).

Further research on the relations between Kerry and Cantor may hold promise for a better understanding of Kerry’s intellectual development. In a footnote to his first article (1885), Kerry writes:

As Prof. Cantor kindly told me, after obtaining the concept of a ‘closed’ point set [*abgeschlossene Punktmenge*], he now wants to define the numerical continuum as a coherent [*zusammenhängende*] and closed point set. (473, n. 1)

Another highly interesting footnote by Kerry is found in his seventh article (1890):

An article by Cantor (which is also mentioned in the *Mittheilungen*, p. 117, n.1) has remained unpublished, although the author was kind enough to send me the proofs thereof. There I have come across a passage, where he characterises the concept of cardinality (of a set) [*Mächtigkeit*], i.e. in our terminology, that of

number [*Anzahl*] “as the most fundamental and both psychologically and methodologically simplest concept [*Stamm-begriff*]” (p. 343, n. 1)

These references suggest that Kerry and Cantor must have had a lively correspondence. Indeed, we would like to know more about its scope and content. As can be gleaned from Michael Hallett’s excellent book *Cantorian Set Theory and Limitation of Size*, at least Cantor’s side of the correspondence is preserved. Hallett refers to a reply by Cantor dated 21 March 1887 (to be found in *Nachlass* VI, 106).² In his reply, Cantor mentions Kerry’s misgivings about the uniqueness of certain general concepts belonging to all sets equivalent with a given set M —which is in turn identical with the set of cardinal numbers—and only those. However, he does not offer a solution. Hallett points out that the same question was to be raised later by Russell in his *Principles* and by Emil Borel in his *Leçons sur la théorie des fonctions* (1898). According to Russell, Cantor’s so-called definition by abstraction is ‘merely a phrase indicating what is to be spoken of, not a true definition. It presupposes that every collection has some such property as that indicated—a property, that is to say, independent of the nature of its terms and of their order; depending, we might feel tempted to add, only upon their number.’³

It is very plausible to assume that Kerry’s qualms in his letter to Cantor are essentially the same he voices in his seventh article, where he states:

According to Cantor, what is meant, say, by the number four is “the” general concept under which all sets of four elements (and only these) fall. . . . Moreover, on a related note, the *definite article* used in the definition in question is not legitimate either: there may be several different general concepts under which all sets of four elements (and only these) fall; if this is the case, then the definition would leave us in the dark as to which of these several possible general concepts we mean by “four”. Let me illustrate the problem with an example. Assume that the redness of a given body is necessarily associated with a certain other property (E) of bodies in general—which may well be the case. Assume further that somebody wanted to define what “red” is as follows: by “red” I mean the general concept under which all bodies exhibiting property E (and only these) fall. Now, what they would thereby define is not *one*, but *two* general concepts (i.e. the concept designated by “red” and the one designated by E). Hence, they would fail in their attempt to define redness. The same holds true for the case at hand.

(1890, VII, 344f.)

As has already been pointed out by Husserl, Kerry’s objection against Cantor could also be levelled against Frege’s definition of number [*Anzahl*]. In fact,

² The letter is not included in Cantor 1991. See also Peckhaus 1994.

³ Russell 1903, 304f.

however, Kerry's objections against Frege's definition are much more radical than Husserl assumes. Kerry disputes the claim that ' n is a number' is equivalent to a statement about the extension of a second-level concept. Kerry worries about Frege's characterization—or rather: the lack of an adequate characterization—of the notion of 'extension' [*Umfang*]. Frege takes it for granted that every concept has a numerically determined extension and fails to consider vague concepts or concepts like 'star', whose extension is indeterminate because we do not know whether they are finite or infinite in number. Kerry's chief objection is essentially that concept extensions are not 'appropriate symbols' for natural numbers, because—as Frege clearly saw (GLA, §69)⁴—relations between concept extensions are not able to fulfil the same task as the relations 'greater than' or 'smaller than', which hold between numbers.

It is, therefore, hardly fortuitous that Kerry's criticism focuses on this particular statement by Frege (fourth article: IV, 276–7). Now, obviously, Frege's subsequent admission that his definition may fail to draw sharp boundaries for the extension 'equinumerous to the concept F ' goes some way towards reinforcing Kerry's doubts. For Frege writes:

Admittedly, the case can still be imagined in which the extension of the concept 'equinumerous to the concept F ' is more inclusive or less inclusive than the extension of another concept, which, according to our definition, could not then be a number [*Anzahl*]; and it not usual to call a number [*Anzahl*] more inclusive or less inclusive than the extension of a concept; but there is also nothing to stop us adopting such a form of speech, should such a case occur.

(GLA, §69, tr. Beaney)

A detailed account of Kerry's instructive remarks about the logical (as opposed to merely psychological) priority of the concepts employed in definitions would require a lengthy book in its own right. Yet, here it is tempting to make at least some guesses.

Firstly, it is not unlikely that Kerry's critical remarks about the notion of extension employed in Frege's GLA may have been partly inspired by Cantor's critical review of GLA, which is quoted by Kerry himself elsewhere (seventh article: VII, 341 n.). In his review, Cantor criticizes the notion of extension as something 'indeterminate' and says:

⁴ The following abbreviations are used to refer to Frege's works (cf. References): *BS* = *Begriffsschrift*; *GLA* = *Grundlagen der Arithmetik*; *FTA* = 'Über formale Theorien der Arithmetik'; *GGA* = *Grundgesetze der Arithmetik*; *BuG* = 'Über Begriff und Gegenstand'; *SuB* = 'Über Sinn und Bedeutung'; *NS* = *Nachgelassene Schriften*; *WB* = *Wissenschaftlicher Briefwechsel*.

... only in certain cases is the 'extension of a concept' quantitatively determinate ... For such a quantitative determination of the 'extension of a concept', however, the concepts 'number' [*Zahl*] and 'cardinality' [*Mächtigkeit*] have to be already given independently in advance, and it is an *inversion of what is correct* to attempt to ground the latter concepts on the concept 'extension of a concept'.

(Cantor 1932, 440, tr. Ebert/Rossberg)⁵

Secondly, it is not overly bold to claim that Kerry realized that Cantor's objection against Frege could also be directed against at least *one* of Cantor's own characterizations of cardinal numbers. This is particularly true of the characterization found in 'Grundlagen einer allgemeinen Mannigfaltigkeitslehre' (1883), where the equivalence class of a set *M* is just the extension of the general concept corresponding to the set *M*, which Cantor calls the cardinality [*Mächtigkeit*] of the set *M* (cf. Grattan-Guinness 1970, 86). However, as Husserl already noticed and Hallett persuasively shows, despite appearances to the contrary, the view that Cantor ultimately holds and is already present in his 'Communications' ('Mittheilungen') is a far cry from the one Frege and Russell take to be correct. In a footnote to his *Philosophie der Arithmetik*, Husserl emphasizes the following statement from Cantor's 'Mittheilungen': 'For the formation of the general concept "five" *only one* set is required... to which this cardinal number belongs' (115, n. 2). The same passage is also quoted by Kerry (VII, 346 n.). As Hallett points out, totalities of such a size as would be required by the approach of both Frege and Russell are ruled out by Cantor's conception of the Absolute. Hallett writes:

The size of the collections chosen by Frege-Russell as numbers in effect puts them beyond the realm of Cantor's transfinite. Certainly they cannot be genuine mathematical objects according to Cantor's *later* theory of consistent and inconsistent (or absolute) collections... For the Frege-Russell collections (0 excepted) are all of the same cardinality as the whole universe of existents (what Cantor in 1899 called 'the totality of everything thinkable'), and thus on Cantor's theory are much too extensive to be sets. (126)

Many passages from Kerry's articles bear out his good instinct for paradox. Whether his concerns about Frege's definition are specifically directed at the quantitative indeterminacy of the concept extensions thus introduced or whether he also sets his sights on indeterminacy understood in terms of 'boundlessness'

⁵ An interpretation of Cantor's criticism of Frege's *GLA*, which differs from the one by Zermelo (see the note appended to Cantor's 1885 review in Cantor 1932, 441f.), is offered by Kreisel in several of his writings. See, for instance, Kreisel 1989.

and inconsistency is anyone's guess.⁶ According to the conception to which Kerry ultimately subscribes, number is essentially a set of units. Hence, his view is in line with that of Cantor, who held that natural numbers are not to be identified with equivalence classes of certain sets, but rather with the elements of these sets that can act as proxies for them.

Yet, between Cantor and Kerry there are also disagreements. They concern first and foremost two points: the first is about the process of abstraction. Kerry has reservations about abstraction as a process that strips objects of all their properties so as to make them indiscernible. The second is about the privileged role played by natural numbers as compared to the other numbers. Apart from the aforementioned statement in the unpublished passage from the *Communications on the Theory of the Transfinite* ['Mittheilungen zur Lehre vom Transfiniten'], which Kerry enthusiastically quotes, Cantor does not seem to have a full appreciation of their special role. Moreover, Kerry has misgivings about Cantor's entire conception of 'free' mathematics (V, 78, 113ff.). He thinks that the consistency of a conceptual content [*Begriffsinhalt*] in mathematics does not allow one to infer the existence of an object falling under the concept [*Begriffsgegenstand*], but also that one cannot content oneself with the mere belief of its being consistent. The correspondence with Cantor presumably dates back to the year 1883. In 1885 Kerry publishes an essay titled 'On G. Cantor's Inquiries into Manifolds' [*Über G. Cantors Mannigfaltigkeitsuntersuchungen*]. Here is Adolf Fraenkel's report about Kerry's essay in his biographical article on Cantor:

The reception of Cantor's achievements was no less hesitant among philosophers than it was among mathematicians. The first detailed and insightful exposition of his work was given by Kerry. Kerry also mentions the insufficiency of the accounts of Cantor by previous philosophers

(Ballauf, Wundt, Laas, H. Cohen). (Cantor 1932, 464)

The young Russell first became aware of Cantor's work by reading Kerry's article: the article is approvingly quoted in Arthur Hannequin's book *Essai critique sur l'hypothèse des atomes dans la science contemporaine* (1895), which Russell had to review for the journal *Mind*.⁷

⁶ The following quotation testifies to Kerry's good instinct for paradox: '[according to Frege, *GLA*, §75] "a stands in relation φ to b" should express the universal form for any judgeable content that is "about object a and object b". By the same token, the sentence "a does not stand in any relation φ whatsoever to b", which is clearly also about a and b, would have to mention a relation φ between a and b, such that one could not say of any two objects a and b that they do not stand in relation to one another, without thereby also stating that they stand in relation with one another. Thus, clearly, the statement of such a relation becomes so general that it is completely pointless' (IV, 290, n. 1).

⁷ Russell 1990, 35–43. On p. 513 of the same volume the editors describe Kerry as a 'German mathematician and philosopher of science'. They also indicate an incorrect date of birth (1858).

As for the influence on Kerry's work, here I will confine myself to mentioning the following authors (besides Frege and Fraenkel): in his *Philosophy of Arithmetic* [*Philosophie der Arithmetik*] Husserl raises objections against Kerry's proposed definition of number in his third article (1886, 424ff.). He also devotes a separate section to Kerry's work, which follows his discussion of Frege. Eley's textual notes in his edition of the *Philosophy of Arithmetic* suggest that Husserl continued engaging with Kerry's essays even after the book was published.⁸

Ehrenfels discusses Kerry in his essay 'On the Philosophy of Mathematics' [*Zur Philosophie der Mathematik*] (1891, 302ff.). References to these authors are found also in the aforementioned review by Höfler, who distances himself from some of the ways Kerry applies the concept of psychic work, which is also criticized in Husserl's book. Moreover, Höfler quotes Kerry in two passages from his *Logic* [*Logik*] (1890, 2nd edition 1922, 167f. and 331f.). The second mention is interesting, because it suggests that Kerry was a relatively well-known author. In that context, Höfler writes: 'The theory and notion of "indirect representation" [*indirekten Vorstellens*] through "transfer of relations" [*Relationsübertragung*] were first developed by Meinong in his "Theory of Relations" [*Relationstheorie*] (1882). Kerry (among others), who in the literature has often been mistakenly indicated as its originator, got it from there.' Höfler seems to have attached great importance to this rectification. For even in a review of works from the schools of Fries and Cohen he wrote in 1909 he does not miss the opportunity to warn the reader that Max Simon, who in his book 'On Mathematics: Expanded Introduction to Didactics' [*Über Mathematik: Erweiterung der Einleitung in die Didaktik*] repeatedly refers to Kerry, went too far with his 'reverence' [*Pietät*].⁹ As Höfler is keen to

⁸ See pp. 511–15. Particularly interesting for my present purposes is the following remark by Husserl: 'In VI. 344 (1890) [recte: 1889] objects against Cantor's definition of number [*Anzahl*] that it merely defines the extension. This is the same objection I raise against Frege, Kerry himself and against the definition of numerical equality [*Zahlengleichheit*] as equivalence [*Äquivalenz*].—One could list all such definitions under the heading "definitions of the concept of number through its extension".—notes to p. 128, more generally, notes to Chapter 7.—Their extension might also belong to different concepts, cf. Kerry, *loc. cit.* As if anticipating my objection, Kerry resists the assimilation of his definitional method to definitions through extension. He understands them as referring to the content quite in the same sense as our definition of the concept "red". VII, 351' (Husserl 1970, "Textkritische Anmerkungen zum Haupttext", p. 514). Eley's emendation is incorrect. A correction of the volume number (and not of the date of publication) would be required: the correct volume number is VII. On the definition of number in Husserl and Frege, cf. the instructive article by Dummett 1987 (reprinted in Dummett 1991), and chapters 12 and 14. On the aforementioned definitions by abstraction, cf. Thiel 1972.

⁹ Kerry, for his part, refers (II, second article, 436f., n.) to Simon's handbook *The Elements of Arithmetic as a Preparation for the Theory of Functions* [*Die Elemente der Arithmetik als Vorbereitung auf die Functionstheorie*] (1884), which, according to Kerry, was influenced by Weierstraß and especially by Cantor. Kerry had received from Simon a copy of Weierstraß's *Collegienhefte*, which Frege criticizes in his posthumous essay 'Logic in Mathematics' [*Logik in der Mathematik*] (NS, 234ff.) These are notes Weierstraß used for his lectures on the theory of functions. Conceivably, they are the same notes that Scholz and Bachmann refer to with the title 'Analytical Functions' [*Analytische Funktionen*] in connection with Frege's posthumous essay.

stress, the key concepts of 'psychic work' and 'indirect representation' do *not* originate from Kerry, but rather from Meinong and himself.¹⁰

Moreover, according to Volker Peckhaus, Leonard Nelson must have had a high opinion of Kerry. The reason for this should become clearer in the second part of my article. Furthermore, in many passages of his *Habilitationsschrift* ('On the Theory of Content and Object of Ideas' [*Zur Lehre vom Inhalt und Gegenstand der Vorstellungen*]) Kasimir Twardowski discusses and critically analyses (cf., for instance, §§4, 6, 15) Kerry's theory of the content and object of ideas at length. In the introduction to the new edition of Twardowski's work, Rudolf Haller states: 'Under the influence of Zimmermann and probably also Kerry, Twardowski gained very early on a sound knowledge of the theory of science [*Wissenschaftslehre*] of the great Bolzano, which found its way into the treatise on the content and object of ideas' (p. vii). Twardowski rejects Bolzano's terminology (as well as his conception) of 'objectless ideas' [*gegenstandslose Vorstellungen*] (§5). He (§14) contrasts Bolzano's claim that not 'every part of an idea [*Vorstellung*] is the idea of a part contained in its object' (*Wissenschaftslehre*, §63) with Kerry's view that 'the object of a concept [*Begriffsgegenstand*] must somehow partake at least of all distinguishing marks [*Merkmale*] of its concept' (Kerry, IV, 422) and judges both inadequate. I will tackle the issues arising in this connection in due course, because they play a role in the controversy between Frege and Kerry.

Now, the obvious question arises why Frege, who had read not only the second and fourth, but presumably *all* of Kerry's articles, was not curious enough to get to know Bolzano's work, which had so many points of contact with his own. Why is Frege silent about Bolzano despite having read authors like Husserl and Kerry, who in turn often quote Bolzano?¹¹ Is it possible that Frege tacitly appropriated Bolzano's theories without mentioning their originator? Such a suspicion turns out to be groundless. It is true, though, that Kerry mentions Bolzano several times—mostly approvingly—both in the text and in the footnotes of these articles of his. In some footnotes Bolzano is even mentioned together with Frege. Yet, Kerry does not examine Bolzano's views thoroughly in their own right, whereas he does when it comes to the theories of Lange, Kronecker, Frege, and Cantor.

Only in the last article in the series, where Kerry summarizes his main claims, does he devote eight pages to Bolzano's notion of 'proposition in itself' [*Satz an sich*]. In that context he tries to dispel the 'mystical' aspects of Bolzano's conception. He states that by 'proposition in itself' one is to understand what can be

¹⁰ A further review dating back to 1912, where Höfler again mentions Simon, Kerry, and Meinong and raises the priority issue, has been found by Auguste Dick. I am indebted to Volker Peckhaus and Christian Thiel for the references.

¹¹ Consider, for instance, the misgivings expressed by Ettore Casari in his interesting article 'Una fonte dimenticata? La teoria bolzaniana del significato' (Casari 1989). Several years ago Jan Sebestik drew my attention to Kerry's articles as a possible source of Frege's knowledge of Bolzano.

abstracted from all propositions that are *de facto* thought: 'Propositions in themselves are nothing but judgements abstracted from the aspect of judgement-making [*ihres Gefällt-Werdens*]' (VIII, 133). However, Kerry concedes that neither Bolzano himself nor a real follower of Bolzano, such as Příhonský, would approve of an interpretation along these lines. For Bolzano admits his inability to explain the notion of 'proposition in itself' any further. As for Příhonský, the best he can come up with is a simple-minded (and to my mind misguided) simile: 'Propositions in themselves are to propositions that are *de facto* thought as flowers are to painted flowers or paintings of flowers, the only difference being that flowers are existent objects, whereas propositions in themselves... are not something that exists.'¹² It is doubtful whether Bolzano would have agreed with Kerry's construal of the concept of truth in itself [*Wahrheit in sich*]. According to Kerry, ' $7 + 5 = 12$ ' would still be a truth—if only a truth in itself—in Bolzano's sense if there had not been a full-fledged arithmetic, i.e. an arithmetic in a 'categorical' sense, as Kerry says. In his own view, this would rather be a case in which there is not yet 'a judgement that has been already made and is accepted as true'.

As Kerry's last article appeared—which did not happen until May 1891—Frege's own article 'On Concept and Object' [*Über Begriff und Gegenstand*] was in all likelihood already in press. The same is true also for 'On Sense and Reference' [*Über Sinn und Bedeutung*], which was written earlier.¹³ Hence, the question whether Bolzano—or rather: Bolzano's *philosophical* theories—may have had an impact on Frege through Kerry is to be answered in the negative at least as far as the particularly influential 1892 papers are concerned. (On the other hand, Bolzano's *Paradoxes of the Infinite* are mentioned also in the works of Cantor and Dedekind, which Frege knew and on which he commented.)¹⁴

¹² Příhonský 1850, 3 (as quoted in Kerry, VIII, 134f., n. 4).

¹³ Unfortunately, the draft of Frege's letter to Walter C. G. Brix, where he replies to a letter of the latter dated 15 November 1890, is not itself dated. Frege writes: 'In his essay on intuition and its psychic processing (*Vierteljahrsschrift für wissenschaftliche Philosophie*, Issue 11) and especially in his fourth article, Mr. B. Kerry assesses my view, albeit without a proper understanding thereof. I will probably be compelled to respond to that with a written rectification of some kind. It will be primarily about what I call "concept". The many different uses of this term make it very hard to reach a mutual understanding' (WB, 12). Brix's article 'The Mathematical Concept of Number and Its Developments' [*Der mathematische Zahlbegriff und seine Entwicklungsformen*] (*Philosophische Studien*, edited by W. Wundt, vol. 5, Leipzig, 1887, 632–77) is mentioned by Husserl in a footnote appearing in the second chapter of his *Philosophy of Arithmetic* [*Philosophie der Arithmetik*] (p. 30).

¹⁴ Compare especially Frege's discussion of Dedekind's proof of the existence of an infinite system (i.e. the system of 'objects of thought' [*Gedankendinge*]) in 'What Are Numbers and What They Should Be?' [*Was sind und was sollen die Zahlen?*] ('Logik' (1897), in NS, 147f.). As is well known, in a letter to Dedekind dated 28 July 1899 Cantor criticizes the idea of a 'totality of everything thinkable' [*Inbegriff alles Denkbaren*] on the ground that such an idea is contradictory (Cantor 1932, 443–7). The tenor of Frege's criticism is different: he thinks that Dedekind's proof is doomed to failure because the singular terms purportedly referring to objects of thought lack a reference. Frege's and Dedekind's arguments are discussed in Hallett 1984 and Boolos 1990.

Importantly, in his last article Kerry remarks that Bolzano's *Theory of Science* [*Wissenschaftslehre*] had sunk into oblivion at the time.¹⁵ His remark can help enhance our understanding of the historical background of the philosophical debate between 1870 and 1890:

Despite the correction we had to make in the previous sections, Bolzano's doctrine of propositions and ideas in themselves have such an epistemological significance that one has good reason to be surprised that neither the doctrine nor the name of its proponent are mentioned in any of the works that have appeared over the past decades in the domain of epistemology. However, the discussions of the issue of transcendence, which are gaining momentum nowadays, should be suspended until one has dealt with that doctrine. Unfortunately, Bolzano's numerous and lengthy writings are almost completely unknown; yet, among them, the mathematical ones are given somewhat more consideration than the philosophical ones. (Cf., in particular, O. Stolz's treatise in *Math. Ann.*, vol. XVIII, p. 257 f.). Although it is beyond doubt that Bolzano is by far the most significant philosopher Austria has produced so far, even his home country seems to have almost forgotten him. Otherwise it is hardly understandable—here I will confine myself to two examples—how Brentano (Vienna) can possibly declare in his *Psychology* (p. 267) that “the true relationship between presenting [*Vorstellen*] and judging [*Urteilen*] has been so far universally misunderstood”: for he could have found a perfectly clear treatment of this issue in § 34 of Bolzano's *Theory of Science* [*Wissenschaftslehre*]. It is equally incomprehensible how he (*op. cit.*, p. 287 f. n.) can possibly declare that the relation of predicates to their subjects by virtue of which the latter are not such much determined, but rather modified by the former, has been “so far overlooked by logicians”: he could have found this very remark in § 23 (I, p. 92) and § 29 (I, p. 138) of Bolzano's *Theory of Science*... In a presentation of Brentano's theory of judgement (cf. this journal, 1884, p. 162), Marty (Prague) cites several of Brentano's precursors...; Bolzano's account of judgement, which goes well beyond a mere “tentative approach” to an adequate theory, is not mentioned at all. And the same goes for a later passage (*op. cit.* p. 178) with regard to the aforementioned remark about modifying predicates.

These pieces of evidence are not meant to detract from anyone's merits. For that would be self-defeating, given that with my references to the notions of

¹⁵ Footnote 2 on pp. 476f. of Kerry's first article is also evidence that already in his days it was difficult to get hold of Bolzano's writings. From this footnote one can also gather that Kerry was in touch with Robert Zimmermann, who was ‘a direct pupil of Bolzano’ and ‘heir to a large *Nachlass* stemming from Bolzano and mostly containing mathematical writings’. Through Zimmermann Kerry gained access to a ‘rarissimum’, i.e. Bolzano's ‘Observations on Some Objects of Elementary Geometry’ [*Betrachtungen über einige Gegenstände der Elementargeometrie*] (1804).

proposition and idea in itself [*Sätze und Vorstellungen an sich*] I have at most only accepted a tenable doctrine by Bolzano. Likewise, I am aware that... even with the best intentions in the world it may happen that someone does not know all of their predecessors in some way (VIII, 135f., n.).

However, such blunt remarks of Kerry's would not have found favour either with the Neo-Kantians in Strasbourg or with Brentano's pupils in Vienna. Considering that neither Brentano nor Marty took notice of Bolzano, it is hardly surprising that Frege also ignored him. Nevertheless, a detail in Kerry's article may arouse the reader's curiosity. In the passage in question (VIII, 129) Kerry mentions a statement by Bolzano in his anonymously published *Wissenschaftslehre und Religionsphilosophie in einer beurtheilenden Übersicht* (p. 26). In that passage Bolzano says that the contrast between 'truths in themselves', on the one hand, and 'thought' or 'known propositions', on the other, is apparent in contexts where one asks whether every truth in itself is known by someone or whether every proposition in itself (including false ones) is thought or presented by someone, that is in contexts where one realizes that

through classes in geometry it is rather the thought or knowledge of the Pythagorean theorem that is reproduced [*vervielfältigt*] in our minds, not the Pythagorean theorem [*Satz*] in itself. Or, after all, also when we say: "If there were not a single rational being, the proposition that there is not a single rational being would be itself a *truth*."

Of course, we cannot rule out that Frege's use of the example of the Pythagorean theorem in his first *Logical Investigation* ('Der Gedanke') to elucidate the difference between subjective idea [*Vorstellung*] and objective content [*Inhalt*] is merely accidental. The terminology employed by Frege is different from that of Bolzano: his use of 'thought' [*Gedanke*] corresponds to Bolzano's 'proposition in itself' [*Satz an sich*], whereas Frege's 'idea' [*Vorstellung*] is equivalent to Bolzano's 'thought proposition' [*gedachter Satz*]. Here is what Frege says in 'Der Gedanke':

Is thought an idea? If other people can assent to the thought I express in the Pythagorean theorem just as I do, then it does not belong to the content of my consciousness, I am not its owner; yet, I can nevertheless acknowledge it as true. However, if what is taken to be the content of the Pythagorean theorem by me and by somebody else is not the same thought at all, we should not really say '*the* Pythagorean theorem', but '*my* Pythagorean theorem', '*his* Pythagorean theorem', and these would be different, for the sense necessarily goes with the sentence... Could the sense of my Pythagorean theorem be true and the sense of his false? (*Der Gedanke*, p. 68; Eng. tr. M. Beaney)

The assumption that truth is ascribed as it were to the ‘reproductions’ [*Vervielfältigungen*] of the Pythagorean theorem leads to an even more radical scepticism. For in that case we could not be certain that the content of my consciousness matches the content of somebody else’s consciousness. And this in turn would entail that one would no longer be entitled to assume that there is something like a ‘common treasure of thoughts’ of mankind (*BuG*, 196). Moreover, the idea of a ‘reproduction’ of one and the same content in the consciousness of different individuals would no longer make sense, for it would not be possible to find out whether the contents grasped (say, the ‘painted flowers’ in Příhonský’s example) come down to one and the same thought or proposition in itself or they follow from it. In view of this impossibility even positing propositions in themselves or other entities belonging to a third realm cannot alleviate the problem. The truth of the matter is that the very way Bolzano and Frege want to spell out the contrast between the subjective and the objective is misguided in the first place. (Yet, this topic would go beyond the scope of the present discussion.)

As I already pointed out, it cannot be ruled out that the use of the same example in the same context by both Frege and Bolzano is merely fortuitous, but it may just as well be that this example got Frege’s attention as he read Kerry’s article and that he employed it several years later in a similar context without having the actual originator of the example in mind.

The eight articles that Kerry published between 1885 and 1891 in the journal edited by Avenarius, *Vierteljahrsschrift für wissenschaftliche Philosophie*, are a very rich source of information as well as a model of scholarship and scientific curiosity. The thinkers addressed in Kerry’s critical study are notably Kant, F. A. Lange, Frege, Kronecker, and the author for whom he always had a preference, Cantor. Kerry had a thorough knowledge of *BS*, *GLA*, and *FTA*, and his critical remarks are never superficial even when they are mistaken or out of place. As for the critical examination of the works of other authors, it seems fair to say that Kerry’s achievement in his articles is greater than, say, Husserl’s analysis in the relevant sections of his *Philosophy of Arithmetic*. The flaws of Kerry’s as well as Husserl’s position become particularly apparent when it comes to the constructive side of their theoretical analysis. There remains a difference between them, though: Kerry has stronger intuitions when it comes to tracking down the special presuppositions—for instance, the axioms of infinity—that go beyond the constraints of the ‘constructive’ conception of mathematics he (and Husserl) upheld.

II

The second part of this study will be devoted to Kerry’s objections to Frege and the replies of the latter to the former. The exchange between these two authors is best

seen against the background of the different philosophical views about the concept of natural number they held, and, in particular, about the role of psychology in the philosophical clarification of mathematical concepts. The misgivings Kerry voices against Frege's view as well as—to some extent—against Cantor basically come down to the following: the way these two authors understand 'natural numbers' is not in line with what we usually mean by 'natural number', for their views fail to reflect the way numbers are grasped and used. In short, the reconstructions of both authors do not merely obscure the psychological reality of the concept of number, they are in tension with it. Kerry challenges Frege's view that numbers are essentially 'objects of reason' [*Vernunftgegenstände*] and that reason can access them with the aid of logic with a psychologically enriched view. He states:

In addition to what is intuitively presented [*dem Anschaulichen*], on the one hand, and the ideas and judgements pertaining to logic, on the other hand, what can constitute the object of scientific treatment are the forms of psychic work [*psychischer Arbeit*] in general; inner perception [*innere Wahrnehmung*]*—*which is conceptually different from intuition [*Anschauung*]*—*becomes acquainted with them, just as it also becomes acquainted with those ideas and judgements that constitute the object of logic. . . . and if one considers that those arithmetical propositions are therefore based on nothing but *instances of psychic work* [*psychische Arbeiten*], it may be that some aspects are thereby cited, which can usher in a more enlightening account of their 'objective validity' than the one offered by the mere appeal to "reason". (IV, 305, 307)

Generally speaking, Kerry defends a more complex version of the position that—using Frege's words in the *GGA*—is influenced by the 'corrupting incursion of psychology into logic'. This judgement reflects Frege's own assessment in a draft of *BuG* preserved in his *Nachlass*: 'As I see it, the reason for the woeful blurring of the difference between concept and object, and between characteristic mark and property in Mr Kerry is the conflation of logical and psychological points of view and questions, which detracts from the value of his treatises' (*NS*, 114). However, Frege attaches some value to Kerry's conception and something should be said about this alleged value, for he is clearly wrong about where the value of Kerry's claim is to be found.

Not only does Kerry criticize Frege's claim that the definite article in front of a concept word indicates that one is not speaking about a concept, but rather about an object; he also points out that there is a certain tension between the grammatical criteria laid down in *GLA* and the earlier characterization of the 'judgeable content' offered in the *BS*. Kerry clearly brings out the tension with the theoretical and terminological views Frege defends in the *GLA* (IV, 273). In his reply, Frege concedes that a clarification of his terminological choices is in order here,

especially in the light of the new semantic ideas presented in the writings that appeared between 1891 and 1893.¹⁶

In *BuG* Frege accepts Kerry's view that the concept 'horse' in the example 'The concept "horse" is a concept easily attained' is one of the objects falling under the concept 'easily attainable concept'. 'Quite so!', says Frege. 'The three words "the concept 'horse'" do designate an object, but on that very account they do not designate a concept, as I am using the word' (*BuG*, 195, tr. Beaney). Not only does Frege take Kerry's proposal literally, but he also generalizes it and fits it into his own theory. Thus, he ends up with a very artificial view.

According to Frege, the phrase 'the concept *F*' serves in our language as an operator that ties a concept with an object of a 'quite special kind' (*BuG*, 201). Frege's wording is vague. He does not explicitly say that these objects are just the extensions of concepts. Arguably, he aims to dodge Kerry's objections and preserve consistency with the surprising claim—indeed, more so in 1892 than it would have been in 1884—that the introduction of extensions does not play a decisive role for the foundation of mathematics. Some authors (e.g. Schirn 1990) even go so far as to claim that the objects mentioned in *BuG* are not the extensions of concepts. Yet, it should be obvious that positing further objects, i.e. objects corresponding to the expression 'the concept *F*', in addition to the already mysterious extensions, leads to an overpopulated ontology and unjustifiably increases the danger of antinomies.

Perhaps one may try to explain the introduction of such peculiar objects considering the example cited by Kerry, where the substitution of expressions that refer to the same concept, but have different senses, may alter the truth-value. Drawing on an example Frege uses in his review of Husserl (p. 320), one could say that the statement that the concept of a conic (understood as the intersection of a plane with the surface of a cone) can be easily attained is true, whereas this does not hold for the statement 'The concept "conic" (defined as a curve with an equation of second degree in two variables) can be easily attained.' In this case, the concepts would not count as equivalent despite having the same extension. Here the concept preserves as it were its individual character and is represented by proxy by an object constructed *ad hoc*.

However, it is a moot point whether this could have been Frege's actual motivation for introducing them. For then he would have seen a way out of Kerry's problem, which is offered by the theory of the indirect sense and reference of linguistic expressions. Indeed, one could argue that Kerry's example is about the sense rather than the reference of expression. Since Frege clearly holds the view that the sense is an object of reference [*Gegenstand der Bezugnahme*], it would not be too difficult to rephrase Kerry's example as follows: 'The sense of the

¹⁶ Cf. letter to Husserl dated 24 May 1891 (*WB*, 94–8); 'Ausführungen über Sinn und Bedeutung' (*NS*, 128–36, notably 130–3); *GGA* I, §4.

concept-word “horse” is easily attainable.’ However, understanding the sense of a word is not the same as grasping the expressible content with the aid of that very word *inter alia*. Hence one should rather tackle the problem on a case-by-case basis.

Had Frege resorted to his theory of the indirect sense and reference, he would have spotted an ambiguity in Kerry’s claim that ‘the concept of the concept “horse”’ has the same distinguishing marks as ‘the concept “horse”’.¹⁷ Kerry’s view is that, if the distinguishing marks of a concept are construed as parts of the concept along Bolzanian lines, then both concepts turn out to have the same parts, they display the same construction, and differ only with respect to the number of repetitions of the expression ‘the concept...’. Such a repetition, according to Kerry, does not add anything to the sense of the expression, but it functions like the repetition of an affirmation (II, 460). This is incompatible with Frege’s point of view—as well as with that of most non-Fregean philosophers: sense and reference of the expression ‘the concept *F*’ are different from sense and reference of the expression ‘the concept of the concept *F*’. For, in the former case one is referring to a concept, whereas in the latter one is referring either to the ways in which different people can grasp the concept *F* or to the relation between the concept in question and the objects falling under it.

These possible readings are incidentally mentioned by Kerry himself (II, 456–8). However, in his view, the expressions previously considered do not have a different sense and a different reference. He takes it that they are related as content and object of the concept, such that ‘the concept “horse”’ is the conceptual object [*Begriffsgegenstand*] corresponding to the conceptual content [*Begriffsinhalt*] of ‘the concept of the concept “horse”’. Frege would have been able to solve the problems arising in this context simply by conceding that different entities are involved, while also denying that they share the same distinguishing marks. It would have sufficed to point out that the doubly indirect sense of ‘the concept of the concept “horse”’ is not identical with the simple indirect reference of ‘the concept “horse”’, which in turn calls for a corresponding distinction.¹⁸ Not only does Frege overlook this option, but he also broadens the scope of the example. The solution for which he opts is that the logical form of the statements

- (1) There is at least one square root of 4,

and

- (2) The concept *square root of 4* is satisfied,

¹⁷ The example of “the concept “horse”” is discussed in the fourth article. In the second article, Kerry addresses the example of “the concept “tree””.

¹⁸ A reliable exposition of Kerry’s claims is provided by Twardowski (1894), §14.

is different, although they express the same content. He argues that the difference in logical form is reflected in the impossibility to substitute the expression 'the concept square root of 4' from the second statement for the expression 'one square root of 4' in the first. Allegedly, unlike 'there is' [*es gibt*] in the first statement, the expression 'is satisfied' in the second statement is a first-level predicate, because a false but meaningful sentence results from substituting, e.g., the proper name 'Julius Caesar' for the expression 'the concept *square root of 4*'.

To be precise, the claim that the predicate 'satisfied concept' (and accordingly also the predicate 'easily attainable concept', which was already mentioned) is a first-level predicate is only found in the first draft of *BuG* (NS, 120). It is conceivable that Frege realized that his solution ends up ascribing a level-ambiguity to many predicates, depending on whether they are applied to objects or concepts. Presumably, this idea was no more appealing to Frege than the suggestion that objects may come under different types. In his correspondence with Russell he mentions (what he seems to regard as) a more acceptable solution to the effect that 'function' and 'concept' are second-level expressions and any attempt to apply them to something else inevitably leads to category mistakes. For instance, in a letter dated 29 July 1902 Frege writes: 'The concept of function must be a second-level concept, whereas in language it always occurs as first-level concept.'¹⁹ If Frege—as seems rather likely—takes the very peculiar objects corresponding to concepts to be the ordinary extensions of the concepts, then their existence is decisively called into question by Russell's paradox. Support for the 'extension' reading comes from several passages in Frege's *Nachlass*. What Frege presumably wants to avoid is an almost 'natural' consequence of Kerry's claims, i.e. that in extensional contexts the difference between a concept in predicate position and a concept in subject position can be drawn by having the latter replaced by a higher-level 'object' and then—employing a method that has meanwhile become standard—ascending from individuals to classes of individuals and then classes of classes, thereby also classifying objects in a way that essentially reflects the hierarchy of functions. However, the correspondence with Russell as well as the appendix to *GGA* II make it abundantly clear that Frege finds a classification of the objects in 'proper' and 'improper' unreasonable.

Frege's reply to Kerry is objectionable in several respects. First of all, he tacitly changes the topic: the very peculiar object corresponding to the concept is not the same as Kerry's object of the concept. By 'object of the concept' Kerry does not

¹⁹ Cf. Frege's article on the foundations of geometry (II), p. 372 n.: 'It is clear that we cannot present a concept as independent, like an object; rather it can occur only in connection. One may say that it can be distinguished within, but that it cannot be separated from the context in which it occurs. All apparent contradictions that one may encounter here derive from the fact that we are tempted to treat a concept like an object, contrary to its unsaturated nature. This is sometimes forced upon us by the nature of our language. Nevertheless, it is merely a linguistic necessity.' This remark is about Russell's *The Principles of Mathematics*. In §§480–3 of this book Russell discusses the controversy between Kerry and Frege.

mean the object that corresponds to the concept as its extension or that can be regarded as a proxy for the concept, but rather the object falling under the concept. It is easy to go wrong here, for the relation involved is between an *individual concept* and the object that *belongs* to it. Kerry's claim is that such an object must exhibit the very distinguishing marks belonging to the concept under which it falls. Since individual concepts are at stake, the function of designating through proper names and the function of describing through individual concepts cannot always be easily kept apart. Moreover, the objects in question are objects of an abstract sort, i.e. numbers, and the individual concepts at hand are generally specified by way of a definition.

The example of the concept 'horse' discussed at length by Frege in the first part of *BuG* plays only a minor role in Kerry's article. Kerry's basic idea is conveyed more clearly by the second example discussed by Frege. Indeed, Kerry generally sticks with the distinction between content [*Inhalt*] and object [*Gegenstand*] of the concept. However, as he concedes, there are cases in which the distinguishing marks of the concept and those of the object are identical. Kerry answers the question whether there are concepts 'that are such that their content [*Begriffsinhalt*] and their object [*Begriffsgegenstand*] contain strictly identical distinguishing marks' in the positive and cites the following example:

We take the number 4 to be the result of the additive combination of 3 and 1. The conceptual object [*Begriffsgegenstand*] of the concept thus specified is the individual number 4, a fully determinate number of the series of natural numbers. This object bears in itself the very distinguishing marks that are mentioned in its concept, and—if we refrain (as we probably have to) from ascribing to it as *propria* the infinitely many *relations* it bears to all the other individual numbers and which from a logical point of view have the status of theorems, since they are *derivable* from the definitions of numbers plus certain axioms—only those: "the" 4 is, likewise, the result of the additive combination of 3 and 1. Of course, if the coincidence of the distinguishing marks is not to be merely apparent, then what is true of the concept itself must hold true for the concept-parts [*Teilbegriffe*] of the concept under consideration. Those concept-parts must also be such that their conceptual objects [*Begriffsgegenstände*] contain exactly the same distinguishing marks as the corresponding conceptual contents [*Begriffsinhalte*]. And this is clearly the case in our example. (II, 424f.)

What Kerry is driving at in this passage is that, while the number 4 is completely characterized by the definition ' $3 + 1$ ' and the content of the concepts '1' and 'increasing by 1', the understanding of the individual concept 'the result of increasing 3 by 1' is not sufficient for acquiring genuine 'knowledge' of the number 4 as an object. Kerry likens the step necessary to reach knowledge to the transition from a nominal to a real definition. First, he wants to draw attention

to the simple and not further reducible nature of the logical relationship between content and object of a concept. Second, he wants to stress that the transition from a concept to its object presupposes a *sui generis* mental act. He thinks that the mental act involved is also the one that, when carried out, establishes a relationship between task and solution, expectation and fulfilment, order and execution.

Twardowski clarifies that Kerry's theories are best understood against the background of Bolzano's ideas. Indeed, Kerry partly aims at raising objections against Bolzano's view that not every component of the idea [*Vorstellung*] of a concept-word (or predicate) belongs to the idea of the parts of the objects falling under the concept. In Kerry's view, there are many cases in which the object of the concept must exhibit the same distinguishing marks (or parts) of the concept under which it falls. The Bolzanian claim he addresses is rather complex and it cannot be discussed here. Among other things, he raises objections against the dictum *nota notae est nota rei ipsius*, which on a given reading of the genitive ('notae') may indeed sound utterly absurd.

In his reply, Frege does not consider the Bolzanian background to Kerry's discussion. Instead, he presents his own theory. Frege rephrases the dictum as follows: (a) The object has as its own property that which belongs to the concept as a distinguishing mark, (b) that which can be said of the concept as a property cannot be ascribed to the object as a property. Generally speaking, the definite article does not refer to the property of an object, but rather to the property of a concept. The relationship between an object and its properties is different from the one between a concept and its distinguishing marks. Rather, it resembles the one between a concept and a higher-level concept under which the former falls. What is the difference? Part of the difference is that the relationship between a concept-word and the sense of its components is a part-whole relationship, whereas the relationship between a concept and the object falling under it is not of that kind.

Frege characterizes the concept as a function. The advantage of this choice (among others) is that it avoids the difficulties one is bound to face when the part-whole model is carried over from the level of sense to the level of reference. In 'On Sense and Reference' (*SuB*, 35f.), Frege does indeed mention the possibility that components of the truth-value may be distinguished and he helps himself to the 'part-whole' model in order to illustrate the idea of compositionality; however, later on he acknowledges that there is an essential difference between the part-whole relation and the functional relation holding at the level of reference. For instance, we cannot say that Sweden and being-the-capital-of-something are components of Stockholm.²⁰ The only level at which the 'part-whole' model can

²⁰ NS, 275: 'We can regard a sentence as a mapping of a thought: corresponding to the whole-part relation of a thought and its parts we have, by and large, the same relation for the sentence and its parts. Things are different in the domain of reference [*Bedeutung*]. We cannot say that Sweden is a part of the capital of Sweden' (trans. Long/White, slightly modified).

be profitably employed is apparently that of the linguistic expressions and of sense. For it is entirely open to us to say that in the concept-word 'positive whole number larger than two' different parts have to be distinguished, each of which corresponds to the expressions 'whole number', 'positive', 'larger than two' respectively, and by means of this explication we can show how the sense of the concept-word is composed of the sense of its components. However, if the parlance of parts is carried over to the concept itself and its distinguishing marks, then it remains unclear what exactly we mean by 'parts'.²¹

Yet, for all their imprecision, Kerry's examples can be read in a way that should have aroused Frege's attention, especially because he brings into play two radically different kinds of objects of a concept [*Begriffsgegenstände*]: on the one hand, a logical object corresponding to the expression 'the concept *F*', on the other hand, the individual object falling under a concept. In his article Frege does not draw a sufficiently clear distinction between two uses of the definite article, that is between its use as a description operator and its purported use in forming names for value-ranges. Kerry's exposition suffers from the same shortcoming, for he states that the 'belonging' [*Zugehörigkeit*] of the object to the concept includes as a special case also the relation between the content of the concept and the *totality of all objects of the concept* [*Inbegriff aller Begriffsgegenstände*],—what is usually called the ("empirical") extension of a concept' (II, 460).²² That Frege was not discomfited by such an ambiguity in the idea of the object of a concept [*Begriffsgegenstand*] may be due to his staggeringly comprehensive concept of object [*Gegenstand*], which is such that everything that is not a concept is an object.

Bearing the ambiguity in mind, it is not difficult to come up with examples that at first glance satisfy the conditions set out by Kerry. Here one may think of concepts that can be predicated of themselves, that is concepts with distinguishing marks that also belong to them as their properties. In this particular respect, Kerry's example is somewhat ill-chosen. However, it is easy to find better ones: it is enough to think of autological and heterological predicates. But also the question whether the extension of the concept under which all self-identical objects (and only those) fall is a self-identical object would already suffice. Had Frege given thought to the peculiar properties of the extensions of some predicates instead of insisting on the difference between distinguishing marks and properties of a concept, it is conceivable that he may have discovered Russell's paradox.

²¹ A classic discussion of the ambiguity of the term 'distinguishing mark' [*Merkmal*] in representative works of logicians of the nineteenth century is found in Twardowski 1894, §13.

²² Cf. I, 463: "There is a dependency between concepts and multiplicities [*Vielheiten*] owing to the fact that every concept has an extension (which, incidentally, can also be 0); assuming it is known *what* (sort of) "objects" are intended, the multiplicity of the objects falling within the extension of the concept is the multiplicity "belonging" to the concept, and the inextricable link [*Zusammengehörigkeit*] between concept and multiplicity is the first and one of the most important relations among those adumbrated." In the footnote to this passage Kerry refers the reader to Bolzano, Ueberweg, and most notably to Frege.

However, instead of directing his attention to the similarities and dissimilarities between the falling of an object under a concept and the falling of a concept within a higher-level concept, Frege focuses on the differences between distinguishing mark and property. In the course of his reply to Kerry's proposal he appeals again to the distinction between properties of the object and distinguishing marks of the concept and stresses that what belongs to the concept as a distinguishing mark also belongs to the object as a property. Nevertheless, even discounting the difficulties already mentioned, Kerry's example strikes a nerve with Frege: he raises problems of which Frege was well aware and which he experienced first-hand. That being said, Kerry's exposition is far from being a paradigm of clarity and it raises several psychological and epistemological questions. Even so Frege need not have had such difficulty in recognizing problems he had already come across himself.

For instance, in the *GLA* (§53) he claims that existence and uniqueness are not mere properties of first-level concepts, but rather that, analogously to what holds for objects, there are also second-level concepts having as their distinguishing marks that which first-level concepts falling within them have as properties. Here one may think that this hierarchy can be continued; yet, it is cut or 'reduced' via the introduction of equivalence classes of second-level concepts. As was already indicated, Kerry's reservations are about (a) the existence and uniqueness of such universal concepts, and (b) the claim that cardinal numbers [*Anzahlen*] are extensions of these concepts.²³

Nevertheless, in his discussion of the difference between the 'result of the additive combination of 3 and 1' and the 'numerical individual / individual number 4' Frege manages to do justice to some aspects of Kerry's reflections. Frege writes:

Accordingly, one has to assume that Mr. K[erry] calls the content of the words 'the result of the additive combination of 3 and 1' concept. I would agree with him if the definite article in front of 'result' were missing. I think Mr. K[erry] will likewise declare the words just mentioned without the definite article to be a concept, so that in this case we would completely agree. And yet, according to K[erry] how would concepts corresponding to entire words with the definite article differ from those corresponding to the same words without the definite article? (NS, 110)

²³ Furthermore, it must be observed that in his discussion of Kerry's example Frege regards the exclusion of 'the infinitely many relations which [the object] bears to all individual numbers' from the *propria* of number 4 as obviously warranted. However, one may wonder how exactly in the logicist definition of cardinal number (or in the case of Peano axioms) one can avoid considering the infinitely many relations an object bears to other objects as properties of the object in question. A natural number is an object belonging to *every* series that begins with 0 and is such that, if it contains an element, then it also contains its successor. It seems to me that the infinitely many relations an object bears to other objects are as it were already implicitly included in the requirement that Peano axioms be categorical. As has been shown by several authors, Peano axioms can be established on the basis of the *GLA*.

Frege thinks that Kerry's example concerns an identity statement, and hence that the words 'the result of the additive combination of 3 and 1' do not designate a concept, but rather an object as long as it is not explicitly allowed that concepts can be different even if their distinguishing marks are the same—a possibility that is still open to Kerry. The definite article cannot indicate a distinguishing mark, but rather only the property of a concept; this is the only respect in which the expressions 'the result' and 'a result' differ. However, the following passage contains yet another characterization of the function of the definite article that has to be taken into account:

Rather, the view that the substitution of the indefinite by the definite article involves the transition from a concept to an object falling under it (provided that there is only one such object) appears to be by far more adequate. Thus I do not follow Mr. K[erry], who says that the object has the distinguishing marks of the concept likewise as its distinguishing marks, but I rather say that the object has the distinguishing marks of the concepts as its own properties. That's what the difference comes down to. For the relation between the distinguish mark and the concept is different from the relation between property and objects (cf. *GLA*, §53). (NS, 113)

Such a description is in line with the way Frege introduces the description operator in the *GGA*, whereas the reference to properties is rather along the lines of *GLA*, where uniqueness is defined as a property of concepts and thus not as an operator picking out from singletons their only element. If there is no such element, the description operator takes its argument as value, that is to say the corresponding extension of the concept. This solution shows how with the aid of the definite article referential success can *always* be secured, although what is referred to in this way may not be the intended referent. Apparently, this is exactly the possibility Kerry wants to rule out. Overinterpreting a bit, one could perhaps say that, according to Kerry, the mere use of the definite article is not enough to guarantee that the concept should have an object [*Begriffsgegenstand*] of whatever kind; his view is rather that the behaviour of the definite article is more accurately portrayed by a partial function. A further solution is mentioned by Russell in the context of this theory of descriptions. Frege's description operator is idle for those who do not accept classes. For, unlike the treatment of descriptions proposed in the *GLA*, the one in the *GGA* depends on the existence of class-like entities, i.e. singletons, from which each of their unique elements can be taken. In the posthumous piece 'On Schoenflies: *Die logischen Paradoxien der Mengenlehre*' Frege deals again with the definite article in the context of his discussion of paradoxes.

Something we need to understand is why Kerry regularly insists that, even when an object is known to fall under the concept, one should make a distinction

between content and object of a concept, in order to reify the corresponding concept and be able to treat it as an object as well. Understanding why he thinks that requires spelling out his general conception in more detail.

The most important target of Kerry's attacks is Kant's view that mathematical propositions are synthetic *a priori* judgements and the latter's idea that the intuition of time fulfils a task that is connected with fundamental 'work' of the mind of a specific kind. Particularly salient among the relevant mental operations are the act of positing [*Setzen*] units and the act of indirect presentation [*indirektes Vorstellen*]. Very roughly, one could say that the notion of indirect presentation is not too dissimilar from Russell's idea of knowledge by description. If the relevant description involves a functional relation, we can be sure that there is an individual object satisfying the description (e.g. 'the father of Socrates', 'the successor of 2'). Kerry claims that, for instance, numbers greater than 10 are not intuitively given, but rather only through indirect presentation. However, in his view, the existence of objects given in this way remains merely hypothetical. For the question still arises how we can be sure that objects given to us through indirect presentation exist as a matter of fact. Apparently, the only answer we can come up with here rests upon the conscious deployment of our capacities as well in the ability to judge whether *the* correct result of a mental activity has been actually achieved.

It is interesting to observe that Kerry agrees with many of Frege's claims and exploits them—except that Kerry puts them to a use different from that of their author. A few examples are the claim that arithmetic is analytical, the condemnation of Kant's conception that the pure intuition of time has a role to play in arithmetic, the rejection of the formalistic theories, which identify numbers with numerals [*Zahlzeichen*]. Kerry sees the last mistake already in Kant's idea that the mathematical sign is a construction prop. Worth mentioning here is also the claim that consistency is no guarantee for the existence of objects falling under the relevant concept as well as the idea that natural numbers are not predicates, but 'individuals' [*Individuen*].²⁴

All of these ideas are conducive to the hypothesis that a clear distinction has to be drawn between hypothetical and categorical arithmetic. Hypothetical arithmetic alone is not sufficient to give us the numbers. 'Supplementary judgements' [*Ergänzungsurteile*] are needed in addition in order for us to arrive at them. These are existential judgements that fulfil by and large the same task as postulates in

²⁴ Particularly interesting is Kerry's outline (and rejection) of the conception of negative and rational numbers as syncategorematic expressions, 'insofar as they do not independently designate the content of a presentation [*Vorgestelltes*], but are capable of signifying only as components of judgements of a certain kind. If that is the case, then the only legitimate question is about the sense and truth of these judgements and the question about the reality of the *concepts* of those numbers does not arise any more' (V, 102 n.). There is a striking similarity between the 'syncategorematic' approach and Frege's context principle.

Euclid: they guarantee that certain constructions are possible [*ausführbar*]. The following statements exemplify such existential judgements: ‘There is such a thing as a 1’, ‘there is only one result of the increasing by 1’, ‘there is the result of the addition of 1’. Kerry clearly does not realize the close connection between these existential judgements, though. Unlike the axiomatic conception that is usual nowadays, Kerry does not establish a systematic connection between his existential judgements. Not only does he question whether the logicist definition of number suffices to guarantee the existence of an infinite set, but he even questions the idea that this definition is enough to ensure the existence of finite sets. Kerry’s interpretation of the axioms concerning the infinity of the series of numbers is also peculiar: he says that they do not refer to the object of concepts like the ordinary axioms, but rather to our capacity to carry out psychic work.

According to Kerry, the construction of categorical arithmetic must be based on psychology. Hence, it must take into account the psychic work that is done in the psychic acts underlying our grasp of numbers. The psychic acts that are essentially required for grasping the distinguishing mark ‘numerical’ [*anzahlmäßig*] are, firstly, the position of the unit [*Einheitssetzung*], which by and large consists in grasping the object as something that is subsumed under a concept; secondly, the ‘indirect presentation’ of numbers; thirdly, the comparison of small sets, which for Kerry constitutes something like a primordial fact [*Urtatsache*]. It is no accident that he has to introduce a distinction between small and large numbers to the effect that the small ones are given by direct presentation and the large ones by indirect presentation. As for the performance of the aforementioned acts, time plays no role in it. They are carried out via some kind of internal reflection—a notion that Kerry borrows from Locke. It is therefore hardly surprising that in the first draft of *BuG* Frege traces back the ‘malady of the times’ [*Zeitkrankheit*], i.e. psychologism, to Locke, Berkeley, and Hume.

Unfortunately, Frege does not address an aspect of Kerry’s criticism, that is, the latter’s questions about the former’s nonchalant use of the notion of extension of a concept [*Begriffsumfang*] as well as the identification of numbers with these very extensions. It is very odd that in *BuG* Frege restates the remark found in his *GLA* that he does not attach any special significance to the extensions of concepts, considering that by this point he had already completed the first volume of the *GGA*. This remark can only partly be justified by the extensional conception of concepts Frege developed in those very years, which—as is clearly stated in the review of Husserl’s *Philosophy of Arithmetic*—says that two predicates designate the same concepts if the corresponding functions each return the same value for the same arguments. Yet, without the additional assumption that it does not make any difference whether one speaks of the extension of the concept ‘equinumerous with the concept F ’ or of the concept ‘equinumerous with the concept F ’, one would not be confronted with numbers as ‘objects’, but rather with a number of second-level concepts, which Frege addresses at the end of the *Notes for Ludwig*

Darmstaedter, and which are not identical with cardinal numbers [*Anzahlen*], as Frege himself is willing to concede:

These second-level concepts form a series and there is a rule in accordance with which, if one of these concepts is given, we can specify the next. But still we do not have in them the numbers of arithmetic; we do not have objects, but concepts. How can we get from these concepts to numbers of arithmetic in a way that cannot be faulted? Or are there simply no numbers in arithmetic? Could the numerals help to form signs for these second-level concepts, and yet not be signs in their own right? (NS, 277; tr. Long/White)

In order to treat numbers as objects one has to presuppose that the definite article in front of the term ‘concept’ turns the latter into the name of the extension of the concept. A footnote to the first draft of *BuG* clearly confirms that this is what Frege thinks. It says: ‘I regard the question whether one should say “the concept” instead of “the extension of concept” as a matter of convenience [*Zweckmäßigsfrage*]’ (NS, 116). Kerry denies precisely this idea.

In conclusion, I want to claim that in his reply Frege focuses on the least interesting aspects of Kerry’s criticism. Most of his answers are contrived and not really convincing. Nevertheless, this does not make his distinction between distinguishing mark and property any less clear and it does not detract from his justification of the emphasis on the predicative nature of concepts. Yet, these are not the points that matter most to Kerry in his remarks on Frege.

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Sigwart, Husserl, and Frege on Truth and Logic, or Is Psychologism Still a Threat?

In §38 of his *Prolegomena zur reinen Logik* which precede the *Logische Untersuchungen* Husserl attributes to Christoph Sigwart's *Logik*¹ the dubious merit of having contributed more than any other work of the time to fostering a psychologistic approach to logic. 'Anthropologism' is the name which Husserl reserves for the brand of sceptical relativism which informs Sigwart's approach to logic. Husserl contends that to the psychologistic philosopher only a relativistic notion of truth is available: truth is made to rest on subjective certainty and no room is left for the idea that there are ideal truths which might transcend our apprehension of them. Along similar lines Gottlob Frege had in the first volume of his *Grundgesetze der Arithmetik* of 1893 criticized Benno Erdmann's *Logic*: idealism and solipsism are, according to Frege's diagnosis, the inevitable consequences of all philosophical approaches to logic which tend to obliterate the gap between being true and being held true. In his opinion, an instance of such a strategy was the attempt, on the part of psychologistic logicians, to explain the meaning and epistemological status of logical laws by appealing to the psychological processes which go on in the head of 'normal' representatives of the species *homo sapiens* at a given stage of its evolution.

Martin Heidegger, in his dissertation of 1913, *Die Lehre vom Urteil im Psychologismus*, employs the term 'naturalism' to indicate a general tendency underlying all psychological treatments of logic, which he identifies in the renunciation of the claims of *a priori* knowledge in favour of an empirical approach, based on the findings of experimental psychology. Heidegger's book can be seen as an updated report on the psychologistic epidemic, described in Husserl's

¹ Christoph Sigwart's *Logik* was first published in 1873 (volume 1) and 1878 (volume 2). The third and fourth editions (published in 1904 and 1911, respectively) were edited by Heinrich Maier, who added to the text his own set of footnotes. In the preface to the first edition Sigwart declares his debt to the works of Mill, Ueberweg, and Trendelenburg. Among the works of kindred spirits mentioned by Sigwart in the preface to the second edition are Windelband's studies on negative judgement, Meinong's treatment of relational concepts, and Volkelt's book *Erfahrung und Denken*. The second edition of Sigwart's *Logik*, published in 1889–93, was translated into English in 1895 by Helen Dendy; it contains a preface by Sigwart to the English translation and a number of appendices, which were either dropped or incorporated into the text of the later two German editions. In what follows I shall quote from Dendy's translation, but I shall also give page references to the German reprint of the fourth edition of the *Logik* of 1921. It is important to bear in mind, however, that the fourth edition differs considerably from the second edition on which the English translation is based.

Prolegomena, which had befallen German philosophy in the second half of the nineteenth century. Appropriately, a chapter of Heidegger's work is devoted to Heinrich Maier's book *Psychologie des emotionalen Denkens*, published in 1908. Maier was a disciple of Sigwart and the editor of the third and fourth editions of Sigwart's *Logik*. Heidegger's criticism is directed at the theory of elementary judgements put forward by Sigwart and endorsed by Maier. It by and large echoes Husserl's criticism of Sigwart's conception of logic as a *normative art of thinking*.

The reason why Sigwart's normative conception of logic should encourage slipping into psychologism is one of the two questions that I should like to explore in what follows; the other is the treatment of elementary judgements offered in the first volume of Sigwart's *Logik*, for a study of this part of Sigwart's doctrine may afford a better understanding of what the threat of psychologism amounts to as far as linguistic *meaning* is concerned. I have argued elsewhere that Frege took the chief defect of psychologism to be that it suggests a mistaken picture of language (natural as well as formal), which turns the objectivity of thoughts into a riddle: by emphasizing the psychological processes whereby we arrive at our judgements psychologistic treatments of logic make us lose sight of the fact that what needs to be done first is to form an adequate conception of their *contents*.² Accordingly, in the first parts of my paper I shall concentrate on Husserl's criticism of Sigwart, while in the later parts I shall speculate on what Frege might have objected to in Sigwart's conception of a judgement and its content. Although Frege never referred to Sigwart in his published work, we know that he was acquainted with Sigwart's *Logik*. Unfortunately, only a small part of Frege's *Nachlass* survived the war, and the notes he took on Sigwart's *Logik* no longer exist.³ Before turning to Sigwart's work, it may be appropriate to reflect briefly on the significance of the controversy over psychologism for present-day philosophy.

Is Psychologism Still a Threat?

In his introduction to the volume *The Revolution in Philosophy* of 1956 Gilbert Ryle singled out as distinctive features of analytical philosophy the revolt against psychologism and the employment of the techniques of modern mathematical logic to approach the problems of meaning. Perhaps it is not a mere chance that both these two revolutionary moves are due to the genius of Gottlob Frege. Frege recognized that the account of thought must be given priority over that of psychological acts of thinking. Since, as he emphasized, human beings grasp

² cf. Picardi (1996). I have speculated elsewhere (Picardi (1987) and (1994)) on what in Sigwart's *Logik* might have aroused Frege's interest.

³ As far as Sigwart is concerned we have only the testimony of Heinrich Scholz; Scholz's list of the contents of Frege's *Nachlass* is published in Veraart (1976).

thoughts only via the medium of language,⁴ it follows that an account of the content and structure of our thoughts can proceed only via the analysis of language. Frege's theory of sense and reference of linguistic expressions (proper names, predicates, logical particles) was the first attempt in the history of philosophy to lay down a *semantics* for a (formalized) language. The concept of truth provided for Frege the key to our understanding both of the thought expressed by a sentence and of the linguistic act of assertion, in performing which we share our knowledge with others. Only after we have formed a picture of the logical structure of sentences can the question be raised whether a purely *logical* justification of the propositions of arithmetic is possible. This is, as is well known, part of the project of a logicist foundation of arithmetic which Frege shared with Russell.

Frege's work did not receive at the time the attention it deserved, neither on the part of logicians nor on the part of philosophers. Indeed, the merit of having defeated psychologism was chiefly ascribed to Edmund Husserl, who in his *Prolegomena* had exposed in detail the many pitfalls of psychologistic treatments of logic. In their *pars destruens* the arguments put forward by Frege and Husserl are very similar; they differ in their *pars construens*. Frege's philosophical insights are inseparable from his invention of a *Begriffsschrift*, a method for representing the logical structure of complex sentences. In a sense Frege's (and Russell's) achievements made obsolete Mill's conception of logic, to which the logic books of Wundt, Erdmann, and Sigwart are indebted.

And yet, there is a sense in which the claims of psychologistic philosophers are not made obsolete by the new logic of Frege and Russell. For their chief ambition was to attempt an approach to the theory of knowledge (which they misleadingly called 'logic') by drawing on the findings of experimental psychology. Quine's ideal of naturalized epistemology can, at least in part, be seen as heir to the claims of psychologistic philosophers. As is well known, Quine's programme of naturalized epistemology comprises two theses: the first is the claim that the role once played by the theory of knowledge should be taken up without residue by the science of psychology; the second thesis is semantical, and intimates that questions of truth, evidence, and ontology are in effect questions concerning language conceived as a social art. It is well to bear in mind that the second part of Quine's naturalistic programme would be unthinkable without the logical achievements of Frege and Russell. Thus, even if, as some suggested,⁵ the time is ripe for reassessing the claims of proto-naturalistic epistemologies, one should not make the mistake

⁴ In a piece of his *Nachlass*, dated 1924, Frege (1983) surmises that there might be beings who, unlike us, can grasp thoughts without the sensible medium of language and without any accompanying idea or representation and concludes: 'The connection of a thought with one particular sentence is not a necessary one; but that a thought of which we are conscious is connected in our mind with some sentence or other is for us men necessary' (p. 288; Engl. transl. p. 269).

⁵ cf. for instance Kusch (1995).

of supposing that the conception of logic (properly so called) of the proto-naturalists stands any chance of revival. When for instance, Kitcher (1992) speculates on a comeback of the old naturalists, his prognosis has to be taken *cum grano salis*. Although the present philosophical climate, dominated by the so-called cognitive paradigm of linguistics and artificial intelligence, resonates with the aims and claims of the psychologistic philosophers of the turn of the century, it is well not to lose sight of the very substantial difference between our times and theirs.

Quite independently of the issue of naturalism, there are several aspects of Frege's legacy which have proved controversial. Many analytic philosophers have called into question the thesis of the relative priority of language over thought. It should be possible—they argue—to offer an account of what thinking a particular thought consists in, or what is involved in the possession of a given concept, which does not take as its starting point the *use* which the speakers of a given language make of the means of expression available in their language for articulating such a concept. Another standard complaint, often voiced, is that adherence to a style of analysis adapted to the requirements of logic has encouraged the neglect of the demonstrative and indexical dimension implicit in the use of natural language. A full appreciation of what is involved in the understanding of personal pronouns and indexicals demands that more attention be given, say, to psychological mechanisms involved in self-location in space and time or to the processing of perceptual information available also to dumb animals and not just to language users. Even if we consider the problem of demarcating logic proper from psychology as settled, this does not mean that we should consider Frege's (or Husserl's) characterization of what is psychological as satisfactory. Frege's identification of the psychological with what is private and hence in principle incommunicable is questionable. Frege's sharp contrast between the subjective and the objective leaves out—as Dummett (1993) has pointed out—the whole range of the intersubjective. Moreover if the only remedy against psychologism were to take refuge in a third realm of thoughts and their ingredient senses, as Frege recommended in *Der Gedanke*, one might almost feel nostalgia for the jungle of psychological acts of representation, creative apperception, double and triple synthesis, psychological abstraction, etc. to which psychologistic logicians resorted to explain how we construct our thoughts out of simpler ideas and concepts. The psychological 'I' has probably no essential role to play in logic, but a philosophical account of the contents of knowledge, belief, and desire which disregards the circumstances that they must be *thinkable* by a human *agent* is seriously defective.

In sum, the general arguments against psychologism employed a century ago by Frege and Husserl seem more than ever in need of a critical reassessment. A first step in this direction is a better understanding of the claims of their psychologistic opponents, which, in its turn, might help to highlight the weakness of certain assumptions shared by both parties.

Logic and Judgement

Sigwart's *Logik* divides into three parts: the first part investigates what its author considers the fundamental function of thinking, viz. judgement; the second is devoted to the conditions and laws of thinking in its 'normal' (i.e. governed by norms) exercise; the third offers a survey of the procedures by means of which untutored thought can be improved upon by means of explicit instruction. Accordingly, Sigwart's logic comprises an analytical, a normative, and a technical part. The normative deals with immediate and mediated judgements and their conditions of justification. This normative part is preceded by an account of what concepts are (in the metaphysical, logical, and empirical sense) and ends with a survey of syllogistic inference. The technical part deals with the canons of inductive method; the most interesting section of the third part is the one devoted to the application of the methodological principles formulated in the second part to the empirical science of psychology. In what follows I shall confine myself to the first two parts of Sigwart's *Logik*.

Sigwart conceives of logic as a corpus of general rules which prescribe how to conduct one's thought so that it might attain knowledge of what there is (*Erkenntnis des Seienden*). The Aristotelian conception of being and the metaphysical realism connected with it are something to which Sigwart is strongly attracted. Striving toward objectivity is in his opinion the deepest craving in all our thinking. However, he denies that, from the standpoint of logic, we can offer a conclusive answer to the sceptic or to the idealist, who ask whence we derive the assurance that in 'subjective' thought we attain true knowledge of being *qua* being and that our intersubjective agreement is grounded in objective features of reality (*Logic*, §1, pp. 7–8; §14, pp. 105–6). Logic alone can tell us nothing about the *metaphysical* validity of our judgements, for it investigates thought as a *subjective* function. Perhaps it is this aspect of Sigwart's philosophical outlook which earned him the epithet 'sceptical relativist' on the part of Husserl, although in his *Prolegomena* he does not touch on that aspect. (Perhaps Husserl himself was not in the best position to criticize Sigwart's suspension of judgement as to whether in our judgements concerning the external world we attain objective validity.) Since logic cannot rely on first truths, the discovery of which—if any—Sigwart assigns as a task to the theory of knowledge, it can be only formal logic. A logic which does not rest on epistemological foundations must of necessity assume a hypothetical shape: all the logic allows us to claim is that if such and such is true, then so and so will also be true. In short, logic is committed to a form of 'if-thenism', for there are no fundamental propositions whose truth can be taken for granted and which could be used as axioms or first truths, from which other truths could be derived.

In assigning to logic a purely formal task Sigwart does not mean to confine logic to a mere algorithmic technique:

We can abstract neither from the particular manner in which our thoughts receive material and contents from sensations and form them into ideas of things, nor yet from the way in which its history has been determined by the community of mankind. We abstract only from the particular nature of the occasions which at any given time give rise to a series of mental phenomena.

(*Logic*, p. 14; *Logik*, p. 15)

Needless to say, both the idea that logic should take into account the actual processes of reasoning and that thoughts have a history which has been determined by the community of mankind were deeply at odds with Husserl's (and Frege's) conceptions.

Sigwart characterizes thinking as a 'Denkenwollen', i.e. as a spontaneous and deliberate activity, which presupposes, however, mental processes which are not accessible to consciousness and which supply the raw material of percepts. Thoughts are the product of an 'innere Lebendigkeit des Vorstellens' ('an inner liveliness of representation'): as ideal subjective formations they differ from the objects which in perception are presented to us as real. All our cognitive activity finds its completion in judgements:

All such thought results finally in judgements, which are expressed inwardly or outwardly as propositions. Every practical consideration of ends and means terminates in judgements, all knowledge consists in judgements; every conviction ends in judgements. All other functions come into consideration only as being a condition of and preparing the way for judgement. Again the judgement can be the object of a scientific investigation only in so far as it is expressed in a proposition; only through the proposition can it become an object of general consideration, and only as a proposition can it aspire to universal validity.

(*Logic*, p. 9; *Logik*, pp. 8–9)

For Sigwart the *activity* of judgement is the driving force in the *production* of thoughts,⁶ which, as we shall see, he refused to conceive as ideal entities outside

⁶ To Frege this eulogy of judgement must have appeared highly suspect. He viewed the emphasis which philosophers laid on the *activity* of judgement as conducive to a form of idealism: 'All psychological treatments of logic arise from the mistaken belief that a thought (a judgement as it is usually called) is something psychological like an idea. This view leads necessarily to an idealist theory of knowledge... Now, since every act of cognition is realized in judgements, this means the breakdown of every bridge leading to what is objective' (Frege (1983) p. 155; Engl. transl. pp. 141–2).

space and time as Husserl had suggested in his *Prolegomena*: to hypostasize sentences as independent essences is sheer mythology.⁷

Sigwart characterizes assertion, viewed as the outer expression of judgement, as the act in which a speaker puts forward a truth claim with the conviction of its objective validity and requires that it be believed by the hearer. As he correctly observes, the definition of judgement as a connection of ideas or concepts ‘overlooks the fact that the import of a statement is never merely to assert the subjective fact that I, at the moment, make such a connection. On the contrary, the judgement by the form it assumes, claims that the connection is true of the objects themselves, and that just for this reason it must be acknowledged by every one’ (*Logic*, 79; *Logik*, pp. 105–6). Moreover, Sigwart holds that every judgement contains the implicit presupposition that the speakers use words according to the conventional meanings that they have in the language, otherwise we could never establish whether, in a given instance, we are confronted with a verbal error or with a substantial error.

Logic is not only regulative. It also presupposes a ‘Wahrdenkenwollen’, i.e. the *will* to think in such a way as to achieve truth. For this will to succeed, we must assume that the following ‘postulate’ is satisfied: that we can with immediate evidence recognize the thought which is objectively necessary and set it apart from that which is not. While leaving open the possibility that our thoughts might not achieve metaphysical validity, still in our subjective exercise of thinking we all experience immediate certainty when we are confronted with certain syntheses of concepts and we ‘should thus be forced to exclude from the community of thought those who should arrive at a different result. In other words, the judgement has objective validity for us because it is necessary to unify ideas which coincide’ (*Logic*, 82; *Logik*, p. 108). No wonder that Husserl will criticize Sigwart for confusing psychological certainty with the evidence we experience when confronted with the ideal truths of logic and mathematics.⁸

Laws of Thought and Laws of Logic

The reason why we need a logic at all, according to Sigwart, is that sometimes thought misses its target. Such failures prompt the search for ‘Normalgesetze des vollkommenen Denkens’: to conceive of logical laws as normative laws of

⁷ cf. C. Sigwart, *Logik*, p. 24, footnote (added to the third edition).

⁸ And no wonder that Sigwart in the third edition of his *Logik* (p. 25) blames Husserl for succumbing to a form of mysticism in postulating two forms of consciousness, an empirical and an ideal one, capable of intuiting atemporal truths. Sigwart also finds an inconsistency between Husserl’s declared anti-psychologism and his characterization of the certainty of logical laws as an ‘Erlebnis’ as well as his appeal to the ‘Evidenz’ with which truth reveals itself to (an ideal) consciousness. An ‘Erlebnis’—Sigwart remarks—is a psychological fact, and ‘Evidenz’ is a state of mind which can be experienced only in time: why make such a fuss when *au fond* agreement prevails?

thinking, with the attendant reduction of logic to an *art* (a *Kunstlehre*) of thinking, is a natural conclusion of this approach. This conclusion is found unpalatable by Husserl, for, in his opinion, it entails a confusion between the *ideal* character of logical propositions and the *normative* function which they are supposed to fulfil in guiding and checking actual reasoning. It looks as if inferential (deductive) practice could be studied from two different perspectives: one of psychological causation, governed by psychological laws, and the other of logical justification, governed by ideal laws, intuitable by means of acts of evident judgement. This division of labour between logic and psychology is far from being satisfactory, but the reasons why this is so are not so easy to discern. Sigwart, too, took it for granted that there are on the one hand psychological laws of thinking, and on the other logical laws of thought: however, he drew the distinction between what belongs to logic and what belongs to psychology with reference to the notion of truth, thus incurring criticism of a different kind from Husserl.

In his *Logik* of 1897 Frege raises the question whether it might be worthwhile to investigate the question whether psychological *Denkgesetze* and logical *Denkgesetze* are in harmony with each other, but then he warns the reader against embarking on such an inquiry.⁹ Such an investigation is doomed to failure and is always on the brink of confusing causal explanation and logical justification. What he deems advisable is a reform of terminology. Logical laws should not be called *Denkgesetze*, for in so doing we engender a double confusion: we encourage the assimilation of logical laws to psychological laws and fail to do justice to the nature of logic itself.

To be sure, there is nothing amiss with the attempt to find logical techniques which increase the efficiency and security of deductive reasoning: there is no particular threat of psychologism in this, for in this kind of research the validity of logical principles is taken for granted, whereas robust psychologism tries to *explain* logical validity with reference to the psychological make-up of the mind. The objection that in so doing the psychologist logician already presupposes the validity of the logical principles that he attempts to justify empirically did not seem to bother the stoutest upholders of psychologism, such as Theodor Lipps or Gerardus Heymans.

Given Sigwart's equation of logical laws with laws of thinking, it was quite natural for him to describe logic as a body of instructions on how to conduct one's thought in such a way that its formal correctness be guaranteed. In this respect, logical laws are more akin to propositions of ethics than to propositions of physics. Sigwart's position differs from that of philosophers like Theodor Lipps and Gerardus Heymans, who defended a full-blooded psychologism according to which logical laws are *identical* with psychological laws, logic being thus conceived

⁹ Frege (1983) pp. 158f.; Engl. transl. pp. 146f.

as part of psychology.¹⁰ The normative conception defended by Sigwart is more diplomatic: there are psychological laws which form the subject matter of psychology, whereas logical laws prescribe how to conduct one's thought in such a way that it may reach its cognitive goal. Moreover, although Sigwart says that logic must take into account how actual reasoning works, he never claims that the ultimate justification of logical principles rests on the psychological constitution of the human mind. Like many philosophers of the time (Frege included) Sigwart draws a parallel between logic and ethics—logic being the ethics rather than the physics of thought. What in Sigwart's opinion sharply distinguishes logic from psychology is that the contrast between true and false plays no role in the latter, whereas it is paramount in the former (*Logic*, p. 10; *Logik*, p. 10).

Wahrdenkenwollen and Wahrsein

Sigwart's contention that in psychology the contrast between true and false has no role to play is open to criticism. All sciences—so the standard objection goes—have as their final goal the establishment of truths, which, once they are recognized, serve to correct errors and to direct further enquiry. Both Husserl in his *Prolegomena* and Johnson in his *Logic* raise this kind of objection against the normative conception.¹¹ In their opinion, logical axioms have the form of imperatives no more than geometrical axioms and physical laws. However, the really critical element of Sigwart's normative characterization is the connection he draws between logic and the will: judgement, as we saw, presupposes the *will* to think in accordance with truth, and logic, being concerned with judgement, must take this volitional element into account in a way in which psychology need not. Heinrich Maier, who in his own work stresses the volitional element of Sigwart's characterization of judgement, will claim that ultimately our *Wahrdenkenwollen* rests on a *Sollen* of a non-cognitive kind.

W. E. Johnson, who is a kindred spirit with Sigwart, and in his *Logic* elaborates certain aspects of Sigwart's doctrines,¹² points out that, in applying the distinction

¹⁰ Sigwart never indulges in arguments of this kind. For instance, he correctly stresses that the application of logical principles presupposes that the concepts to which they relate have sharp boundaries, and this, in its turn, requires a certain amount of idealization and definitional activity. He also puts forward an interesting suggestion as to the relation between the meaning of logical particles and the logical principles which govern them: the principles of non-contradiction and of double negation determine (in the sense of being *constitutive of*) the meaning of negation. From the principles of double negation and of non-contradiction the law of excluded middle can be derived, and need not, therefore, be assumed as a primitive logical principle. All these observations, however, do not form a systematic picture.

¹¹ Johnson's *Logic*, which comprises three volumes (published, respectively, in 1921, 1922, and 1923), is written in the spirit of the Millian tradition. It is strikingly old-fashioned compared to the logic which was being done at that time (and place) by Russell and Ramsey.

¹² *Ibid.*, ch. 10. Here Johnson elaborates on Sigwart's distinction between narrative, existential, and subsstantial propositions.

between the positive and the normative to the sciences which deal with 'psychological material', we have to distinguish two senses of normativity: '... the positive or descriptive treatment of mind—in its thinking, feeling or acting aspect—is (like all sciences) normative in the sense of being potentially corrective of false judgements on the topics directly dealt with; while the treatment in Logic, Aesthetics and Ethics of these same processes is normative in the more special sense that these sciences examine and criticize the norms of thought, feeling or action themselves.'¹³ And he correctly goes on to point out that one should not confuse causal description with evaluation of standards. Once these two notions of normativity have been distinguished, we are in a position to appreciate whence the imperative character of logical principles derives: it is due to the circumstance that logic deals with mental facts considered in relation to certain norms and standards. 'Truth' and 'falsity' carry with them a reference to an imperative, and hence to a standard:

The idea of truth and falsity, in my view, carries with it the notion of an imperative, namely of acceptance and rejection—a corollary from the theory which insists on the reference of judgement and assertion to the thinker. For it is only so far as assertion is recognized to be a mental act, that the notion of an imperative becomes relevant. An imperative of reason implies a restraint upon the voluntary act of assertion—a restraint which does not however infringe the freedom that characterises every volition, since the obligation to think in accordance with the truth is self-imposed. (Johnson 1921, p. 224)

While Johnson regards a reference to mental acts as somehow built into the notion of truth, to Husserl this conception would probably have appeared faulty because it contained a contamination of truth with human interests. However, it is not this aspect of Husserl's view which I want to explore here. It is rather on Sigwart's volitional conception of thinking, amplified in Johnson's claim that the 'obligation to think in accordance with the truth is self-imposed', that I want to dwell.

What is, I think, a correct insight is that the notion of truth which we employ when describing the *goal* of our assertions indicates an ideal which we try to attain and regard as worth pursuing. We may and do criticize others for falling short of it, and we expect them to respond to our criticism and modify their beliefs. The words 'right' and 'wrong' denote objective standards of assessment: whether an assertion of mine fulfils its goal is something which my hearer is entitled to judge as much as myself. There is no sense in which I may claim that an *assertion* of mine is true for me but perhaps not true for others, for, as Sigwart correctly points

¹³ Ibid., pp. 225–6.

out, to make an assertion is to advance a truth claim which demands belief on the part of my hearer.

This insight of Sigwart's was partly lost on Husserl, who conceived the faculty of evident judgement of ideal propositions in accordance with the model of vision, and, moreover, of the vision of ideal beings not subject to human limitations. My seeing a house is something that involves only myself and the house: the appeal to objective and shared standards of correctness is irrelevant for establishing whether my perception is veridical. Applied to the notion of truth, conceived as the goal of assertion, this picture has untenable consequences. There may be true sentences—Husserl argues in his *Prolegomena*—which we human beings may never be able to understand, let alone accept or question. Even concerning the truths that we do understand, it hardly matters that there happen to be beings to whom we can *communicate* what we take to be knowledge with the expectation that they share it with us. The notion of truth advocated by the absolutist logician (as Husserl labels his own position) does not admit a reference to human capacities of judgement and recognition: we accept a sentence as true as we would accept a revelation, oblivious of the grounds and consequences of so doing.

If Husserl's picture has untenable consequences, Sigwart's and Johnson's conceptions do not, on the whole, fare much better. In fact, when we apply the volitional picture of *Wahrdenkenwollen* to the logical laws themselves it is not clear that we have got hold of a genuine insight. For, what does it mean that the will or duty to think in accordance with the truth is self-imposed? Do we really in thinking have no choice but to comply with the 'laws of thought', which, as Sigwart and Johnson conceived them, partly coincide with the logical laws which govern deductive inference? The oddity of this question makes us aware of the inadequacy of the analogy between complying with a logical law and choosing to follow a certain norm or instruction. What would it be like to choose not to think in accordance with a law of logic?

The most thorough rejection of the conception of logic as a normative science is to be found in Wittgenstein's *Tractatus Logico-Philosophicus*. Wittgenstein rejected the entire phraseology of laws of thought and laws of inference which had still been used by Russell and Frege. By connecting logic and language in the closest possible way, he made it virtually impossible to formulate the question as to the normative vs. ideal status of logical propositions along the lines of Sigwart and Husserl. On the one hand, the laws of logic are downgraded from laws of thought to tautologies; on the other, logic is built into the very fabric of language and thought. There is no stepping outside the limits of our language and, hence, the limits of our logic.¹⁴ Logic fills the world: logical form *shows itself*, possibly

¹⁴ In *Der Gedanke* Frege tentatively surmises that logic might be characterized as being concerned with *Geist*: 'Neither logic nor mathematics has the task of investigating minds and contents of consciousness owned by individual men. Their task could perhaps be represented rather as the

with the help of a suitable notation.¹⁵ It is with a certain relief that we acquiesce, at least provisionally, in the Tractarian distinction between saying and showing as a way out of the controversy concerning the status (normative vs. ideal) of logical laws. Logical laws are neither outside nor inside the mind: if we insist that they should have a home, it can be only the use we make of language in arguing, inferring, guessing, etc.

Frege, for one, was not willing to follow Wittgenstein and deny logical propositions a content, although they both agreed that dragging in the psychological subject, with its strivings and failings, tells us nothing about the notions of truth and logical validity. Indeed, all attempts to bridge the gap between the realm of our ordinary inferential practice and the realm of atemporal, ideal laws were seen by Frege and Husserl as implying a confusion of utterly different spheres of reality. However, the contrast between psychological laws of holding true and logical laws of being true, to which they resorted for defending their positions from the assaults of psychologistic philosophers, is deeply unsatisfactory. For a sceptic might ask: how can Husserl and Frege be so sure of having correctly identified the laws of being true? Could it not have happened that they promoted the laws of holding true to the rank of laws of being true? Is not all this talk of two kinds of laws mystifying? Neither Frege nor Husserl ever paused to spell out what these psychological laws of holding true might be: internalized logical rules? inductive principles? routines of problem solving? Frege and Husserl, like their opponents, took it for granted that there are such laws, which it is the task of psychology to discover. And yet, we have not the foggiest idea of what such laws of holding true might be like, how they differ from the laws of logic proper, and whether, once discovered, they throw any light on the logical laws, properly so called. This lacuna is particularly striking in the preface to the first volume of *Grundgesetze der Arithmetik*, where Frege makes extensive use of the contrast between *Gesetze des Fürwahrhaltens* and *Gesetze des Wahrseins*. Here Frege goes so far as to suggest that we might without contradiction imagine the possible existence of people whose laws of holding true differ from ours. Although if confronted with such people we would have the impression of facing a hitherto unknown form of madness, the supposition that there may be such people does not go against the 'Denknotwendigkeit' invoked by Erdmann, nor, we may add, the 'postulate' invoked by Sigwart. Frege has no difficulty to concede that no *a priori* contradiction is involved in supposing that there may be beings who accept as sound inferential principles different from ours; a discrepancy between their laws of holding true and ours could not be discarded as spurious by an appeal to the

investigation of *the* mind [*Geist*]; of *the* mind, not of *minds*' (Frege (1918–19) p. 74; Engl. transl. p. 369). This statement has a strikingly idealistic ring: logic is well on its way to becoming a key to all mythology.

¹⁵ cf. Wittgenstein, *Tractatus Logico-Philosophicus*, 5.61.

conditions of the possibility of thought in general. By contraposition, our acceptance of the principle of identity or the law of non-contradiction cannot be used as a justification of the validity of logical principles. What Frege says is that meeting a deviant tribe, though deeply puzzling, would not make us *doubt* whether *we* are wrong in accepting such laws, nor would it make us call into question our inferential practices. However, this unshakable commitment to certain inferential principles which we find in ourselves cannot be turned into a justification of the ultimate grounds on which the necessity of the logical laws themselves rests. If one tried to turn it into such a justification, then this would amount to acknowledging that logical necessity is grounded in human nature, in our mental and physical constitution.¹⁶ But we may accept Frege's distinction between causal explanation and logical justification without endorsing the questionable contrast between logical laws of being true and psychological laws of holding true. It is only in *Der Gedanke*, published in 1918, that Frege, while still talking of psychological laws of holding true, seems uncomfortable with this distinction, and suggests that we should bypass it:

A derivation from these [psychological] laws, an explanation of a mental process that ends in taking something to be true, can never take the place of proving what is taken to be true. But may not logical laws also have played a part in this mental process? I do not want to dispute this, but if it is a question of truth this possibility is not enough. For it is also possible that something non-logical played a part in the process and made it swerve from the truth. We can decide only after we have come to know the laws of truth [*Wahrsein*]; but then we can probably do without the derivation and explanation of the mental process, if our concern is to decide whether the process terminates in *justifiably* taking something to be true. . . .

I assign to logic the task of discovering the laws of truth [*Wahrsein*], not the laws of taking things to be true or of thinking. The meaning of the word 'true' is spelled out in the laws of truth [*Wahrsein*].

(Frege 1918–19, p. 59; Engl. transl. pp. 351–2)

This way of presenting the issue contains the seeds of a more fruitful development—more fruitful, than e.g. Husserl's positing of ideal laws or Frege's expedition into a third realm. Once the notions of truth, meaning, and justification are placed at the centre of our philosophical inquiry, we can ask whether the notion of truth which informs our judgement and constitutes the norm by which we judge other people's assertions actually bears the absolutist and transcendent interpretation which Husserl (and Frege) placed on it. It is arguable that the

¹⁶ Frege (1893) p. xviii.

distinction between being true and being held true can be maintained without endorsing an absolutist conception of truth. Again, once the concepts of truth and justification are brought to the fore and connected with the meaning of the sentences we utter, we may begin to investigate the question whether a justification of a logical law which goes beyond reduction to more fundamental laws is possible in the first place. However, for tackling these questions better ammunition is required than that available to Sigwart or Husserl.¹⁷

From the way in which Husserl expresses himself in his *Prolegomena* it looks as if there is nothing in the content of a proposition which suggests that it might become the object of conscious scrutiny on the part of beings with the physical and mental constitution of human beings.¹⁸ Husserl actually alludes to angels with keener insight than ourselves who might grasp truths which are inaccessible to us. In this almost theological perspective it is understandable that the chief defect of psychologistic logicians is seen in their constant appeal to the actual constitution of the human mind, or, worse still, to human interests, which, as such, are subject to change. For Husserl not only logical principles but also the contents of sentences are ideal entities outside time and space. What might have a history is our way of coming to grasp a thought, but not the thought itself. This is one of the cornerstones of Husserl's criticism of Sigwart:

According to Sigwart it is illusory that a judgement could be true, independently of whether there is an intelligence who thinks this judgement . . . The judgement which expresses the law of gravity would not have been true before Newton; actually it would have been contradictory and utterly false: for apparently unlimited validity for all times forms part of the intended meaning of its assertion. (*Prolegomena*, §39, pp. 127–8)

Quite correctly Sigwart retorts:

Of course, the planets did move long before Newton in conformity with the law of gravity. But before Newton put forward his theory there was, within human knowledge, no sentence about that which might have been true. Now of course the sentence holds, in virtue of its content, also for the past. From the fact that of a sentence which has so far never been thought one cannot claim that it is true, Husserl infers that previously it had been false. Can one say of an unborn baby that it is healthy or that it is ill? And if it is not true that the baby is healthy, because it does not exist at all, must we conclude that it is ill? But this is precisely the way in which Husserl argues: he mixes up truth and reality, and confuses mere negation with the assertion of the contrary statement. (*Logik*, pp. 24–5)

¹⁷ For a full discussion of this issue see Dummett (1991).

¹⁸ The contrast between ideal (logical) laws and norms is discussed by Husserl in his *Prolegomena*, §48.

There are, I think, strong intuitions that speak in favour of Sigwart's conception, and others, perhaps equally strong, in support of Husserl's view. The realism vs. anti-realism controversy centres precisely on the question whether an absolutist conception of truth is the only admissible one, given the meaning we attach to our words—including the logical constants.

Simple Judgements

On account of the connection he drew between truth, sense, and reference Frege was in a better position than Husserl to perceive where the real weakness of psychologism resides, i.e. in an inadequate picture of the meaning of our utterances, which turns the communication of thoughts via language into a mystery. The 'objectivity' which Frege claims for the sense expressed by a sentence consists in its being communicable and capable of becoming a 'possession of many'. The question he asks himself is: what makes this objectivity possible, how does it come about that a sentence has the same sense for those who utter it and for those who understand it? If the sense of a sentence consisted in the ideas which it evokes in people's minds, how could people ever know that they mean the same when uttering the same words? Although Frege appreciated only in part the role played by the common language in securing the desired objectivity of meaning, still his reflections on sense and reference point in the right direction.

By 'objectivity' Sigwart means something quite different. The question he asks is: how is it possible that the subjective creations of the mind—the judgements—have objective validity, i.e. correspond to features of a reality outside the mind? Sigwart takes it for granted that there is uniformity and accord between the contents of people's judgements: it is our physical and mental constitution which secures such accord.

From the very beginning, Sigwart makes it clear that he is chiefly interested in the genetic aspect of judgement. After having remarked that *objectives Dasein* belongs not to the judgement itself but to the sign—the written or spoken sentence—he goes on to remark that the sentence can be considered from two different perspectives:

On the one hand it refers to its source, the mental processes of the person who gives utterance to it, and in so doing reveals his thoughts. On the other hand it addresses itself to the hearer by whom it is to be understood; he is called upon to interpret the outward signs, and out of them to reconstruct the thought expressed by the speaker. But the functions of the person understanding the spoken words are not the same as those of the speaker; although if we assume complete understanding, the final result in the mind of the hearer must coincide with that from which the speaker started...

Grammar and Hermeneutics, which start from the spoken or written word, naturally tend mainly to the standpoint of the hearer; they direct their attention to those functions which are active in understanding, and treat of them in the order in which they occur in the hearer. But in psychological analysis, where our aim is to investigate the nature of thought in judging, the other side—the action of the speaker—is of primary importance; and this the more so because communication to others is not necessarily the object of all Thought which clothes itself in words. (*Logic*, p. 26; *Logik*, pp. 30–1)

I have quoted this passage in full because it is very revealing of the style of Sigwart's *Logik*. There are two aspects of Sigwart's approach to logic that give it a markedly 'psychologistic' flavour: the first is the emphasis he places on the first-person point of view, the second is his constant focusing on sentences with indexicals. The first-person point of view is the natural one to take if one wants to analyse judgement in its genetic aspect, for from the hearer's point of view words appear as signs in need of interpretation and thus require one to take the hermeneutic stance. W. E. Johnson points out in a footnote of his *Logic*, in the context of his discussion of Sigwart's subdivision of judgements, that 'according as logicians exclusively interpret propositions from the point of view of the speaker (writer) or hearer, they are to be classed respectively as conceptualists or nominalists. The difference between these two points of view lies at the root of many logical controversies.'¹⁹ According to this characterization, Sigwart falls squarely within the conceptualists' camp. Communication will have achieved its aim if the words uttered by the speaker enable the hearer to reconstruct in his mind the thought which was present to the speaker's mind and prompted him to formulate his assertion. How the hearer performs this task or what makes it possible for him to perform his task is not a question to which Sigwart gives much thought. Although Sigwart is fully convinced of Mill's dictum that when we entertain a proposition our thought does not revolve around the ideas corresponding to its several terms, but rather refers to the things which those ideas are about, yet the way he describes what makes successful linguistic exchange possible is reminiscent of the ideational conception of the British empiricists. The function of words seems to be chiefly that of calling the 'right' images to the mind of a hearer—where an image is the right image if it resembles the one in the speaker's mind. What perhaps distinguishes Sigwart's position from that of the British empiricists is his conviction that even in the simplest perceptual judgements we have a predication, and hence conceptualization.

¹⁹ Johnson (1921) p. 168. Johnson applies Sigwart's notion of narrative judgement to offer an account of how indefinite expressions and definite descriptions perform their referential role in the course of a (fictional or historical) narrative.

Frege would have objected that he who forms a judgement of perception—Sigwart's favoured examples, as we shall see, tend to bear on perception—and he who apprehends it as a 'communicated judgement' share neither the sensory impressions nor the accompanying memory images, nor need they view things from the same perspective (though, of course, they might). Therefore, under the assumption that linguistic communication tends to be successful, understanding the content of a sentence cannot possibly consist in the hearer's recreating in his own mind the ideas and perceptions present to the mind of the speaker who addresses him. Rather, their mutual understanding seems to go via a grasp of the sense and *reference* which the words they use have in the context of the sentences they utter. Frege does not deny that words might evoke images and ideas; what he denies is (a) that the latter constitute the communicable meaning of a sentence, and (b) that a complex of representations can be a truth-bearer. If the sense expressed by a sentence qualifies as true at all, 'true' cannot here possibly be construed as 'true for me': since ideas, as Frege conceives them, are completely private, there is no way of eliminating from their 'contents' any reference to their 'bearer'. There are, admittedly, sentences which require 'completion' from the circumstances of their utterance, but this does not turn the thoughts they express into representations. As I said, Sigwart sees no problem of privacy in his constant appeal to images, feelings, sensations, or perceptions when elucidating the activity of judgement—quite rightly, in a way, but not for the reasons he gives. His very classification of judgements is centred on those judgements which contain features of contextual dependence. Sigwart's treatment of logic does not begin with categoric judgements or with judgements which express relations among concepts, but with 'elementary' judgements, which closely resemble the *occasion sentences* we encounter in chapter 2 of Quine's *Word and Object*.

In fact, the most original part of Sigwart's *Logik* is the one concerning simple judgements: denominative judgements (*Benennungsurteile*), impersonal judgements, judgements of recognition (*Wiedererkennungsurteile*). Simple judgements divide into narrative and explicative. Narrative judgements always contain a temporal reference and hence an element of contextual dependence: they name particular objects of intuition. At the opposite end there are explicative judgements, which deal exclusively with predicates and concepts.²⁰ Examples of explicative judgements are 'Blood is red and snow is white': they are not about this or that particular sample of blood or snow and contain no temporal reference. Among narrative judgements a special place is occupied by judgements of naming. Sigwart explains his terminological choice thus:

²⁰ Sigwart, as a rule, employs the words 'subject' and 'predicate' to indicate, not the linguistic expressions which play these logical roles, but what the judgement is about, i.e. the individual referred to and the property ascribed to it.

I choose this expression [*Benennungsurteile*] in order that I may have one denotation to cover statements which elsewhere are sometimes treated as judgements of subsumption (when the predicate is a more general idea), sometimes judgements of identity (when the predicate completely coincides with the subject). In the simplest cases there is no clear line between the two; in both the process is essentially the same, i.e. we are conscious that what is presented is in its totality one with a previously known idea. (*Logic*, p. 57; *Logik*, p. 74)

Examples of narrative judgements of naming are 'This is blood', 'This is snow', 'This is Socrates', or even 'Fire', 'The stork', all uttered as expressions of immediate recognition: no inference is involved in the immediate recognition of snow or blood. This process of immediate recognition is to be distinguished from what Stumpf had called 'fusion' and from episodes of involuntary memory in which one is presented with similar images. The characteristic mark of an act of immediate recognition is that we apprehend a predicate as 'being one' with an object. Sigwart endorses the Aristotelian doctrine that predicates are *in* the object, and tries to make this intuition fit the empiricist conception of 'comparing' a mental image (*Bild*, as he calls it) of a fire or of a stork with the *perceived* image of a fire or of a stork. The constant activity of what, for want of a better word, we may call 'comparison' between the mental image and the presented image is, according to Sigwart, at the basis of all our other cognitive activities.

Cognition is above all recognition, and as such involves predication and conceptualization. There is of course a difference between recognizing an animal as a stork, or a specific conflagration as a fire, and being taught to apply these words to indicated specimens: only in the former case is it appropriate to talk of 'recognition'. Thus Sigwart agrees, for instance, with Mill that there are words which 'denote things as such', but he denies that they work in a fashion utterly different from general names.²¹ He also disagrees with Mill when the latter calls adjectives such as 'white', 'heavy', and even demonstrative expressions such as 'this' *names* of things. These words do not fulfil a naming function in a fashion similar to a pointing gesture: 'When a particular thing is denoted by means of a word it is always because it is recognized as agreeing with the general idea expressed by the word; I can denote the thing lying before me as this book, or my book, only because the general meaning of the word is applicable to it' (*Logic*, p. 50; *Logik*, p. 64).

A special category of judgements which falls under the heading of narrative judgements and which Sigwart singles out for special treatment is that of 'immediate judgements'. A judgement is called 'immediate' if we cannot state it without

²¹ In fact, a proper name is distinguished from a general word 'only by the consciousness which accompanies it that the corresponding reality is a particular thing and always the same' (*Logic*, p. 50; *Logik*, p. 64).

being aware of its objective validity. Immediate judgements comprise both explicative judgements and judgements which rest on immediate intuition either of the external world or of the world of inner experience. Sigwart believes that he has discovered a fundamental difference, within the latter judgements, between conceptual judgements and judgements of immediate experience. It may be worth quoting in full this passage from Sigwart's *Logik*, because there is a certain parallel between the ways Sigwart and Frege approach the subject of judgements in the first person. Besides, it seems to me that Sigwart's *Logik* contains a formulation of the cognitive asymmetry between (present tense) statements in the first-person singular and statements in the second person which is strikingly similar to the one which was to become standard in the analytical tradition:

Conceptual judgements have subjects which are assumed to be thought by every one in the same way. The judgement 'I see' has a subject which can be present to no one as it is to me. . . . the contents of that which is denoted by 'I' can never be exhaustively stated: it is given to us in a manner which is absolutely incomparable with all other objects of thought. The conceptual judgement says 'If I think of A, I must think of it as having the determination B'; in the judgement of self-consciousness there is no if. The subject is simply thought whenever anything is thought, and the fact that it is thought is the absolute presupposition of all other thought.

. . . We can always ask whether that which is contained in a concept exists; I can never ask whether I exist. The characteristics of the concept are unvarying; the predicates of the ego, with the exception of the ego itself, change from moment to moment, and yet every judgement as it is formed possesses an immediate certainty of truth, which can only be acknowledged, and never proved.

. . . the idea which *someone one else* has of *me* is different from that which I have of myself; it refers to the same subject, but not in the same way. Thus the affirmation of being in its most original form is an act peculiar to the individual, and dependent upon conditions peculiar to the individual. Any judgement concerning another ego is necessarily mediated, whether it is the acknowledgment of his being or belief in his utterances. (*Logic*, pp. 302–3; *Logik*, p. 408)

In the first Logical Inquiry Frege mentions a variety of thoughts—'I'-thoughts for short—which seem to be both private (and hence, according to his own characterization, incommunicable) and yet fully objective. Their privacy, as Frege construes it, does not consist in their requiring a bearer (as representations, for instance, do), but in a peculiarity of the first-person pronoun 'I'. This pronoun has, as it were, two senses: a public and communicable one, and a private one, which relates to the peculiar way in which everyone is given to himself. I doubt that Frege's construal of the sense of the first-person pronoun is tenable. Be this as

it may, all I want to point out here is that on the rare occasions when Frege discusses examples of sentences with features of contextual dependence—which in Sigwart's *Logik* are the rule—his distinctions between sense, reference, and 'representation' show a certain strain.²²

Certainly, that Sigwart should give such an account of immediate judgements in the first person is much less surprising than Frege's claim that the *sense* of the word 'I' has something to do 'with the primitive and special way in which everyone is presented to himself and to no-one else'.²³ Whereas Sigwart follows Kant more or less closely, and refrains from inferring from the certainty of 'I'-thoughts any conclusion as to their metaphysical import, Frege offers an argument to the effect that there is at least one *object* (i.e. the one to which 'I' refers in my mouth) which is the bearer of contents of consciousness.

Impersonal Judgements and Individual Events

Among narrative judgements a special place is occupied by impersonal judgements. In 1888 Sigwart had published a short monograph, entitled *Impersonalien: Eine logische Untersuchung*, to rebut the misunderstandings which his account of impersonal judgements had met with on the part of philosophers of the Brentano school, and of Anton Marty in particular. In the second and third editions of his *Logik* Sigwart again takes up the question how *judgements* such as 'It rains', 'It thunders' should be construed. The analysis, needless to say, is carried out from the standpoint of the speaker, who issues such judgements as denominative judgements of perception. Sigwart's observations could be viewed as an attempt to analyse that which, for want of a better word, could be called the logical form of impersonal judgements.

In German (and English) the grammatical form of impersonal judgements is that of a predication with an indeterminate subject, indicated by 'es' (and 'it', respectively). It sometimes happens that we hear a noise or perceive a smell and say 'Es rauscht' ('It's rustling'), 'Es riecht' ('It smells'), before having grasped what it is that is rustling or smells. Along the same grammatical pattern we form sentences like 'Es regnet' ('It is raining'), 'Es schneit' ('It snows'). All these sentences can be used to express denominative judgements in which a given percept is assimilated to a familiar idea. However, if we want to bring to the fore the logical form of 'genuine' impersonal judgements, we must not take their grammatical form as an infallible guide.

In fact, in Sigwart's opinion, there are important differences between sentences formed along the same pattern: whereas in the case of 'It is rustling' or 'It smells'

²² For a defence of Frege's conception cf. Evans (1985).

²³ G. Frege, *Der Gedanke*, p. 66, Engl. transl. p. 359.

the attempt to determine the subject is legitimate, in the case of sentences like 'It is raining' it makes no sense to enquire after the 'it' that is raining. Only the latter are, in Sigwart's opinion, genuine impersonal judgements. A sentence expressing a genuine impersonal judgement contains no indication of a specific *thing* in which the event takes place, but points to a phenomenon present at the moment: 'all that we do is to name the event perceived without proceeding further to the denotation of the subject in which it takes place' (*Logic* p. 64; *Logik*, p. 84). However, these so-called 'propositions without a subject' lack a subject-thing only in the narrow sense, and thus constitute no exception to the general account of predication which Sigwart defends. Impersonal judgements too contain a synthesis of a known general idea with a present phenomenon: it is this phenomenon which constitutes the subject of the judgement. The pronoun 'es' which occurs in 'Es blitzt' names nothing but this particular flash of lightning. But impersonal judgements differ, in Sigwart's opinion, from existential statements proper:

When we say 'It lightens!' we do not mean to predicate actuality of the lightning, but to predicate lightning of something actual. The naming of the present impression is the fundamental act, without which the proposition, as the expression of the present perception, could never come about.

(*Logic*, p. 65; *Logik*, p. 86)

Sigwart concedes that if the proposition is analysed from the point of view of the hearer, we seem to be dealing with an existential judgement which ascribes actuality to a certain event of lightning. The hearer first apprehends the general idea of a lightning from the word and then by its inflexion 'he is called upon to think of the lightning as particular and present' (*Logic*, p. 65; *Logik*, p. 85). However—Sigwart insists—from the speaker-perceiver point of view, the right order is the reverse: we see a flash in the sky and of it we say that *it* is a lightning. We thus form a proposition of which the logical subject is nothing more 'than an event or state perceived at the moment' (*Logic*, p. 65; *Logik*, p. 85).

Using modern terminology we might try to analyse impersonal judgements in such a way as to do justice both to the hearer's and to the speaker's points of views. Without being unfaithful to Sigwart's account, we might say that impersonal judgements contain a hidden variable ranging over individual events, of which the properties, say, of being a lightning, of happening in a certain region of space, at a certain time, with a certain duration etc., are predicated. The existential import of the impersonal judgement is captured by binding the free individual variable by means of an existential quantifier. The logical form thus individuated is a bit remote from the superficial grammatical form. It helps however to throw light on the logical role of the pronoun 'it', both in the context of the sentence 'It lightens' and in anaphoric back-reference—e.g. when we go on to say 'And *it* did not last long'.

The search for suitable ways of representing the *logical* form of sentences of a natural language was the starting point of Frege's and Russell's enterprises. Sigwart stumbled almost by chance on this kind of problem: his main aim was to call attention to the various syntheses of judgement which precede the formulation of impersonal judgements. The emphasis he placed on the genetic aspect of thinking and judging could certainly encourage pernicious forms of meaning psychologism; however, with hindsight, this should not prevent us from also seeing certain virtues of Sigwart's way of looking at things.

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Frege's Anti-Psychologism

There is hardly a piece of writing by Frege—be it book or letter, article or review—where he misses the opportunity to stigmatize the evil of psychologism. An unmistakable mark that a philosopher has fallen prey to this infection is his tendency to blur certain distinctions, for example, between the laws of thinking and the laws of thought, between the sense of linguistic expressions and that which they can be used to talk about, between sense and mental representations. As far as mathematicians are concerned, even those (few) who refrain in their writings from psychologistic phraseology betray symptoms of psychologism in their careless employment of the word ‘sign’: they claim to pay attention only to the sign but in reality switch freely from the sense of a sign to that which it stands for and even to the calculation techniques commonly associated with it. In a spirit similar to Frege’s Georg Cantor in his *Mitteilungen zur Lehre vom Transfiniten* of 1887 denounces the *akademisch-positivistische Skepsis* (personified by Helmholtz and Kronecker) as being guilty of conceiving numbers as mere signs and moreover ‘not as signs of concepts which refer to sets, but as signs of the individual things counted in the subjective process of counting’ (‘Zeichen für die beim subjektiven Zählprozeß gezählten Einzeldinge’).¹ A source of this error resides in the genetic fallacy of confusing the meaning and import of mathematical propositions with the mental acts whose performance is allegedly required for apprehending them (i.e. acts of psychological abstraction, of construction, of pattern recognition). But—Frege urges—the content of a sentence (what the sentence says) and our forming a judgement about it are two utterly different things: the use of the word ‘judgement’ (‘Urteil’) to refer both to the content and to the act of judgement and the careless employment of the word ‘idea’ (‘Vorstellung’) are but two examples of the conceptual confusions which mar all psychologistic accounts of logic.

Frege’s anti-psychologism has many targets and facets, some of which I have explored elsewhere by way of comparing Frege’s conception of logic with certain aspects of the logical doctrines of Erdmann, Wundt, Sigwart, and Kerry.² Frege’s

¹ G. Cantor, *Abhandlungen mathematischen und philosophischen Inhalts*, ed. by E. Zermelo, Hildesheim, Olms, 1932, repr. 1966, pp. 378–439, pp. 382–3.

² Cf. Picardi, ‘The Logics of Frege’s Contemporaries or “der verderbliche Einbruch der Psychologie in die Logik”’, in D. Buzzetti, M. Ferriani (eds.), *Speculative Grammar, Universal Grammar and Philosophical Analysis of Language*, Amsterdam, Benjamins, 1987, pp. 173–204; ‘Kerry und Frege über Begriff und Gegenstand’, *History and Philosophy of Logic*, 15 (1994), pp. 9–32, and *La chimica dei concetti: linguaggio, logica, psicologia 1879–1927*, Bologna, Il Mulino, 1994, chs. 1 and 2. I have mentioned Kerry, Wundt, and Sigwart because they are among the authors whose work we know to

most sustained discussions of psychologism are to be found in his preface to GGA, in his review of Husserl's *Philosophie der Arithmetik*, and in the First Logical Inquiry, 'Der Gedanke'. In 1893 the scapegoat of psychologism is Benno Erdmann, whereas in 1918 no particular miscreant is mentioned. Frege's target might very well have been a corpus of doctrines and not a specific author. However, internal textual evidence has led me to arrive at a conjecture concerning a specific author who may have been at the back of Frege's mind.

With the exception of this philological curiosity, which I will take up in the Appendix, I do not here intend to plunge deeper into the historical background of Frege's work, but I want rather to explore what I believe Frege took the main fault of psychologism to be, i.e. a mistaken picture of language which turns the objectivity of sense and the communication of thoughts into a mystery. The confusion between logical laws and psychological laws of thinking, which Frege discusses in the preface to the first volume of GGA, is yet another facet of that mistaken picture. The assimilation of logical laws to psychological laws can be seen as the result of an extreme form of naturalism:³ according to Frege, at the root of this confusion is the lack, on the part of psychologistic philosophers, of an adequate concept of truth. To the psychologistic philosopher only a relativistic notion of truth is available: truth is made to coincide with universal agreement and is implicitly relativized to the mental equipment of the 'normal' representatives of the species *homo sapiens* at a given stage of its psychological and social evolution.⁴ The chief defect of a naturalistic approach to logic is not just that it disregards the claims of *a priori* knowledge without offering any alternative account, but that, by embodying a relativistic notion of truth, it issues in a form of extreme subjectivism

have been actually studied by Frege. As is well known, Frege's article *Über Begriff und Gegenstand* is a reaction to the criticism which Benno Kerry had levelled against Frege's account of the notions of a concept and an extension of a concept in *Grundlagen der Arithmetik*. Between 1885 and 1891 the Austrian philosopher Benno Kerry (1858–1889) had published in the *Vierteljahrsschrift für wissenschaftliche Philosophie*, a review edited by R. Avenarius, a series of eight articles under the title 'Über Anschauung und ihre psychische Verarbeitung'. In these articles Kerry discusses in detail the philosophical and mathematical doctrines put forward by G. Cantor, F. A. Lange, L. Kronecker, and B. Bolzano. Even apart from their intrinsic value, Kerry's articles constitute a mine of information as regards the philosophical atmosphere in which Frege's work was written and received. Wilhelm Wundt's *Logik* is mentioned by Frege in two places of his *Nachlaß*, whereas as far as Sigwart is concerned we have only the testimony of Heinrich Scholz; it appears that Frege had taken notes or excerpts, now lost, from the *Logik* of Christoph Sigwart. Scholz's list is published in vol. 1 of M. Schirn (ed.), *Studien zu Frege/Studies on Frege*, Bad-Cannstatt, Frommann-Holzboog, 1976, as an appendix to A. Veraart's article which describes the history of Frege's *Nachlaß*.

³ 'Naturalism' is not Frege's term, but Heidegger's; cf. M. Heidegger, *Die Lehre vom Urteil im Psychologismus* (1913), in *Gesamtausgabe*, vol. 1, *Frühe Schriften*, Frankfurt a. M., Klostermann, 1978. Heidegger's criticism of psychologistic theories of judgement follows by and large the arguments outlined by E. Husserl in his *Prolegomena zur reinen Logik* (1900) which precede the *Logische Untersuchungen*.

⁴ This is the gist of Frege's criticism of Benno Erdmann's *Logik* in the Preface to GGA. Husserl's critical survey of Sigwart's and Erdmann's conception of logic in §§38–40 of his *Prolegomena zur reinen Logik* closely resembles the line of argument employed by Frege in GGA. Both Frege and Husserl identify the chief error of psychologism in the endorsement of a relativistic conception of truth. Both Sigwart's and Erdmann's books on logic were influential in Frege's times.

as regards meaning. It is Frege's conception of truth which, in my opinion, provides the link between his anti-psychologism in logic and his anti-psychologism in the account of meaning. The thesis I am attributing to Frege is a strong and controversial one: it amounts to the claim that nothing short of the classical notion of truth can give us a correct account of the meaning we attach to our utterances. Frege held that truth and meaning stand in the closest connection to each other: the account of meaning which he had outlined in his essays 'Über Sinn und Bedeutung', 'Über Begriff und Gegenstand', and 'Über Funktion und Begriff' provides in my opinion the most effective remedy against psychologism.

If what I have said is correct, then it is not only unpromising but positively misleading to tackle the issue of Frege's anti-psychologism by appealing to his conception of epistemology—a conception of which we can at best surmise its bare outlines. Philip Kitcher, for instance, has suggested that Frege implicitly relied on Kant's account of knowledge as far as the ultimate justification of logical principles is concerned, thereby himself succumbing to a form of psychologism, for, in Kitcher's opinion, Kant's theory of knowledge is psychologistic.⁵ Kitcher is not the first to advance the suggestion that Frege had slipped inadvertently into psychologism. The same charge, on very different grounds, was made by Russell and Jourdain. Before examining Kitcher's claim, it may be appropriate to review Frege's own rebuttal of the charge brought against him by Jourdain and Russell.

A New Form of Psychologism?

It must have been galling for Frege to read in Russell's letter of 12 December 1904: 'We do not assert the thought, for this is a private psychological matter [eine psychologische Privatsache], we assert the object of the thought, and that is to my mind a certain complex';⁶ and equally galling it must have been for him to be queried some ten years later by Philip Jourdain whether he 'now regard[ed] assertion ($\vdash$) as merely psychological' and whether 'in view of what seems to be a fact, namely, that Russell has shown that propositions can be analysed into a form which only assumes that a name has a "Bedeutung", & not a "Sinn"' he would now hold that "'Sinn" was merely a psychological property of a name'.⁷

This is by no means a minor clash of mentalities between the founding fathers of logicism; on the contrary, it is a sign of a deep philosophical difference. The charge that assertion is purely psychological is very likely Wittgenstein's. It seems quite probable that already in 1913–14 Wittgenstein had come to this conclusion

⁵ P. Kitcher, 'Frege's Epistemology', *Philosophical Review*, 88 (1979), pp. 235–62.

⁶ G. Frege, *Wissenschaftlicher Briefwechsel* (= WB), ed. by G. Gabriel, H. Hermes, F. Kambartel, C. Thiel, A. Veraart, Meiner, Hamburg, 1976, p. 251; Engl. edition abridged by B. McGuinness and translated by H. Kaal, *Philosophical and Mathematical Correspondence* (= PMC), Blackwell, Oxford, 1980, p. 169.

⁷ Jourdain to Frege, 15 January 1914, WB, p. 126, PMC, p. 78.

and talked to Jourdain about it. Unfortunately the letters exchanged between Frege and Wittgenstein concerning the *Logisch-philosophische Abhandlung* never arrived at Wittgenstein's remark 4.441, where he expresses his view on assertion.

Had Russell (and Wittgenstein) uncovered a form of psychologism which had escaped the most vehement advocate of anti-psychologism? Had Frege himself fallen prey to the very same *Zeitkrankheit* which he had so ruthlessly exposed in other philosophers and mathematicians?

In his reply to Jourdain Frege touches neither on the subject of sense nor on that of assertion but only points out certain obscurities which in his opinion beset Russell's notion of a variable. An earlier draft of the letter, however, is entirely devoted to these matters. Here Frege explains once more the reasons why he believes that the notions of sense and assertion, far from involving any concession to psychologism, are an effective antidote against it. One may wonder why in the letter Frege actually sent to Jourdain neither the notion of sense nor that of assertion is mentioned. Perhaps remembering his utter failure in 1904 to get across to Russell what he meant by sense, thought, cognitive value, Frege simply deemed it a waste of time to repeat the experiment with Jourdain.

I shall quote two passages from the draft of the letter to Jourdain for they are in my opinion the best and most concise account of what Frege took the bearing of the notion of sense on the issue of psychologism to be. The first argument concerns the contribution of the sense of a proper name to the sense of a sentence in which it occurs, in harmony with the principle of compositionality; the second concerns sense as the mode of presentation of the referent and its relevance to explaining the cognitive significance of identity statements. The first argument runs as follows:

The possibility of our understanding propositions which we have never heard before rests evidently on this, that we construct the sense of a proposition out of the parts that correspond to the words. If we find the same word in two propositions e.g. 'Etna', then we also recognize something common to the corresponding thoughts, something corresponding to this word. Without this language in the proper sense would be impossible. We could indeed adopt the convention that certain signs were to express certain thoughts like railway signals ... but in this way we could not form a completely new proposition, one which would be understood by another person though no special convention had been adopted beforehand for this case. ... Without meaning [*Bedeutung*] we could indeed have thought, but only a mythological or literary thought, not a thought that could further scientific knowledge. Without a sense we would have no thought and hence also nothing that we could recognize as true.⁸

⁸ WB, pp. 127–8; PMC, pp. 79–80.

After having described the Aphla–Ateb example—that is, the example of one and the same mountain designated by different names by different people living in its vicinity—and after having remarked that the thought expressed by ‘Ateb is at least 5000 metres high’ differs from that of ‘Aphla is at least 5000 metres high’ since one can judge the former true without judging the latter true, and vice versa—as two acts of knowledge (‘Erkenntnisthaten’) are involved here, and not just one—Frege concludes:

Now if the sense of a name was something subjective then the sense of the proposition in which the name occurs and hence the thought would also be something subjective and the thought one man connects with this sentence would be different from the thought another man connects with it; a common store of thoughts, a common science would be impossible. It would be impossible for something one man said to contradict what another man said, because the two would not express the same thought, but each his own. For these reasons I believe that the sense of a name is not something subjective [crossed out: in one’s mental life], that it does not therefore belong to psychology and that it is indispensable.

Similar arguments can be given in support of the non-psychological character of the sense of predicates and relational expressions, but for reasons which I will explore later proper names constitute a delicate test case for our intuitions. I do not here want to discuss the question whether Frege’s notion of sense as something which can be shared and communicated, and which stands in the closest relation with truth, is ultimately defensible as it stands. I simply want to point out that he took the threat of psychologism to consist in an intolerable form of subjectivism, which in GGA he had labelled ‘idealism’ and ‘solipsism’. However, if all that Frege had said about sense had consisted in the formulation of principles of compositionality underlying our understanding of sentences, such an insight, though valuable, would give us no clue how to answer the question what it is for a hearer to recognize a sentence as bearing a given sense, or, indeed, as bearing the same sense that the speaker who addresses him confers upon it. To answer the latter question we must appeal to the concept of truth: the sense of a linguistic expression can then be characterized as its contribution to the determination of the semantic value of the sentence in the context of which it occurs. It is via the notion of truth and reference that we attain a conception of what it is for a sentence to express a thought, i.e. to possess a definite content which we may reasonably aim at communicating to others and to establish as true or not true to the satisfaction of all parties concerned. In default of such an account of meaning, of which I have here given a very rough sketch,⁹ we are powerless against

⁹ I am here following Dummett in his interpretation of Frege’s conception of semantics. Cf. e.g. M. Dummett, *The Interpretation of Frege’s Philosophy*, London, Duckworth, 1981.

psychologism, and no amount of Kantian transcendentalism could restore our faith in objectivity. Or so it seems to me.

In his book on Frege, Hans Sluga has claimed that 'it would require a strong argument to show that Kantian transcendentalism is insufficient to account for the desired anti-psychologism, objectivism and apriorism. No such argument can be found in Frege's writings.'¹⁰ Since, as I construe them, Frege's complaints against psychologism are essentially semantic in nature, I am at a loss to see how a mere appeal to Kantian transcendentalism could have secured the desired objectivity of content. Had Frege thought that Kantian transcendentalism was sufficient, he would perhaps have said so, and very probably he would not have bothered to devise his own a theory of sense and reference.

As both Cantor and Windelband recognized, psychologism can be seen as an offspring of a naturalistic reading of Kant. Windelband's assessment of the German philosophical scene between 1870 and 1890 is worth quoting:

It was the fate of the revival of Kantianism that because of the interests of the scientific way of thinking it was at first restricted to the theory of knowledge, whose extreme developments toward a decidedly empiricist position led to positivistic transformations on the one hand and to the dissolution of philosophical problems into psychological problems on the other. Thus, just as during the time before Kant, the disastrous predominance of psychologism became widespread, for a few decades, particularly at German universities.¹¹

Given the many different trends of Kantianism in German philosophy, the claim that Kantian transcendentalism of itself provides a barrier against psychologism stands in need of qualification and circumstantial evidence. Of course, I am not denying that Frege (young Frege especially) made use of concepts and formulations which derive from Kant. For instance, his celebrated characterization of sense as 'die Art des Gegebenseins eines Gegenstandes', his claim in 'Der Gedanke' that perceptual experience involves a nonsensible element, his ascription of conceptual priority to the sentence used to express a judgement over its component parts, all these ring a familiar (Kantian) bell. But adverting to these features helps us not at all in understanding Frege's conception of logic and his anti-psychologism. Besides, psychologist philosophers were only too ready to assign to judgement (as content and act, if they managed to keep them apart) the pride of place in their accounts of logic; Benno Erdmann's *Logik* is a case in point, and its debt to Kant's philosophy is quite obvious to the reader.¹²

¹⁰ H. Sluga, *Gottlob Frege*, London, Routledge, 1980, p. 60.

¹¹ W. Windelband, *Lehrbuch der Geschichte der Philosophie*, 1892, p. 539 (repr. of the 13th ed. of 1935, Tübingen, J. C. B. Mohr, 1980).

¹² Not very surprisingly, perhaps, if we remember that Benno Erdmann was not only an eminent Kant scholar, who edited several volumes of the Akademie-Ausgabe, but also the author of a book called *Kant's Kritizismus*, published in 1878.

In order to understand Frege's foundational project we have to bear in mind a distinction which psychologistic logicians did not or could not draw, namely the distinction between our entitlement for holding a sentence true and the ultimate grounds on which the justification for its truth rests. As I said, Frege's realistic conception of truth is an essential ingredient of his conception of logic. Sluga rightly perceives that the adoption of Platonism is irreconcilable with Kantian *Kritizismus* and sets out to show, unsuccessfully in my opinion, that Frege was not a realist in Dummett's sense. A line of argument similar to Sluga's is put forward by Philip Kitcher in his paper 'Frege's Epistemology' of 1979. Kitcher's interpretation rests on his reading of a statement Frege makes in the preface to GGA in the context of his discussion of Erdmann's doctrines. Frege says:

The question why and with what right we acknowledge a law of logic to be true, logic can answer only by reducing it to another law of logic. Where this is not possible logic can give no answer.¹³

Kitcher reads this statement as evidence that Frege thought that the question with what right we acknowledge a law of logic had already been settled. 'A philosopher whom he greatly admired, had set out a general scheme for human knowledge. . . . The philosopher was Kant.'¹⁴ In Kitcher's opinion Kantian transcendentalism provides the key to the understanding of Frege's foundational programme. I myself don't see that this follows from the passage quoted above, nor from what Frege says elsewhere.

In a well-known passage of *Grundlagen*, where Frege declares that the Kantian distinctions between the *a priori* and the *a posteriori*, between the synthetic and the analytic, do not concern the content of a judgement but our justification for making it, he qualifies his statement thus:

When a proposition is called *a posteriori* or analytic in my sense, this is not a judgement about the conditions, psychological, physiological and physical, which have made it possible to form the content of the proposition in our consciousness; nor is it a judgement about the way in which some other man has come, perhaps erroneously, to believe it true; rather, it is a judgement about the ultimate ground upon which rests the justification for holding it to be true.

(GLA, §3)¹⁵

¹³ G. Frege, *Grundgesetze der Arithmetik*, vol. 1, 1893 (= GGA, I), vol. 2, 1903 (= GGA, II), Jena, Pohle, repr. Hildesheim, Olms, 1962; Engl. part. transl. by M. Furth, *The Basic Laws of Arithmetic*, Los Angeles, University of California Press, 1967, p. 15.

¹⁴ Kitcher, 'Frege's Epistemology', p. 241.

¹⁵ G. Frege, *Die Grundlagen der Arithmetik* (= GLA), Breslau, Koebner, 1884, *Centenar Ausgabe* edited by C. Thiel, Meiner, Hamburg, 1986; Engl. transl. by J. L. Austin, 2nd. rev. ed., Oxford, Blackwell, 1967.

If a proof of the analytical (i.e. purely conceptual) nature of arithmetical propositions could be secured, Kant's appeal to an *a priori* intuition to justify their necessity could be dropped as redundant. Frege shares Kant's opinion that an appeal to intuition is indispensable in the case of geometrical propositions, on the ground that the denial of geometrical axioms (unlike that of analytical propositions) does not lead to inconsistency (GLA, §14). Whereas the acceptance of specific geometrical axioms is forced on us by the peculiarity of our pure intuition of space, no appeal to intuition is required in the case of arithmetical propositions. Having discarded Kant's account of necessity,¹⁶ Frege sets himself two formidable tasks: (a) to defend the thesis that pure conceptual thought, issuing in analytical propositions, can extend our knowledge, and (b) to show that logic, far from being barren, can beget peculiar 'objects', i.e. numbers. Having rejected Kant's thesis that without sensibility no objects are given to us, he squarely faces the question: 'How then are numbers given to us, if we cannot have any idea or intuition of them?' (GLA, §62). The surprising, indeed revolutionary, answer he gives to this question is that we should rather concentrate our attention on the assertions in the context of which number-words occur and investigate the grounds on which the justification of their truth rests. This way of recasting epistemological issues in semantic terms (i.e. as questions concerning meaning, reference, and conditions of justification) constitutes, in my opinion, the background against which Frege's remedy against the 'corrupting incursion of psychology into logic'¹⁷ has to be found.

To the three fundamental principles enunciated in the preface to GLA Frege adhered all his life. And yet, after 1884, he never again made use of the terms 'analytic' and 'synthetic'. Dummett surmises that perhaps he no longer thought that 'arithmetical truths in the plural are analytic, but that arithmetic in the singular is a branch of logic and need not to draw upon experience or intuition for the basis of its proofs'.¹⁸ In his *Logik in der Mathematik* of 1914,¹⁹ after having raised the question how we explain the difference in cognitive significance between two identity statements like ' $606 = 606$ ' and ' $137 + 469 = 606$ ', he does

¹⁶ Contrast the above-quoted statement from GLA with Kant's characterization of necessity in KrV (Einleitung, B17). It seems to me that Kant here wants to draw attention to the actual processes whereby we apprehend the content of synthetic (mathematical) judgements: the relevant question to be asked in this context is not, according to Kant, one about what we ought to add in thought to a given concept, but rather what we *really* think in thinking that thought, albeit obscurely, for only then does it become clear that the necessity of a certain predicate inhering in a given concept is brought about by an intuition which must be added to that concept. Frege's notion of objective justification, on the other hand, relates to the logical form of sentences expressing arithmetical propositions, and thus, in a sense, to that which we 'ought to think'. The further question of whether and how the content of such logically analysed sentences is related to that which we *really* think is discarded by him as *logically* irrelevant.

¹⁷ G. Frege, GGA, I, p. xiv, Engl. transl. p. 12.

¹⁸ M. Dummett, 'Frege and the Paradox of Analysis', in *Frege and Other Philosophers*, Oxford, OUP, 1981, pp. 17–52, p. 28.

¹⁹ G. Frege, *Nachgelassene Schriften* (= NS), ed. by H. Hermes, F. Kambartel, F. Kaulbach, 2nd revised and enlarged ed., Hamburg, Meiner, 1983; Engl. transl. of the 1st edition (1969) by P. Long and R. White, *Posthumous Writings* (= PW), Oxford, Blackwell, 1979.

not answer it by appealing to the activity of counting, calculating, constructing, etc., as a Kantian would have done, nor does he mention his own previous account of analyticity, put forward in GLA. He now appeals exclusively to the argument he had given in 'Über Sinn und Bedeutung'. After having mentioned examples of numbers, planets, stars, mountains, all on the same footing, he pauses to remark on the marvellous potentialities of language, repeating the argument which we already found in the draft of the letter to Jourdain and which recurs at the beginning of the Third Logical Inquiry.

On the other hand, appealing to the manipulation of signs, to techniques of calculation, and to constructions would have been regarded by Frege as playing into the hands of the formalists. Whatever he may have meant in 1924 by a 'logical source of knowledge', I am sure he did not mean the ability to manipulate signs and carrying out constructions. Benno Kerry, who concurred with Frege in mistrusting naive formalism, sees the latter as stemming from Kant's emphasis on construction. Hilbert himself—not a *naive* formalist, to be sure—acknowledges, in his essay *Über das Unendliche*, his debt to Kant's philosophy. 'Contentual' meta-mathematics is, in Hilbert's eyes, an embodiment of Kantian insights, and as far as I understand Hilbert and Kant, Hilbert's claim does not seem far-fetched at all. The following statement is extremely instructive:

Kant already taught—and indeed it is part and parcel of his doctrine—that mathematics has at its disposal a content secured independently of all logic and hence can never be provided with a foundation by means of logic alone; that is why the efforts of Frege and Dedekind were bound to fail. Rather, as a condition for the use of logical inferences and the performance of logical operations, something must already be given to our faculty of representation [*in der Vorstellung*], certain extralogical concrete objects that are intuitively [*anschaulich*] present as immediate experience prior to all thought. . . in mathematics, in particular, what we consider is the concrete signs themselves, whose shape, according to the conception we have adopted, is immediately clear and recognizable.²⁰

Even a superficial reader of GGA soon realizes that without the account of how a sentence of the formal language is determined as true—the sense expressed by an assertion of a sentence of GGA being the thought that its truth-conditions are fulfilled (GGA, I, §32)—Frege would have not a shade of an argument for opposing the doctrines both of psychologistic logicians and of formalist mathematicians. There would be no explanation of what it is for a sentence to be objectively true or false 'independently of the judging subject' (GGA, I, p. xvii).

²⁰ D. Hilbert, 'Über das Unendliche', Engl. transl. in *From Frege to Gödel*, ed. by J. van Heijenoort, Cambridge, Harvard University Press, 1967, p. 376.

The psychologistic logicians add to a thought all sorts of irrelevant paraphernalia, since they lack the notion of objective truth and have access only to that of 'true for us' ('us' meaning something like mankind at a given stage of its evolution). The formal mathematicians (e.g. Heine and Thomae) claim to be able to strip the signs of all content and they also employ a very crude notion of a sign and the sameness of a sign (GGA, II, §90); no wonder that, as soon as within the theory of a game they want to say something significant about admissible and inadmissible configurations of signs they are forced to reintroduce through the back door the contentual understanding of the rules governing the use of the signs concerned.

Frege thought he could prove that such an approach was misguided by showing how the thought expressed by a sentence of GGA is uniquely determined by the truth-conditions of the sentence, no matter whether *we* are or ever will be in a position to recognize whether or not they obtain. The originality of Frege's Platonism does not reside in his belief that there is a mathematical reality external to our minds but in his view that the sense and reference of mathematical statements are determined once their truth-conditions have been laid down. The disrupting effect of Russell's paradox consisted, among other things, in showing that this kind of requirement was not fulfilled by certain sentences of GGA, thereby jeopardizing Frege's entire conception of semantics.²¹

I claimed that it is misleading to suppose, as Sluga and Kitcher do, that in Frege's opinion a generic appeal to transcendentalism would provide an answer to psychologism and a groundwork for objectivity. How, then, is the passage of GGA quoted by Kitcher to be interpreted? If we read the quoted statement in the context where it belongs (and how else should we read it?), it will ring a very different bell. For Frege continues:

If we step away from logic, we may say: we are compelled to make judgements by our own nature and by external circumstances; and if we do so, we cannot reject the law—of Identity, for example; we must acknowledge it unless we wish to reduce our thought to confusion and finally renounce all judgement whatever. I shall neither dispute nor support this view: I shall merely remark that what we have here is not a logical consequence. What is given is not a reason for something's being true, but for our taking it to be true. Not only that: this impossibility of our rejecting the law in question hinders us not at all in supposing beings who do reject it; where it hinders us is in supposing that these beings are right in so doing, it hinders us in having doubts whether we or they are right.²²

Far from being reticent, as Kitcher insinuates, Frege is here refusing to endorse the view that Kitcher attributes to him. Frege has no difficulty in conceding that no *a*

²¹ Or, at any rate, his conception of logical objects.

²² GGA, I, p. xvii; Engl. transl. p. 15.

priori contradiction is involved in supposing that there may be beings who accept as sound inferential principles different from ours; a discrepancy between their laws of holding true and ours could not be discarded as spurious by appeal to the conditions of the possibility of thought in general. One need not think of exotic tribes to find examples of the situation here envisaged. It is enough to suppose that the beings in question suspend judgement on specific applications of certain principles which we accept; they need accept as true the negation of the law of identity or of a classical tautology. If we think of mathematics, the situation is even more clear-cut: Kronecker and Cantor could be seen as representatives of such different tribes. What Frege says is that meeting a deviant tribe, though deeply puzzling, would not make us *doubt* whether *we* are wrong in accepting such laws, nor would it make us call into question our inferential practices. However, this unshakable commitment to certain inferential principles which we find in ourselves cannot be turned into a justification of the ultimate grounds on which the necessity of the logical laws themselves rests. If one tried to turn it into such a justification, then this would amount to acknowledging that *logical necessity stems from us*, from our human nature, from our mental and physical constitution. Must we then conclude with Wundt (or Kitcher) that logic needs the theory of knowledge for its foundation and methodology for its completion, for otherwise 'logical laws would hold as given and unexplainable matters of fact'?²³ Frege held that logical laws are neither capable nor in need of justification: must we rest content with this statement?

Twenty years after the *Begriffsschrift*²⁴ Frege comes back to his simile according to which the logical rules are 'images' [*Abbilder*] of logical principles which because of their forming the basis of our thinking cannot be expressed in the concept-script, and he now says: 'In the domain of objects there hold certain laws, and one can understand how they are mirrored in the rules that hold good for the corresponding symbols.'²⁵ The possibility of a logical justification of an arithmetical sentence ultimately depends on there being objects standing in certain logical relations. But is there a way of forging a link between the grounds on which the laws of truth rest and our conscious striving to heed 'the rules which hold for the corresponding signs'? In GGA Frege himself gives us a clue as to where a possible link can be found when he sums up the discussion by remarking that the source of the dispute between the psychologistic logicians and himself lies in their different

²³ W. Wundt, *Logik*, vol. 1, Stuttgart, Enke, 2nd ed. 1893, p. 2.

²⁴ G. Frege, *Begriffsschrift: Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*, Halle, Nebert, §13; repr. in *Begriffsschrift und andere Aufsätze*, ed. by I. Angelelli, Hildesheim, Olms. Engl. transl. and ed. by T. W. Bynum, Oxford, OUP, 1972.

²⁵ G. Frege, *Über die Zahlen des Herrn Schubert*, Jena, Pohle, 1899, repr. in *Kleine Schriften*, ed. by I. Angelelli, Hildesheim, Olms, 2nd. rev. ed. 1990, pp. 240–61, p. 253; cf. also GGA, II, §§138–55.

conceptions of the True. I suggest that we interpret Frege's clue as follows: it is the notion of truth which informs our judgements and shapes our understanding of their contents which forces us to accept those propositions which we consider neither capable nor in need of justification and choose as axioms in our logical system. The reason why we acknowledge as valid the rules which we have, as it were, extracted from the axioms is that they can be used as inferential principles which preserve truth: this is why 'anyone who has once acknowledged a law of truth has by the same token acknowledged a law that prescribes the way in which we ought to judge, no matter where, or when, or by whom the judgement is made'.²⁶ We cannot transcend our notion of *truth*, and, for this reason we cannot transcend our logic either. Frege's notion of truth is realistic in Dummett's sense: truth is not only objective but 'independent of the judging subject'.

Anti-Psychologism and the Third Realm

In the previous section I have explored the path which leads to a defeat of psychologism through the theory of meaning. However, in many of his writings Frege alludes to a different path to securing the objectivity of content, which does not go via language but is straightforwardly metaphysical in its nature. So far we have been dealing with thoughts conceived as the senses of declarative sentences; however, in 1918 thoughts are characterized as self-subsistent entities, independent in the strongest possible sense of our apprehension of them.²⁷ All thoughts collectively are said to form a third realm. In the First Logical Inquiry the answer to the threat of psychologism seems to rest ultimately on a metaphysical postulation. One might go even further and claim that, since the whole theory of sense and reference is absent from the *Logische Untersuchungen*, Frege has here called into question his previous conception of semantics. To be sure, on account of Russell's contradiction, certain features of Frege's semantics had to undergo modification, but it is hard to believe that by 1918 Frege had abandoned also his entire conception of the sense and reference of linguistic expressions. Such a surmise—of which I do not know whether it has ever been put forward—would, I think, be untenable. In the précis of his logical views which Frege sent to Darmstaedter in July 1919 the theory of sense and reference still occupies the place of honour. Besides, in his *Logische Untersuchungen* Frege offers very detailed discussions of the way language functions (indexicals, negation, and truth-functional connectives) including an analysis of assertoric force.

²⁶ GGA, I, Preface, p. xvii; Engl. transl. p. 15.

²⁷ Cf. M. Dummett, 'Frege's Myth of the Third Realm' (1986), in *Frege and Other Philosophers*, pp. 249–62.

Now, independently of how we feel about third realms, as interpreters of Frege's philosophy we are faced with the question whether by 1918 Frege had reached the conclusion that a flight into a third realm was the only route to secure the desired objectivity of thoughts and thus the only defence against psychologism. Could it be that in the meantime Frege had become sceptical as regards the justification he had given in his previous writings of the role of the notion of sense in his account of logic and language? Could the way thought is exiled into a third realm in 'Der Gedanke' be seen as an admission of defeat? What evidence is there in Frege's writing?

Here is a suggestion, which I myself consider extremely feeble and which I explore mainly in order to dramatize certain difficulties that Frege's doctrine of sense encounters on its own ground. Frege might have come to the conclusion that the sense of a proper name is something that belongs to psychology; perhaps he found himself unable to rebut Russell's challenge, perhaps he could find no argument for explaining why a word—and a proper name in particular—bears in the language the 'objective' sense which Frege ascribes to it.²⁸ As a consequence he may then have assigned the function of securing objectivity to non-actual, a-temporal, a-spacial, self-subsistent entities.

These entities are intended to work like the gold that backs the paper currency of sentences, to borrow Geach's apt metaphor.²⁹ However, as Frege realizes, sheer postulation is useless unless accompanied by an account of how the market works, i.e. how the gold is related to the paper currency. At the close of 'Der Gedanke' he offers a streamlined account of judgements of perception, which perhaps could be generalized. It goes roughly as follows: a non-sensible element, belonging presumably to the third realm and therefore itself thought-like in character, has to be acknowledged, for it is its presence that discloses the outer world and turns sense impressions into perception and judgement. Perhaps the mutual understanding which we seem to reach through language is made possible by the presence of this non-sensible element: it turns our apprehension of the sentences of natural language (conceived as physical entities) into a grasp of the thoughts they help to convey. The thoughts thereby grasped transcend the world of inner experience and the outer world of sense impression. Perhaps following the same train of thought Frege conjectures in 1924 that there may be beings who, unlike us, can grasp these thoughts without the sensible medium of language and without any accompanying idea or representation:

²⁸ Cf. M. Dummett, 'Frege's Distinction between Sense and Reference' (1975), repr. in *Truth and Other Enigmas*, London, Duckworth, 1978, pp. 116–44, p. 130.

²⁹ P. Geach, 'The Identity of Propositions' (1967), repr. in *Logic Matters*, Oxford, Blackwell, 1972, pp. 166–74, p. 171.

The connection of a thought with one particular sentence is not a necessary one; but that a thought of which we are conscious is connected in our mind with some sentence or other is for us men necessary.³⁰

In 1924 the relative independence of thinking and speaking is seen by Frege as the only way of explaining how logical paradoxes are possible in the first place. This is not a matter of a struggle in which thinking is at war with itself, but the paradoxes are due to the imperfection of the linguistic means of expression which even creeps into the formal language. As a diagnosis of the source of the contradiction Frege's analysis must strike us as unconvincing; I merely mention it as testimony to a development of the view put forward in the First Logical Investigation. The exile of thoughts in the third realm could then be seen as issuing from Frege's failure to see how the objectivity of thoughts and their constituent senses could be secured in the absence of a viable semantics.

A symptom of a very different kind of dissatisfaction could be detected in the way Frege handles the example of Gustav Lauben in 'Der Gedanke'. He there comes to the conclusion that Leo Peter and Herbert Garner, who attach different definite descriptions to that name, 'speak a different language', for although they use the name 'Gustav Lauben' to refer to the same man they are unaware of so doing. Perceiving this as a defect, Frege modifies the example and decrees that 'Dr. Gustav Lauben' and 'Gustav Lauben' should be treated as different names. Now, the circumstance that two speakers might attach different definite descriptions to the same proper name does not turn its sense into something psychological. The different definite descriptions can be spelled out and become 'possession of many'. The name 'Gustav Lauben' in the mouth of Leo Peter and Herbert Garner still differs in status from an interjection like 'ouch!'—Frege's example of something psychological par excellence (GGA, I, p. xix).

It is undeniable, however, that the account of proper names given by Frege in 'Der Gedanke' bears a certain similarity to Russell's. In 1918 Russell takes the whole point of the proper names of a natural language to be that of calling up ideas to the mind of speaker and hearer. In his lectures on logical atomism of 1918 Russell candidly states:

When one person uses a word, he does not mean by it the same thing as another person means by it. I have often heard it said that that is a misfortune. That is a mistake. It would be absolutely fatal if people meant the same things by their words. . . . We should have to talk only about logic—a not wholly undesirable result.³¹

³⁰ NS, p. 288; PW, p. 269.

³¹ B. Russell, *The Philosophy of Logical Atomism*, repr. in D. Pears (ed.), *Russell's Logical Atomism*, London, Fontana, 1972, Lecture II, p. 50.

Frege would of course have heartily disagreed with this statement. And yet, Frege's failure to draw a line between modes of presentation of an object grounded in the specific information of individual speakers and modes of presentation immanent in language makes the contrast between his account of names and Russell's less sharp than it may look at first sight.

In footnote 2 of 'Über Sinn und Bedeutung' Frege had pointed out that different speakers may attach different descriptions to the name 'Aristotle' and that therefore the same statement would have a different significance for different speakers: 'Aristotle was born in Stagira' carries no news for those who attach to the name 'Aristotle' the definite description 'The greatest philosopher of antiquity born in Stagira', whereas it may be news for someone who uses that name to refer to Aristotle while unaware that Aristotle is the unique satisfier of that description. In 1892 Frege says that no harm ensues as long as these speakers attach the same reference to the name; in an ideal language, however, these oscillations of sense should be avoided. But in 1918 he says that people who attach the same reference to a name without being aware of so doing speak a different language.

Now, obviously each of us has a different set of proper names and attaches different information to the same proper names; and each of us knows it and cannot possibly intend his words to carry for the hearer the very same detailed information which they carry at a certain time for himself. However, we should not describe the situation by saying that each of us speaks a different language, but rather that each of us has different beliefs concerning the bearer of the name. At least we should say so from a Fregean standpoint: communication rests on that ingredient of sense which is shareable, communicable, and can be the common possession of many. The lack of a distinction between the information of a single speaker and community-wide information is a defect of Frege's doctrine but I believe that it is a defect which can be corrected without depriving the notion of sense of its cognitive significance. I think that Frege was not aware of the tension involved in this and that one should discard any conjecture according to which a perplexity as to the wholly psychological character of the senses of proper names might have shaken his faith in the non-psychological character of the senses of declarative sentences themselves in which those names occur.

But could an intervention of thoughts from the third realm help to disentangle the objective and the subjective ingredients in the thoughts conceived as senses of declarative sentences? The answer must, in my opinion, be negative. Not only have we no conception of how these entities are related to the thoughts expressed by the sentences of natural language, but we should not even know where to look for an answer to the question whether two people who utter the same sentence attach the same a-temporal thought to it. How can we know whether it is the same self-subsistent thought which two people get hold of when uttering the same sentence (without indexicals)? We cannot. Here an act of faith would be required. Frege should have remembered that at an analogous point he himself had detected

a decisive weakness of psychologism, against which he had fought throughout his philosophical career. If the sense of a sentence consisted in the representation which each of us attaches to it we would be at a loss what to make of the disagreement among different speakers of the language: for what looks like a disagreement could then turn on a hidden difference of representational content, while what looks like agreement could depend on ignoring such a difference. If what the speaker takes the sentences to mean is equated with the complex of representations associated with the sentence in the speaker's mind, then it is unintelligible how anybody can ever hope to get his meaning across. Of course, when thinking or speaking there may occur in our minds all sorts of images, feelings, and recollections, but they do not constitute the semantic contents which we intend to communicate.

A first step to take in order to get out of this tangle is to revert to the conception of sense as the sense of a linguistic expression. A second step, which Frege only hinted at, is to try to understand how, in detail, the notion of truth enters into the account of the practice of assertion. As I said, Frege's conviction was that only a concept of truth realistically understood could feature in an adequate account of assertion—quite independently of the threat of psychologism. This, however, is a very large claim which I do not propose to take up here.

Appendix

After having distinguished between the three realms—of thoughts, of things in the external world, and of ideas—Frege sets out to forestall a possible objection which could be raised against his neat tripartition.

The section of 'Der Gedanke' devoted to rebutting the objection to the three realms bears many similarities to a section of his *Logik* of 1897,³² where he complains about the sliding into idealism of physiological psychology and remarks on the strange coincidence between idealism and realism. The editors of the *Nachlaß* append a footnote in which they refer to Wundt's work *Grundzüge der physiologischen Psychologie* of 1874. For reasons I will not enter here I doubt very much that Wundt was Frege's target in 1897, and I am sure that Wundt cannot be the thinker censured in 1918. In a similar vein Frege refers in 'Der Gedanke' to the 'Sinnespsychologie' and notices the odd transformation of realism into idealism. Who is the philosopher whose 'odd objection' he seems to hear? My surmise is that the philosopher in question is Ernst Mach.

It seems very plausible that what Frege had in mind was the author of the *Analyse der Empfindungen*, whose first edition was published in 1886. In Germany

³² NS, pp. 156–7; PW, pp. 144–5.

Wilhelm Schuppe and Richard Avenarius, the editor of the *Vierteljahrsschrift für wissenschaftliche Philosophie*, the journal in which Frege published his 'Über Begriff und Gegenstand', put forward views very similar to those of Mach. Mach is the thinker from whom derives the notion of monism (that is, the coincidence of idealism and realism) and the thesis that we start with sensations of external things only to discover that external things are posits useful in the economy of thinking. Much the same applies to the 'I' conceived as a substance. Hence the well-known slogan 'Das Ich ist unrettbar'—'The I is irrecoverable'. Apart from these analogies with Frege's objector, what really clinches the case is the following statement from 'Der Gedanke':

Does everything need an owner without which it can have no existence? I have considered myself as the owner of my ideas but am I not myself an idea? It seems to me as if I were lying in a deckchair, as if I could see the toes of a pair of polished boots, the front part of a pair of trousers, the waistcoat, buttons, some hair of a beard, the blurred outline of a nose. Am I myself this entire complex of visual impressions, this aggregate idea? It also seems to me as if I saw a chair over there. That is an idea. I am not actually much different from the chair myself, for am I not myself just a complex of sense-impressions, an idea? Why need this chosen owner of these ideas be the idea I like to call "I"?... The dependence which I found myself induced to ascribe to the sensations as contrasted with the sentient being disappears if there is no longer any owner. What I called ideas are then independent objects. No reason remains for granting an exceptional position to that object which I call "I".³³

I am sure that Frege's contemporaries had no difficulty in reading between the lines and recalling the famous drawing in chapter 1 of Mach's *Analyse der Empfindungen*.

Gottfried Gabriel has pointed out to me that this suggestion is not new, but has already been put forward by Gregory Currie in the last chapter of his book on Frege.³⁴ Currie, however, does not greatly elaborate his own surmise, and this is somewhat surprising, if we remember that the declared aim of his book is to show that Frege conceived of his own work in philosophy as a contribution to (Kantian) epistemology. Currie correctly observes that the author of the First Logical Inquiry 'was capable of deviating markedly from Kant's position',³⁵ and this too reinforces

³³ G. Frege, *Der Gedanke: Eine logische Untersuchung*, repr. in *Kleine Schriften*, pp. 342–62, p. 356; Engl. transl. Oxford, Blackwell, 1984, p. 365.

³⁴ The surmise is formulated at p. 182 of Currie's book *Frege: An Introduction to His Philosophy*, Brighton, Harvester Press, 1982, and is spelled out in some detail in the last footnote of the last chapter of the book.

³⁵ *Ibid.*, p. 185.

the impression that a comparison between Mach's and Frege's respective contributions to epistemology would have been highly appropriate.

In 'Der Gedanke' Frege tries to meet the challenge of the sceptic who questions the way in which the distinction between ideas and thoughts is drawn. Having a thought differs dramatically, in Frege's opinion, from our having a *Vorstellung*: an idea or a sensation. Ideas, unlike thoughts, require a bearer, actually, in Frege's opinion, *one* bearer, an individual self. The sceptic questions not only our naive belief in an external world, independent of the inner world of consciousness and ideas. He also questions the belief that there is such a thing as a self which may count as the bearer of representations and is clearly different from them. The sceptic whom Frege alludes to between the lines by putting forward a no-ownership thesis for feelings, sensations, and ideas, undermines the distinction between what is subjective (and necessarily private) and what is objective (and potentially accessible to all). If ideas need no bearer, then they are not different from thoughts. But is the sceptic right in suggesting that the contents of my consciousness alone can be objects of my thought? If at least one object could be found which does not belong to the contents of my consciousness, then the sceptic's reduction of the world to *my* world of ideas, and of the latter to nobody's world in particular (on account of the no-ownership thesis), would be blocked. In his reply Frege does not distinguish sharply between the thesis that ideas need no bearer and the thesis that ideas need no one (single) bearer. He argues that there is at least one *object* (namely myself) which can be the object of my thinking without being an idea. It may very well be that I may think of myself under this or that idea, but I am not myself the content of that idea.

Frege correctly points out that whenever I reflect upon myself it is myself and not an idea of myself which is the object at which my reflection is aimed. Yet one may grant the point without also granting that the self at which I seem to be directing my thought is itself an object—i.e. a self-subsistent substance. The gist of Kant's criticism of rational psychology was precisely that the latter turns the self of transcendental apperception into a substance, conceived as a simple, enduring, and immutable object. But—Kant contends—there is no such object for us to know, no object which is to be found by inspecting the world of inner experience or the outer world. There is no trace of Kant's argument in Frege's purported demonstration of having found at least one thing which can be made the object of one's thought without being turned into an idea under which one thinks of such an object. Be that as it may, the self does not stand a good chance of fulfilling Frege's requirement for what to count as an object, i.e. something which subsists independently of its ever being thought of by any thinking being.

Frege's answer is so brief, that it would be rash to classify it under this or that well-known label. It is however an answer which is distinctly unsympathetic to a naturalistic dismissal of the self. Frege also hints that certain thoughts, on account of their intrinsic nature, are not open to sceptical doubts: thus I may be in doubt

whether there is a green leaf over there, but not whether I am having a visual impression of something green. It would be hasty to assimilate the kind of certainty Frege alludes to here to the certainty Russell claimed for sense data. In fact, as we saw above, Frege's central claim is that a non-sensible element is necessary to transform sensation into perception. This applies, I think, to entities and events in our inner world, as well as to those of the external world. It is the presence of a non-sensible element which opens up the outer world to us, 'for without this non-sensible something, everyone would remain shut up in his inner world'.³⁶ The distinction between what is true and what is true for me would be lost, and the idea, so dear to Frege, that mankind has a common treasure of (true) thoughts would be unintelligible. Whether this non-sensible element is to be construed as a thought-like ingredient, or as the faculty of reason *tout court*, is a question which may here remain unanswered.

In conclusion, Frege's so-called epistemology does not provide him with any good argument for deflecting the sceptical challenge. Here, too, a more promising line of attack on sceptical doubts can be found by turning our attention to the role language plays in shaping our idea of objectivity: I doubt, however, that such a line of argument could persuade a realist of Frege's calibre.

³⁶ Frege, *Der Gedanke*, p. 360; Engl. transl. p. 369.

Frege, Peano, and Russell on the Primitive Ideas of Logic

The logic that is taught today in the departments of mathematics and philosophy is rooted in the work of Frege, Peano, and Russell. Each of these pioneers conceived of logic in a way that was quite different from the other two, and these differences emerge with striking vividness in the epistolary exchanges and in the writings from the period between 1892 and 1913. Peano wrote reviews of the first volume of the *Grundgesetze der Arithmetik* (= GGA) and of the first two volumes of *Principia Mathematica* (= PM); Russell, in *Principles of Mathematics* (= PoM), dedicated a chapter to Peano and an appendix to Frege. From the scientific exchange with Peano Frege received the motivation to make his views more precise, without however making substantive changes. From the correspondence with Russell Frege became convinced that the contradiction discovered by Russell fatally compromised any proposal of logical foundation of arithmetic—not only the one presented in GGA.

Although today there is a flourishing of specialized studies on Frege, Russell, and, to a lesser extent, Peano, there is not, to my knowledge, any substantial study in which these three protagonists of modern logic are studied in their mutual relations. The foundations for a serious study have already been laid: think, for example, of the introductions by Hans Hermes to the correspondence between Frege and Peano and by Christian Thiel to the correspondence between Frege and Russell and Frege and Jourdain (Frege 1969), or, as far as Russell is concerned, of the third volume of the *Collected Papers*, published in 1993, which bears the title *Towards the 'Principles of Mathematics': 1900–1902*, edited by Gregory Moore. The third part of this volume is entitled 'After Peano: Foundations of Mathematics', and contains, among other things, an article by Russell, 'Recent Italian Work on the Foundations of Mathematics' (1901), probably intended for the journal *Mind*, which remained unpublished for unknown reasons (until 1993). The purpose of the article was to make known to the English-speaking public the work of Peano and his collaborators whom Russell had met in person in August 1900 at the International Congress of Philosophy in Paris, of which the section on logic was organized by Louis Couturat. As far as Peano is concerned, except for the works by Kennedy—the monograph on Peano and the article of 1975 in which nine of Peano's letters to Russell are published—and some works by Quine (1987a and 1995a, ch. 3 and Appendix) and, in Italy, by Gabriele Lolli (1983), I do not

think that there is a fervour of study comparable to that which today surrounds the work of Frege and Russell. In short, much has been done, but a lot remains to be done on the historical and philosophical side to refine a synoptic overview of the intertwining ideas, suggestions, and reciprocal borrowings which took place during this decade between Peano, Russell, and Frege. In the following, I will try to develop this comparative study by drawing on some of my previous work (Picardi 1998, and Picardi 2000).

1. 1892–1914: A Synoptic Overview

A brief survey of the timeline may not be out of place. Frege's *Begriffsschrift* (BS) was published in 1879, and ten years later, in 1889, Peano's *Arithmetices Principia, Nova Methodo Exposita* was printed, containing the first axiomatization of arithmetic (Peano's famous five axioms, plus the three concerning the property of identity). Similar formulations of the axioms are found in Frege's *Grundlagen der Arithmetik* (1884) and in Dedekind's writings (both in his 1888 paper, *Was sind und was sollen die Zahlen?*, and in his famous letter to Keferstein of 27 February 1890, where Dedekind explicitly acknowledges the similarity between his own characterization of natural numbers and Frege's). In 1893 the first volume of the *Grundgesetze der Arithmetik* was published; between 1894 and 1899 three versions of the *Formulaire des mathématiques* were published. In 1895, in the fifth volume of *Rivista di matematica* (= RdM), Peano's review of the first volume of GGA appeared; in September 1896 Frege sent Peano a letter to better explain the content of his book, which was published along with a short reply by Peano in the sixth volume of RdM, printed only in 1899. Peano's real answer to Frege's objections is contained in *Studii di logica matematica*. It is not known exactly when the correspondence between Frege and Peano began; in all probability around 1892, since in the draft of a letter from Frege to Peano there is an allusion to Peano's article of 1891, 'Sul concetto di numero'. In the lecture given in Leipzig in 1896 and published in 1897, under the title *Über die Begriffsschrift des Herrn Peano und meine eigene*, Frege alluded to an earlier lecture on the same subject given in Lübeck. Frege's interest in Peano's work did not stop in 1903 with the end of correspondence, but remained at least until 1914, as evidenced by several passages contained in the writings published posthumously (*Scritti postumi* = SP) in particular *Logical Defects of Mathematics* (1899–1903), *Reasons for the Particular Rigor of My Canons in Defining* (1897/8), and *The Logic in Mathematics* (spring 1914). Also in the notes that Rudolf Carnap took of the lectures given by Frege in Jena on BS in the Wintersemester of 1910–11 and in the Sommersemester of 1913, which, thanks to Gottfried Gabriel's transcription work, have become accessible to scholars in 1996, there is an explicit reference to Peano concerning the notion of apparent variable and a tacit one regarding the topic of definitions.

Peano, in turn, in the years between 1895 and 1901, rewrote multiple times the definitions, proofs, and the choice of primitives in the subsequent versions of his

Formulario, but he did not substantively change his ideas either on the philosophical side (indeed he punctiliously abstained from having, or at least expressing them) or on the foundational side (where he had some, though they were expressed in an incomplete and unsystematic way). Peano embodies, in some respects, the prototype of the mathematician ‘untouched by philosophy’, rather intolerant of philosophy, dialoguing only with mathematicians and, after 1910, not even with them, but preferably with the experts of artificial scientific languages (Esperanto, Volapük, Latino sine flexione, Interlingua). He too, however, probably benefited from the exchange with Frege and Russell—for example, by recognizing the importance of the logic of relations, with which Russell aimed to ‘complete’ the work of Peano’s *Formulario*. It is possible, for example, that it was Frege’s remarks that convinced Peano of the need to distinguish between ‘equal by definition’ and ‘equal’ (an amendment found in the *Formulario* of 1897), in order not to incur the criticism of circularity in the definition of identity. Perhaps his pessimism about the possibility of identifying the primitive ideas of logic also has its roots in the controversy with Frege. Unfortunately, the part of the epistolary exchange between Frege and Couturat concerning Frege in the years 1899–1906 has been lost. In July 1899 Couturat, a tireless organizer of conferences and an apostle of the ‘logique algorithmique’ in the *Revue de Métaphysique et de Morale*, invited Frege to participate in the Congrès International de Philosophie held in Paris in August 1900—a conference that Russell and Peano and his collaborators attended too. Frege turned down the offer but accepted to be part of the Comité de Patronage.

There are objective reasons that explain the different degree to which each of the three profited from the ideas of the other two: in 1903 Peano and Frege had reached and passed the acme of their creative phase, while Russell was entering into his most productive period. However, even though Russell was the one who had the most to learn, he didn’t always learn the right thing. To give just two examples: the encounter with Peano entrenched in him the (debatable) belief in the foundational importance of the notion of *variable*, and in the dialogue with Frege he did not fully appreciate Frege’s main idea, namely that of the construction of a *semantics* for a (formalized) language, in harmony with the context principle and the compositionality principle.

As far as Russell and Peano are concerned, it is crucial to recall Russell’s claim (1967, p. 217) that ‘[t]he Congress was the turning point of my intellectual life, because I there met Peano’. In Paris, Russell presented a talk at Louis Couturat’s invitation, titled ‘Is Position in Time Absolute or Relative?’ (which Hans Freudenthal, mercilessly, would characterize as ‘a philosophical article in the worst sense of the word’, cf. Kennedy 1973), which marks Russell’s abandonment of the philosophy he had learnt from Bradley.¹ The encounter with Peano

¹ In sending Frege a copy of this article Russell motivates sending it by saying that it contains ‘a fierce criticism of psychologism and Kantism’ and is therefore similar in spirit to Frege’s point of view (Russell to Frege, 10 July 1902, WB, p. 220).

convinced Russell of the fundamental importance of the ideas of Italian logicians. In particular, Russell realized that he needed to revise parts IV and VI, and to write parts I and II for the first time (cf. Grattan-Guinness 1996–7). Complicating the final editing of PoM was the ‘discovery’ in 1902 of Frege’s logic, and the consequent transfer of Russell’s interest and admiration from Peano to Frege.

The chronology of the Frege–Russell epistolary is too well known to be recalled here (see, for example, Picardi 1982). Volume 3 of Russell’s *Collected Papers* contains important material to understand better both the impact that the meeting with Peano had on the young Russell, and the process through which, thanks to the study of Peano’s notation and ideas, Russell better understood Frege’s work, and became convinced of the philosophical and theoretical superiority of Frege’s approach over Peano’s. Tangible signs of this recognition can be found in the writings of those years. Both in the article devoted to Peano and his school (Russell 1993, p. 352), and in §§ 21 and 32 of PoM, and in the popular writing of 1901, *Recent Work on the Principles of Mathematics*, Russell appended footnotes in which reference is made to Frege (Russell 1992, p. 268, footnote added in 1917, on occasion of the reprint of the article in *Mysticism and Logic* (1918)). These notes serve as a corrective to claims that attribute to Peano and his school complete credit for the edification of mathematical logic.

2. The Primitives of Logic: Peano and Russell

What then, by his own admission, had Russell learned from Peano? In part the choice of primitive notions, namely the universal quantifier, the relation of individuals’ membership in a class, and the notion of class abstraction. But it’s not just that, though this is already quite a lot. Russell was indebted to Peano also regarding the key role to be assigned to set theory and to the idea of expressibility of mathematical theories in the symbolic language of the new logic. In a note from October 1900 Russell states that he understood from Peano that the part/whole relation has no place in logical calculus, which is rather based on the notions of implication, “and”, and “not” (Moore 1993, p. xxviii). Russell thus describes his debt to Peano and his school:

The first advance consisted in separating propositions of the form “Socrates is mortal” from propositions of the form “All men are mortal” . . . Neither logic nor arithmetic can get far until the two forms are seen as completely different. . . . The full statement of “All Greeks are mortal” is “For all possible values of x , if x is Greek, then x is mortal” . . . The statement says nothing about Greeks in particular, but is a statement about everything in the universe. (Russell 1959, p. 66)

This reconstruction of what he had learned from Peano is suffused with a Fregean ‘colouring’ that Peano would not have liked. It underlines the complete generality

of logic, defined as the study of those constants that recur in propositions concerning everything in the universe; it is also said that in the formulation of the propositions of 'pure' mathematics only logical constants occur. Neither logicism nor the insistence on the universality of logic are themes we find in Peano. In Peano's eyes the *complete* generality of logic, to which Frege and the young Russell aspired, is an exigency that does not arise from mathematical practice. A logic that focuses on 'everything in the universe' does not interest Peano. Only the logic that he first called 'mathematical', i.e. the logic designed for the applications of mathematics, is of interest to him. But in these applications it is always necessary to indicate—and here is one of the points of irreconcilable disagreement with Frege—for which objects both the operations introduced by definition and the signs of equality and implication appearing in the formulas are defined. Even the logical constants must have indices. Purely 'logical' constants, i.e. without indices, are admissible for Peano only in propositional calculus, where, precisely, there are no variables (cf. Peano, *Notations*, p. 139). We will return to the topic of the variable in a moment.

Peano's real philosophical contribution (the gist of the lesson Russell learned from Peano) is embedded in his symbolic notation—in the epsilon indicating the relation of membership of an individual in a class, as distinct from that of class inclusion, in the inverted iota, in the distinction (introduced in 1897) between individual and singleton, in the sign for the existential quantifier, in the notation for class abstraction, 'the class of x 's satisfying a certain condition F ', just to name a few (cf. Quine 1987a). Thanks also to PM the notation devised by Peano has become part of the current logical notation, and Peano's tacit philosophy is accessible to all those who master this notation (cf. Lolli 1983, on Hilbert and Peano).

3. The Task of Ideography: Frege vs. Peano

Reviewing in 1913, in *latino sine flexione*, the first two volumes of PM, Peano (Peano 1958, p. 391) observes:

In *Formulario*, logica-mathematica es solo instrumento pro exprime et tracta propositiones de mathematica commune, non es fine de se; logica-mathematica es explicato in 16 pagina, una hora de studio suffice pro cognosce quod es necessario in applicationes de isto novo scientia ad mathematica. Libro de nostro Auctores tracta logica mathematica ut scientia in se, et sua applicationes ad theoria de numero transfinito de vario ordine; quod exige symbolismo multo plus amplo.²

² 'In *Formulario*, mathematical logic is only an instrument to express and treat propositions of common mathematics, it is not an end in itself; mathematical logic is explained in 16 pages, one hour of study suffices to know what is needed in the applications of this new science to mathematics. The book

In PM, unlike in the *Formulario*, mathematical logic is investigated as ‘scientia in se’, and this explains the amount of prose devoted by the authors to logical-philosophical questions. In the review, Peano is careful not to go into the merits of all this prose and limits himself essentially to show how to translate in the *Formulario* the notation of PM and where PM is indebted to the *Formulario*. Not a single mention is devoted to the theory of types, incomplete symbols, or the principle of the vicious circle.

The previous remark contains *in nuce* Peano’s entire philosophy: the logical notation is not investigated as such, as ‘scientia in se’ or as part of a philosophical logic, but only inasmuch as it serves its applications to mathematics. By ‘applications to mathematics’ Peano essentially means the work undertaken in the *Formulario*, which is halfway between an encyclopaedia of the most significant mathematical propositions and a review of the various proofs of which a single proposition is liable (cf. Lolli 1983). The diversity of the demonstrative methods depends, in turn, on two factors: the system of primitives and definitions adopted on the one hand, and the kind of mathematical theory in which a certain theorem is applied (analysis, algebra, geometry) on the other. The latter feature sets Peano apart from both those who, like Hilbert, will pursue the ideal of the *Methodenreinheit* (‘the purity of methods’ of proof), and those, like Frege and Russell, who pursue a foundational ideal.

Peano did not cultivate any foundational project, and explicitly denied that the concept of number could be defined by the use of notions and demonstrative principles belonging to logic. On the other hand, Peano considers the notion of class and the relation of belonging of an element to a class as primitive logical notions. Even those who do not share Quine’s point of view, who sees in the sign of membership the first non-logical operator one encounters almost ‘spontaneously’ in the most elementary regions of logic, will have to agree that the demarcation between logic and mathematics suggested by Peano isn’t entirely satisfactory. Peano takes it for granted that these notions belong to mathematical logic in the strict sense, but, precisely for this reason, it is not clear which notions pure mathematics should contain other than those that fall under the jurisdiction of logic. For Frege and Russell the notion of class and the range of values of a function belong, derivatively, to logic: this assumption is one of the pillars on which the logicist construction is based. Frege and Russell (with due distinctions) consider the notions of propositional function in intension and of concept (understood as an application that leads from objects to truth values) as primitive ideas of logic. The notion of class and the notion of extension of a concept (and of the range of values of a function) are derivatives of the former, though of crucial

of our Authors [Russell and Whitehead] treats mathematical logic as a science in itself, and its applications to the theory of transfinite numbers of various orders; this requires a much richer symbolism’ [translated by the editor].

importance for the logicist foundation of arithmetic. For Frege the abandonment of logicism results in a new version of the ideography; among the primitives there is no longer the second-level function indicating the transition from the concept to the corresponding range of values (cf. Frege 1996).

Also, in the review of the first two volumes of PM there is, *en passant*, a remark about the assertion sign that says a lot regarding Peano's opinion about the philosophical ideas that were so dear to Frege and Russell. As the first step in the translation of the formulas from PM into the language of the *Formulario* Peano mandates the removal of the sign of assertion: 'signo de assertione es implicito in numero de propositione' ('The assertion sign is implicit in the number of the proposition'), and as such it can be dropped. Evidently Peano not only did not appreciate the recognition given to Frege in PM's notation, but he saw in it an implicit allegiance with Frege in the controversy concerning the primitive notions of logic.

In the list of primitive signs of the respective ideographies it was precisely the compound sign of judgement (*Urteilsstrich*) present in the BS and absent in the *Formulario* that made Peano say (a bit hastily) that the Fregean ideography employs five primitive ideas (negation, implication, universal quantifier, vertical stroke of judgement, and horizontal stroke) compared to the three in the *Formulario* (conjunction, negation, and the inverted 'C' sign for formal implication) and that therefore 'the system of the *Formulario* corresponds to a deeper analysis'. But in what sense—Frege asks in the Letter to the Editor—would the analysis of the *Formulario* be deeper than that of the *Begriffsschrift*? The comparison presupposes homogeneity of intentions, and Frege rightly observes that Peano's aims in the *Formulario* are different from his in the *Begriffsschrift*. What then were Peano's purposes?

In answering this question, one needs to keep in mind that the *Formulario* is a work in progress: in the 1897 paper Frege refers to Peano's 1894 work, *Notations de logique mathématique*, which had appeared as the introduction to the *Formulaire de mathématiques*; in the posthumously published writings Frege critically discusses the 1897 version of the *Formulario*. Peano's short reply to Frege's letter and his real answer, contained in *Studii di logica matematica*, are formulated from this new perspective.

In *Notations de logique mathématique* Peano states that it was his success in the axiomatization of arithmetic that drove him to a similar search for the primitives of mathematical logic. In his review of GGA I Peano declares himself convinced that however one chooses the primitive notions and the axioms, all ideographies will converge on a single set of logical propositions:

Since mathematical Logic does not consist of a series of arbitrary conventions varying at the whim of the author, but rather in the analysis of ideas and of primitive and derived propositions. *And this analysis is unique.*

(Peano 1958, p. 190).

However, in later editions of the *Formulario* Peano confesses that ‘it is easier to recognize how many and what are the primitive ideas in Arithmetic and Geometry, than in Logic’ (Peano 1899, 1958, p. 204). He nevertheless continues to be convinced that all adequate ideographies are only notational variants of each other, although he never showed whether and how such a conviction can be proved. One cannot exclude the possibility that Frege’s remarks, made in the correspondence, about what is to be considered a primitive notion of logic, whether primitive ideas introduced by stipulation (class, membership relation, etc.) belong there, and whether the concepts of *truth* and assertion can aspire to this status, even when they do not find expression in symbolic notation, also contributed to Peano’s scepticism. Though the question ‘What is a logical notion?’ is raised by Frege in 1896, Tarski will search for an answer well into the 1970s (Corcoran 1987).

Often Peano expresses himself as if what discouraged him from this research was not only the difficulty of getting to the bottom of the problem but also the belief that the choice of primitive notions with their axioms and of the defined notions with their theorems is a matter of mere convention. In contrast, Russell, a fervent follower of Peano in 1900, complains about Peano’s abandonment of the search for the minimum number of primitive notions of logic. It is not by chance that in the second introduction to PM he will consider the notation using the Sheffer stroke one of the most significant discoveries made in logic between 1913 and 1925.

Frege agrees with Peano that the ideography should be used to represent mathematical reasoning. But, unlike Peano, Frege doesn’t see himself as just building a language capable of symbolizing the modes of inference employed by mathematicians; it is necessary to show that these modes of inference can be executed by means of rules of a purely formal nature. To show that arithmetic is independent from intuition it is necessary to build a formalized language that allows one to show first that every inference performed in an informal way is executable based on a formal rule and therefore does not contain an appeal to intuition; and second that those inferences that apparently seem to require the intervention of intuition are actually representable by formal rules. The ambiguous role of Peano’s mathematical logic is effectively noted by Frege. In his 1896 paper, BSP, Frege took up the (Leibnizian) distinction between *lingua characteristica* and *calculus ratiocinator* he had already employed in ‘Anti-Boole’ (unpublished until 1969) and applied it to Peano:

In Leibniz’ terminology one could say: Boole’s logic is a calculus ratiocinator, but not a *lingua characteristic*. Peano’s mathematical logic is above all a *lingua characteristic*, besides also a calculus ratiocinator, while my conceptual writing is supposed to be both with equal emphasis. Actually Peano should not leave the Boolean notation unaltered, for it is not very apt to grasp the content—particularly mathematical content. (Frege 2001, p. 103; trans. by the editor)

Frege's impression that in Peano the logical calculus is subordinated to the *lingua characteristic* is entirely justified by the lack of a systematic formulation of the rules of inference, proof, and substitution. In Peano, the logical machinery for carrying out proofs is mostly implicit and does not blend with the logical alphabet used to express the content, to the point that some have seen in it a simple tachygraphy and shorthand (cf. Couturat's 1906 defence of Peano against Poincaré's criticism). The greatest advantage of the ideography—as Frege had observed in 'Booles rechnende Logik', in response to Schröder's review of the BS—is when 'the content is not only indicated but constructed from its component parts by means of those same signs employed to calculate' (PS, p. 13). By 'calculation' Frege does not mean a mechanical procedure, that is, an actual procedure of calculation, executable even by a machine, or 'without thinking' as he puts it. One reason why logic is not reducible to a purely mechanical process is that it involves the extraction of *new* complex concepts, not explicitly contained in the premises of a deduction (cf. Dummett 1991). This also helps to explain in what sense logical deduction can 'broaden' our knowledge without involving elements inferred, in a Kantian way, from the pure intuition of time or from sensory experience *tout court*. Also, for this reason the invention of a *lingua characteristic* and that of a *calculus ratiocinator* must, according to Frege, proceed in step.

4. The Variable: A Primitive Logical Idea?

In a letter to Frege dated 10 July 1902 Russell writes:

My book is already in print: I will treat your work in an Appendix, because it is now too late to talk about it in detail in the text. When I read your *Grundgesetze* for the first time, I could not understand the conceptual notation: only when I began to notice the gaps in Peano's notation did I succeed. Unfortunately, by then my book was already complete. (WB, p. 220)

This is not a mannered statement, although it is not easy to say exactly to what extent and in what respect Russell profited from reading Frege's work.³ Perhaps it is possible to see the merits of others without making them one's own, or rather to use the ideas of others to develop one's own by contraposition. In the correspondence between Frege and Russell there are substantial reasons for

³ Judging from the summary of the article on Peano and his school, which has remained unpublished, it should have contained a section entitled 'Philosophical merits: fundamental nature of epsilon: Padoa è. Comparison with Frege. Kinds of Df' (Russell 1993). This part is not developed in the article Russell sent to Stout. Probably the failure to develop this section can be explained by Russell's statement in the letter to Frege, quoted above, that he did not fully understand its meaning at the time (in 1901). Russell had first read the first volume of GGA in 1897, apparently without much profit.

philosophical disagreement, and these will deepen over the years. These philosophical disagreements made Russell's fruitful appropriation of Frege's ideas virtually impossible.

Perhaps for this reason even in PM, and not only in PoM, Peano's influence continues to be more visible than Frege's. Emblematic of Russell's synthesis of ideas taken from Peano and Frege is the notation for class abstraction: the circumflex accent on the variable of the operator is a tribute to the smooth breathing that in GGA indicates the second-level function that indicates the transition from the concept to its extension, while the logical scope of the operator is rendered through the Peanian 'the x s such that they satisfy a certain condition' (cf. Peano 1896–7). Russell also affixes the circumflex to the notation for propositional function, perhaps also here following a suggestion of Frege's, who in a letter proposes the use of the rough (instead of smooth) breathing to indicate the function (the *Bedeutung* of a functional expression, i.e. something unsaturated), divorced from its arguments (Frege WB, p. 218, cf. also the posthumously published piece, 'Observations on Sense and Meaning', now in Frege 2001, p. 79, for a related notational proposal). The explanation that Russell gives of this notation is at odds with the one that Frege would have given: in PM we read in fact that a propositional function 'ambiguously denotes its values' or even that it has an ambiguous denotation. A similar terminology is used in the illustration of real variables: the primitive notion seems to be that of 'ambiguous assertion', or 'ambiguous member' of the values of the propositional function.

In his writings Frege employs the distinction between 'andeuten' (to allude, to indicate) and 'bezeichnen' (to designate) to illustrate one of the roles of variables in logic and mathematics with respect to constants and proper names. In no case is the task of the variable (real or apparent) to designate ('bezeichnen') anything. One of its uses is to indicate the number and type of arguments with which a function is saturated. For Frege there are neither 'ambiguous assertions' nor 'ambiguous values' designated by the x of ' $f(x)$ ': the variable has here the only purpose of alluding to the scheme of the function, deduced from a series of equiform atomic statements. The variable—Frege tirelessly points out—is not part of the function. In order to grasp the essence of the function, it is necessary to bring out, with the help of an appropriate notation, its unsaturation. This requires a reformation of the usual mathematical notation. The functional expression must always be accompanied by the indication of the number of its argument-places and the level of its arguments. In GGA these argument-places are indicated with lower-case Greek consonants, letters that do not belong to the object language but to the metalanguage ('Darlegungssprache') in which the expressions of the object language are articulated. Only with these ideographic devices will those definitions (nowadays obsolete, but not at Frege's and Peano's time) in which the function is characterized as a variable quantity or even as an indefinite variable number be avoided.

But that is not the only role of the variable. The variable, observes Frege, resembles a pronoun, with the help of which we obtain the accounting of the coreferential links that exist between the various occurrences of the same letter (indicating the variable) in a formula. Since in Frege the function of the variables is closely related to that of the universal quantifier, he did not reach the combinatorial intuition that Quine attributes to Schönfinkel to clearly separate the idea of generality from that of combinatorial device. Unlike Schönfinkel's combinatorial functions, Quine's 'predicate functors' behave like adverbs that when applied to verbs or enunciative clauses give complex verbs and enunciative clauses. 'It is in predicate functors that the analysis of the variable is extricated from set-theoretic entanglements and rendered coextensive with elementary logic' (Quine 1995, p. 101). Discussing Peano's notation for predicative abstraction ('the x s such that') Quine states:

The variable comes to appear in its true light as purely a means of identifying and distinguishing the referential places in a sentence, and has nothing to do with 'all' or 'some', which is the business of ' $\forall$ ' and ' $\exists$ '. As for the etymology of 'variable', it should be studiously ignored: it rests on a pernicious old mathematical metaphor that is happily fading away. (Quine 1995, p. 32)

One of the reasons why Russell remained, at least until 1925, of the opinion that the free variable (and its linguistic equivalent 'any') was a primitive notion of logic is to be found in the use of free variables as parameters, to indicate an arbitrary element of the domain. Relative to this domain, parameters behave as proper names (or constants). If a propositional function holds for this arbitrarily chosen element, we can assert its universal closure (subject to the usual caveats about other constrained variables that the propositional function possibly contains). The use of free variables as parameters is particularly insightful in demonstrations that discharge hypotheses, i.e. in systems of natural deduction, although the idea behind this way of reasoning is familiar from geometric demonstrations where one reasons 'on any triangle' to prove theorems that hold for all triangles. Russell, an attentive reader of Berkeley, translates into logical terms the controversy endemic in empiricist circles as to what is to be understood by a generic triangle, obtained by (psychological) abstraction from the specific triangles on which the demonstration takes place. Frege in the *Foundations of Arithmetic* (1884, §47) had dismissed the entire issue by saying that 'undetermined object' is nothing more than the equivalent of 'concept', and moreover an unhappy and contradictory synonym. Russell, based on discussions with Ramsey and Wittgenstein, will come to a similar conclusion.

There is a third idea that Russell proposes when he insists on the foundational importance of the notion of variable, namely that in (constructive) demonstrations of existence theorems one must exhibit a specific example of the number that

enjoys a given property (cf. PM, Introduction, ch. 1, 'Ambiguous Assertions and the Real Variable'). Here too, more than a specific use of variables, we should speak of different kinds of proofs. Another idea that Russell (and Peano) built into the notion of the variable was a kind of logical validity—the primitive idea of 'true in all interpretations'. In the absence of the (model-theoretic) conception of interpretation, the substitutional conception of validity may have occurred to Russell and Peano. A formula is logically valid (in the substitutional interpretation) if all the statements obtained by substituting expressions of the appropriate category for the schematic letters are true. But here the relevant notion is not that of variable, but that of schematic letter and substitution-example. Naturally the variable is also given the task of expressing the notions of 'all' and 'some', i.e. the generality. The problem once again has nothing to do with the variable, but concerns the appropriate way to understand the universal quantifier and the existential one (infinite conjunctions and disjunctions? modal operators of necessity and possibility? second-level concepts?). Talk in terms of 'ambiguous assertion' only confuses ideas.

Only in 1925, in the second introduction to PM, does 'any' lose its privileged role as a primitive idea of logic. Of the variable it is now said: 'the essential use of the variable is to pick out a certain assemblage of elementary propositions, and to enable us to assert that all members of this assemblage are true, or that at least one member is true' (PM, p. xxviii). This conception of the variable as a device for grouping propositions exhibiting a common pattern is closer to Frege's conception than the characterizations that Russell gave in his earlier works. In the 1925 introduction there is no mention of 'ambiguous assertion', 'ambiguous value', or 'ambiguous denotation'. Even the notion of elementary proposition sketched in 1925 owes much to the *Tractatus Logico-Philosophicus* (TLP) and to Ramsey's ideas.

A further burden carried by the variable was to signal the functional dependence between arguments and values, or, as they say, the law that makes each argument correspond to a unique value. This idea, i.e. the notion of a function, is indeed primitive and indefinable even for Frege, and is encapsulated in the notation for applying a function to its arguments to obtain a unique value (GGA, I, §1). While Peano had used the membership sign for this case as well, Frege uses the simple juxtaposition of the functional expression to the expression indicating the argument. What is true on the level of expression is also true on the ontological level: concept (a special case of function) and object are, so to speak, made for each other, and in order to complete each other they need neither the verb to be in the role of copula nor the sign of belonging. Frege's system has the merit of explicitly pointing out the primitive and indefinable character of the relationship between objects and concepts, and of not trying to hide it by appealing to other extraneous notions, such as variable or indeterminate quantity.

Finally, there is another role played by the variable concerning the order in which the arguments are chosen, depending on the choices previously made. For example, the order of the quantifiers of a formula imposes an order in the choice of the objects of the domain, depending on the choices previously made. Frege's, Russell's, and Peano's ideographies do not provide for any other possibility of dependence.

Note that with the above remarks I am not arguing that, since today we know how to 'eliminate' variables by means of the combinatorial notation, there is no reason to insist on the character of logical primitive of the variable, nor am I suggesting that, since we know how to define the ordered pair in a set, the logic of Russell's relations does not have the foundational role that he believed. Incidentally, such a possibility was also known to Russell, since his student Norbert Wiener mentioned it in his doctoral dissertation written at Harvard (cf. Grattan-Guinness 1975). At that time, however, Russell was convinced of the correctness of the intensional approach in logic and therefore the possibility of a set-theoretic reduction did not interest him. Peano's proposal to characterize relations as classes of ordered pairs is formulated from an extensional point of view, but obviously it does not eliminate the idea of ordering, and therefore it does not constitute a set-theoretic 'reduction'. I am just observing that some important distinctions about the concept of variable had already been made by Frege, but neither Russell nor Peano appreciated this call for conceptual analysis constantly issued by Frege (think, for example, of the 1904 article 'Was ist eine Funktion?'). In both cases a further obstacle was the adherence to the (then) current terminology in mathematics textbooks. Russell, like Peano and Couturat, remained faithful to the way mathematicians expressed themselves. And as Couturat found it inconceivable to abandon talk of 'indeterminate numbers' (cf. letter of Couturat to Frege, 13 October 1901, in WB, p. 24), so Russell continued to speak until 1925 of 'ambiguous value', 'ambiguous assertion', and 'ambiguous denotation' of a propositional function.

There are still other ambiguities surrounding the notion of a variable that we will encounter in examining the controversy between Frege and Peano. In short, far from being a primitive and indefinable idea of logic, it is a patchwork of ideas (we have counted at least seven above). It is the task of logic to distinguish these ideas with the help of a perspicuous notation. Only in this way will ideography, in the words of Peano, durably give 'Alas ad mente de homo' (wings to the mind of man).

5. Incomplete Definitions

The notion of a variable, the distinction between index and variable, and the way of indicating the scope of variables in formulas are the recurrent themes of the

correspondence between Peano, Frege, and Russell. We find a resumption of the discussion on the notion of variable in the correspondence between Frege and Jourdain, which in 1914 (WB, in particular pp. 131ff.) reiterates and expands more or less faithfully the ideas espoused by Russell in PM, some of which had already emerged *in nuce* in the correspondence between Frege and Russell twelve years earlier.

Perhaps unintentional echoes of this debate between Frege, Peano, and Russell concerning the notion of variable can be found in Wittgenstein's TLP, where he speaks of Russell's failure (following Peano) to distinguish between index and argument (TLP, 5.02) of an operation and underlines the necessity that for the *Urzeichen* (primitive signs) of logic (those that occur in the indication of the general form of the proposition indicated in TLP, 6), *mutatis mutandis*, the conditions Frege required for the introduction in a formalized language of *new* signs through definitions be applied (TLP, 5.451). This remark by Wittgenstein brings us to the main topic of the dispute between Peano and Frege, namely the characterization of logical primitives in their respective ideographies and the role of definitions in logic and mathematics. These two topics, as we will see, are intertwined.

The gist of the controversy can be summarized as follows: assumed (but not granted, for Wittgenstein) that the sign of identity should be counted among the *Urzeichen* of logic, it is wrong to consider the meaning as only partially defined and extensible as the scope of the arguments is enlarged. For the *Urzeichen*, observes Wittgenstein, the requirements that Frege posits for the definitions of *new* signs hold, namely that they are introduced at once through an explicit definition, which satisfies the requirements of eliminability and non-creativity, and not through conditional or partial definitions. These are, incidentally, two different things: the conditional definitions ('bedingte Definitionen') contain a clause specifying the kind of entities on which the newly introduced sign is defined, partial or 'stückweise' definitions concern the possibility of extending and modifying *even* the meanings of the terms assumed as primitive (the logical constants and, for Peano, the concept of number) as the numerical field expands. Frege often lumps them together under the label of 'incomplete' definitions: in one case the incompleteness concerns the failure to define the function for all possible values, in the other the implicit indeterminacy of meaning of the primitive notions and operations (number, addition, etc.). Partial and conditional definitions should be kept distinct from definitions by abstraction (theorized by Peano and other mathematicians) and from contextual definitions, employed, for example, by Russell to characterize class description and abstraction operators. Even for the latter type of definitions Russell owes a great deal to Peano (Picardi 2000, pp. 180–1).⁴

⁴ A classical introduction to the topic of definition is ch. 8 of Suppes (1957); for what concerns the topic of definitions by abstraction in mathematics see Thiel (1995a), ch. 6 'Konstruktion und Abstraktion', and Kutschera (1989), in particular ch. 9, dedicated to the topic of definition.

Frege reproaches Peano for making the *definition* of identity depend on the sort of *index* affixed to the sign of identity. It is as if the identity acquires a new meaning when it is between negative integers, real numbers, complex numbers. It is not only circularity that Frege objects to: in order to understand 'equal' in 'equal by definition' one must already know what equality is. Peano parried this objection by accepting the suggestion of two of his students Burali Forti and Pieri to consider the equal by definition as a sign in itself, which indicates one of the ideas or rather the 'primitive activities' of the mathematician. Once this objection had been countered, there remained the more relevant one concerning the way in which Peano gives indices to the signs of relation and, in particular, to the sign of identity and the sign of implication. For Frege the identity and the conditional are primitive logical relations, represented in the BS by a first-level function with two arguments: the identity takes the value True when the argument-places of the function are occupied by names of the same object and False if the antecedent is a name of the True and the consequent is a name of the False, and the value True in all other cases. To the progressive enlargement of the numerical field does not correspond, according to Frege, any change in the meaning of the sign of identity, but only the indication of new criteria to determine whether two expressions indicate the same object or not. In Peano's notation the index, which has the task of indicating the sort of numbers on which the operation is defined, is affixed to the identity sign, thus giving rise to a new compound identity sign that we might call *sortal identity*. Moreover, in Peano the use of indices is ambiguous: sometimes they serve to indicate the sort of numbers among which the operations are to be carried out, sometimes to indicate logical generality, which Frege indicates with two styles of letters, Latin and Gothic. Sometimes, when there are ambiguities of scope, Peano puts an index to the sign of implication to indicate the part of the formula that is bound to the variable. In this way the notation for implication, like that for identity, is amalgamated with the notation for generality, and the latter is restricted by means of indices. Guilty of this state of affairs is, according to Frege, the inadequate way of conceiving both the variable and generality. The generality signalled by the use of Roman letters in Frege's notation always extends to the whole formula; these letters, according to the indications contained in GGA, §8, never occur in formulas not prefixed by the assertion sign. The Gothic letters instead can appear in subordinate parts of a formula and thus serve to indicate the part of the formula governed by the variable of the (universal) quantifier. Frege says of them, for example, in the lectures held in Jena in the Sommersemester of 1913, that they can be used without the assertion sign, since their task is to indicate the scope of the variable of the operator concerned.

The following can be said about the debate between Frege and Peano on this point: Peano is wrong in overloading the signs of identity and implication with indices, but he is right in requiring that mathematical operations and functions in general be accompanied by conditional clauses indicating the scope of the

definition of the signs of operation and function. Frege, for his part, does not grasp the scope of conditional clauses at all, as confirmed by a passage in the *Nachlass* (Frege 1969, pp. 166ff.), in which he mistakenly applies the contraposition to the conditional clause itself, treating it as if it were the antecedent of the conditional—treating, that is, a conditional definition as a definition that has the form of a conditional. Clearly, this is not what conditional definitions are all about. They stipulate that if the arguments are not the appropriate ones, the function is not defined. Frege instead, in GGA, sets ‘default values’ that ensure a value in all cases.

In *Studii di logica matematica* (1897) Peano rebuts Frege’s objections: in his view, what is incomplete are just Frege’s formulas. The equation ‘ $A \times B = B \times A$ ’ does not make any sense, indeed it is incomplete until we are told the kind of numbers to which the multiplication applies. In the reply to Frege’s letter published in *Rivista di matematica* Peano writes:

Mr. Frege wants only one definition for each sign. And such is also my opinion if it is a sign not containing variable letters (F₂ 1 P7). But if what is defined contains variable letters, i.e. it is a function of these letters, then I see the general necessity of giving conditional definitions of that expression, or definitions with hypotheses (id. P7’), and of giving as many definitions as there are species of entities on which we perform that operation. Thus the formula $a + b$ will be defined first when a and b are integer, then a second time when they are fractions, then when they are irrational or complex. One encounters the same sign between infinite or transfinite numbers (F₁, VI) and then one must give a new definition. You encounter it between two vectors, and it will be defined again, and so on. And as science progresses it extends more and more the meaning of the same formula. The various meanings of the writing $a + b$ have common properties, but these are insufficient to specify all the values that expression can have.

The same happens with the formula $a = b$; in some cases its meaning can be expressed as a primitive idea; in others it is defined... With the progress of science one feels more and more the need to extend the meaning of the formula $a = b$. The various meanings of it have common properties; but I do not see how they are sufficient to specify all the possible meanings of equality.

(Peano (1899) 1958, vol. II, p. 295)

The disagreement between the two could not be more complete. As I anticipated, Frege’s interest in the topic of definitions did not cease in 1906, with the abandonment of the logicist project, as shown by the passages contained in the *Posthumous Writings* I mentioned above. In the lectures of 1913, published by Gabriel in 1996, we find in this regard an interesting statement, which appears as an admission of defeat in the face of Peano’s arguments. Here Frege states that while in the *Begriffsschrift* (which in the 1913 version no longer contains the

notation for ranges of values) functions must be defined for all objects as possible arguments, in mathematics things are different.

Ideography differs from arithmetic and analysis in that it does not deal exclusively with numbers. In mathematics, the boundaries of what a number really is, what can be indicated by a variable letter, are never precisely defined (the numerical field is progressively extended: negative numbers, fractional numbers, complex numbers). It is not known how far the concept of number is still extensible. In this way the sense of all sentences becomes indeterminate. In fact, as the numerical field is extended, the theorems proved previously are no longer valid without qualification. In fact, what we have proved is the sense, not the form, and the sense must be proved again. We can understand why, to make sure that the statements have unlimited validity, the focus has been placed more and more on the form, which has ended up being considered essential.

(Frege 1996, p. 38, my translation)

In the *Logic of Mathematics*, the text of the lectures given in the following year, these considerations regarding the difference between BS and mathematics and of the indeterminacy of meaning of all the propositions of mathematics do not recur. They simply say that incomplete definitions must be avoided and that defined concepts must always have sharp boundaries. As the numerical field is extended, it is necessary to find new signs to indicate, for example, addition, between complex numbers, instead of integers.

Note that the danger that worried Frege exists only if one subscribes to the principle that every function must be (a) defined for every object as an argument and that (b) the principle of *Tertium non datur* is satisfied. These are two different requirements, but Frege treats them as being on a par. The first one can be remedied with a sortal quantification, while the *Tertium non datur* may not hold even with the sortal restriction. The implicit aspect of Frege's argument seems to be this: the sense of an expression must be such as to fix its extension, thus determining how the expression is to be applied in every arbitrary case that may arise. We may not know it, but if the expression has a determined sense, it is already fixed (by mathematical reality, so to speak) what truth value the sentence in which it appears takes in such a circumstance. If no stipulation is made concerning all the cases that may arise, the sense of the expression is indeterminate.

But there is no reason at all to accept this reasoning. The meaning of an observational predicate, for example, is determined even if it is not now determined how we will apply it in all possible cases that may arise in the future. There may be conditions, not foreseeable today, that may require the introduction of new criteria for making a semantic evaluation, or there may be no basis for a

decision. This possibility does not make the meaning of the predicate, as we currently employ it, indeterminate or vague. We can admit that the extension of a conceptual term is not completely determined and to some extent vague, without affecting the indeterminacy of the sense of the conceptual term. But Peano's claim, that the open-endedness of the extension of the concept of number would show that the concept of number is indeterminate, should also be corrected. If we take Peano literally, we should conclude that we know what a natural number is only if we know what kinds of numbers there are and we know moreover that they exhaust the domain of numbers. Peano (1899) in his reply to Frege embraces this conclusion, while Frege rejects it: the problematic premise they each share is that an indefiniteness in the extension of a conceptual term is *ipso facto* an indefiniteness in the sense of the term, as we now use it. But there is no reason at all to subscribe to this premise.

6. Interlude: The Context Principle

Wittgenstein's remark about *Urzeichen*, which we mentioned above when talking about the notion of variable, sets the stage for some remarks about the way Frege in GGA, in harmony with the context principle presented in the GLA, tries to fix, through an (almost inductive) stipulation, the *Bedeutung* of the primitive expressions of language, by enumerating the typical contexts in which they can be used.⁵

As I have indicated elsewhere (Picardi 1982), Frege's first reaction to the contradiction discovered by Russell is to think that it reveals a serious, but perhaps remediable, defect in his own BS, a defect analogous, but worse, to those he had found in the work of other mathematicians, first and foremost Peano. The defect consists in this: that GGA's stipulations are not sufficient to give a *Bedeutung* to the primitive symbols of the system, and not only to those introduced by definition, as he had reproached Peano for a few years before. While partial or incomplete definitions (in Peano's sense) only risk not to determine the *Bedeutung* of the new signs introduced by definition in all cases, the contradiction discovered by Russell shows a gap concerning the stipulations for primitive symbols. Hardly in Frege's eyes could one imagine a logical defect worse than this! I believe Frege came to this specific diagnosis of the origin of the contradiction just following the line of arguments used during the controversy with Peano about the legitimacy of incomplete definitions. The looming defect in Peano's proceedings had materialized in GGA. In a few years Frege had become firm in his

⁵ This, incidentally, shows how short-sighted is the opposition between 'logic as language' and 'logic as calculus' formulated by Jean van Heijenoort in the famous 1967 article. Van Heijenoort's distinction is (also) aimed at showing how Frege's conception (like Wittgenstein's and Russell's) of 'logic as language' is inadequate because it makes semantics 'ineffable' (in the Tarskian sense), since it does not provide a clear distinction between object language and metalanguage (theory and metatheory).

conviction that the defect highlighted by the contradiction discovered by Russell was much more serious than the one diagnosed in the spur of the moment: there was no way to save the notion of range of values, and there were no clauses to add, such as the one devised in the appendix, to avoid the contradiction, and thus fix the *Bedeutung* of the class operator.

In the letter to Peano of 29 September 1896, we read:

Our spoken languages have essentially already accomplished their task when people in normal linguistic commerce attach the same thought, or roughly the same thought, to the same sentence. For this purpose it is not necessary that the words each have a sense and meaning in isolation, but it is sufficient that the whole sentence has a sense. Things are different when it comes to draw deductive inferences; in fact in cases like these it is essential that the same expression appears in both sentences and that it has exactly the same meaning in both. The expression must therefore have a meaning in isolation and independently of the other parts of the sentence. (Frege 2001, p. 87)

This statement therefore should not be taken, I believe, as a retraction of the context principle as applied to a formalized language, but as an observation specifically concerning definitions. Here Frege observes that for scientific purposes (as opposed to those of everyday speech) it is not enough that the sentence as a whole has a *Sinn* and a *Bedeutung*, but that the words taken separately and prior to the definition must also have it. It is not the principle of context formulated in the *Grundlagen der Arithmetik* (1884) that needs to be revoked, it is the conditions for definitions of new terms introduced by explicit definition that need clarification. For the reference of the new term which serves as *definiendum* to be determined it is necessary (at least) that the terms used in the *definiens* each have a *Sinn* and a *Bedeutung* established prior to their occurrence in the definition; in the case of *Urzeichen* the reference will have been fixed by means of the quasi-inductive reasoning inspired by the context principle.

According to Frege's initial diagnosis of the contradiction, its emergence indirectly casts doubt on the validity of the context principle as a general method for fixing the reference of the primitive terms of a fundamental mathematical theory, such as arithmetic.⁶ We might ask, following Dummett (1995), what other methods exist to fix the reference of these terms? The answer he and Wright (1983) are inclined to give is that the contextualist strategy is the only viable one. Kutschera (1989, p. 68n) believes that the context principle does not play a systematic role in Frege's writings; in particular, he points out that in view of Skolem's and Löwenheim's results there is no guarantee that the meaning of a

⁶ For a discussion of this topic see, for example, Kutschera (1989), chs. 7 and 8.

primitive expression has been fixed once the truth value of the utterances in which it may occur has been fixed: a proper name (a constant), for example, can have different *Bedeutungen* in different models, and this indirectly shows not only that the sense of the name fails to fix the reference of the term, but that the sense of the name is inherently indeterminate. According to Kutschera, the point of view expressed by Frege in his correspondence with Peano should be seen as a retraction of the context principle (a very wise move, from a point of view that is not historically Frege's). Obviously to discuss this point would require that we address topics that belong to the theory of reference and the philosophy of mathematics, and in this context we cannot do anything like that. We limit ourselves only to point out the extent and difficulty of the problems implicit in the idea that it is possible to fix the reference of the primitive expressions of a theory by means of the strategy implicit in definitions by abstraction and in axiomatic definitions.

7. Sentences and Logical Objects: Frege vs. Peano and Russell

Frege, however, sees both differences as well as similarities between Peano's ideography and his. Peano's notation has one constant for the absurd (the upper-case lambda) and one, unused, for the true (the inverted lambda). In the *Formulario* we can write ' $A = \text{Falsum}$ ', and from here we will also derive ' $\text{Non } A = \text{Verum}$ '. In this Frege sees an analogy between Peano's point of view and his own, and regrets that Peano does not draw the inevitable conclusion, implicit in the introduction of the constant for the absurd (the false) and the sign of equality between statements. Frege is convinced that one of his main discoveries in the field of logic consists in considering statements as (complex) names of truth values. It is for this reason that sentences can appear alongside the identity sign. The distinction between *Sinn* and *Bedeutung* also serves to remedy the counterintuitive consequence that all true sentences designate the same object. On this point Frege insists vigorously both in his correspondence with Peano and in his posthumously published writings related to the dispute with Peano. It is the difference in sense that makes true sentences contribute to the expansion of our knowledge, while it is because they coincide as to their *Bedeutung* that we can apply to them the same principles of substitution *salva veritate* applicable to proper names.

Peano, on the other hand, refuses to see an analogy between his own treatment of propositions and the one advanced by Frege, and limits himself to pointing out that the symbol for the true does not play any role in the *Formulario* and is therefore completely redundant. Probably the admission of the analogy would have implied, in Peano's eyes, a retraction of the statement that the horizontal stroke is not redundant, contrary to what was stated in the 1895 review. In fact, by applying the abstraction operation to the horizontal stroke Frege forms two ranges

of values—the one under which only one object, i.e. the True, falls, and the one under which every object except the True falls—and proposes to identify these two classes with the two logical objects (True and False), previously characterized only informally as the values of certain functions (concepts and relations) for the appropriate arguments. Incidentally, here we find a further analogy between Frege's procedure and that of Peano before 1897, namely the proposal to identify, in the case of individuals, an individual with its singleton (the True with the range of values of which it is the only element) (cf. Quine 1987 on this topic). Peano doesn't at all consider the distinction between *Sinn* and *Bedeutung* ('senso' and 'significazione', as he proposes to translate them in Italian), but limits himself to say that the absurd (indicated by the capital lambda) is treated in the *Formulario* as a primitive notion, not characterized as a set, even if sometimes it is convenient to assimilate it to the empty class.

While Peano adduces no argument for rejecting Frege's proposal to consider sentences as names of logical objects, Russell critically examines it. Already at the time of his correspondence with Frege Russell finds unacceptable the consequence that Frege does not hesitate to embrace, that all true sentences have the same *Bedeutung* (ditto of false ones). This consequence is tempered by the notion of *Sinn*, which accounts for the diversity of cognitive content. It is in opposition to Frege that Russell specifies his conception of 'singular propositions', containing individuals and propositional functions as constituents, and not the senses of the component linguistic expressions. According to Russell's interpretation, the content stroke (this was the name of the horizontal stroke at the time of BS) should serve to present a thought without affirming it as true ('the thought that a cold wind blows'), as distinct from the assertion of the content, 'A cold wind blows' (indicated by the vertical judgement stroke). This distinction is understood by Russell as indicating, in a Meinongian fashion, two ways of thinking or directing oneself onto an 'Objekt' (or target) ('eine verschiedene Weise des Gerichteseins auf ein Objekt' ['a different way of being judged on an object']): once as the object of representation ('the coldness of the wind'), and once as the complex, which can be true or false ('the wind is cold'). In this way, however, Russell objects, there is the risk of making truth a constituent of the asserted proposition, while truth is not a constituent of the Truth, as green is a constituent of a green tree ('Wahrheit ist nicht Bestandteil des Wahren, wie Grün Bestandteil ist eines grünen Baums'). In order to understand this somewhat sibylline affirmation of Russell's, it is worth remembering the affirmation contained in *Sense and Reference*, to which Russell refers, that is the passage where it is said that to judge is 'to distinguish parts in the truth-value'. When we judge a thought to be true, we are not discerning the truth in it as if it were a constituent of the fact (or 'Objektiv') indicated by the sentence. Russell, who at that time already thought of the proposition as a fact, an *Objektiv*, refuses (correctly) to conceive of truth as an ingredient of the represented fact, similar to the green of the grass. This move would have had the further undesired consequence of transforming falsity into something that has an objective way of

being. Although this was not Frege's intention, since he did not conceive of truth as a logical object that exhausts the *Bedeutung* of the proposition, nevertheless Russell's remark highlights a defect in the (metaphorical) characterization that Frege gives in *Sense and Reference* of judging ('to distinguish parts in the truth-value'), namely the inadequacy of the part-whole model applied to the level of *Bedeutung*.⁷ Russell concludes that truth is not 'part' of the true thought (which in his terminology is equivalent to 'fact', which, in turn, is equated with 'asserted proposition') and is external to the thought, but agrees that the notion of truth is indispensable in the foundation of logic. If it is wrong, according to Russell, to assimilate sentences to names of logical objects, this does not mean, however, that the notion of truth has no logical citizenship. Not only that: the denotation relation is also part of logic, unlike the relation of *Sinn*, which Russell tends to relegate to psychology. In short, some of Frege's formulations hit the mark, but are formulated in a philosophically unsatisfactory way. In the first paragraph of PoM, where Russell defines what he means by pure mathematics (i.e. the class of all propositions of the form ' p entails q ', where p and q are propositions containing one or more variables, the same in both, and neither of them containing constants, except logical constants, where by 'logical constants' are meant those constants definable in terms of primitive logical notions) he says: 'In addition to these, mathematics uses a notion which is not a constituent of the proposition which it considers, namely the notion of truth.' It is worth dwelling on Russell's statement: mathematics uses the notion of truth, which, as such, must be mentioned among the primitive notions. However, truth is not a constituent of propositions and therefore does not need a special sign, as, for example, the (compound) sign of judgement employed by Frege. The tacit premise is, of course, that each component sign of a proposition of PoM corresponds to a *constituent* outside the proposition. Russell will later change his mind about this sign, and in fact it is used in PM, but with a different meaning from Frege's (cf. Picardi 1994).

8. A *Sui Generis* Logical Primitive: The Judgement Sign

Peano, regarding the judgement sign (or assertion sign, according to the terminology introduced by Russell in PM), noted in his review of GGA that:

⁷ Frege would later recognize that the part-whole model is inadequate when transposed to the level of reference, while it remains valid at the level of sense. There is no evidence, however, of the weight that Russell's observations may have had in this regard. It is relevant to note that this point was already clear to Frege in 1913 and in fact in Carnap's notes we read: 'The meanings [*Bedeutungen*] of the parts of the sentence are not parts of the meaning [*Bedeutung*] of the sentence. But: The sense of a part of the sentence is part of the sense of the sentence. The meaning of the sentence is the truth value.' One could hardly find a clearer retraction of the passage in SuB, p. 43, where it says, 'One could also say that judging is the distinguishing of parts within the truth value.'

Given the proposition a , our Author (p. 9) introduces a notation $\vdash a$ to say ' a is true'; and another notation ' a ' to indicate 'the truth of a '. I don't see the use of these conventions, which have no corresponding ones in the *formulario*. Indeed, the various positions that a proposition can have in a formula completely indicate what is stated about it. (Peano, 1895a, 1958)

Peano's reading of the compound sign of judgement fits the sign of BS better than the sign of GGA. However, his rendering of it is inaccurate: to express the scope of the sign of judgement (force sign) it is unnecessary to employ the word 'true', but it is necessary to say something about the assertoric force with which we pronounce words. As for the horizontal stroke, it should be rendered with the periphrases used by Frege, 'i.e. the truth value of being 2 to the power of two equal to 4', or 'the truth value of being to a name of the true'. I am not reproaching Peano for not having correctly grasped the interpretation of the horizontal stroke—an interpretation really fraught with difficulties within Frege's system—but for not having had the flair to grasp the originality of the conception of a semantics for a formalized language, present in GGA.

Peano gives three examples of conditionals and comments: 'the latter [$a \rightarrow b \rightarrow c$] does not indicate the truth of a , b , c nor of $a \rightarrow b$, but only the truth of the relation indicated between these propositions'. This concession by Peano is sufficient for Frege to say, in the Letter to the Editor:

For example, I have introduced the sign $!$, or judgement stroke, which is used to assert something as true. You have no corresponding sign, but you too recognize the difference between the case where one only expresses a thought without asserting it as true and the case where one asserts it. (Frege 2001, p. 91)

The absence of the judgement sign from the *Formulario* does not testify to a deeper analysis, as Peano had unwisely concluded, but only means that an important logical distinction is not reflected in the symbolism. Of course, it is only in the light of Frege's project of a semantics for a formalized language that one can fully grasp the need for this distinction and, in particular, the importance of identifying something that can serve as the *semantic value* of a sentence, by which the sentence can contribute to the semantic value of compound sentences. Different semantic theories can give different answers to this question: for Frege the semantic value coincides with the truth value, and this proposal, in turn, is liable to formal developments different from those that Frege had in mind. In the symbolic language of GGA the vertical judgement stroke affixed to a sentence that figures as the argument of the function indicated by the horizontal stroke indicates the externalization of the recognition of its truth by the subject of the judgement (better: by the writer), but at the same time it intimates that the judging subject is completely secondary with respect to the content that figures as the argument of the horizontal stroke. It is not of a psychological act of judgement that we want to

inform the listener when we assert that ice floats on water, but rather of the (true) thought that ice floats on water.

Frege offers three arguments, not all equally cogent, to explain the difference between a sentence used with or without assertoric force. The first argument is based on the contrast between narrative and scientific discourse, the second one concerns the difference between asserting that *p* and asking if *p* (i.e. the existence of different kinds of linguistic acts and different illocutionary forces—to use the terminology introduced by Austin), the third one relies on the occurrence of an unasserted sentence as a component of a compound sentence presented with assertoric force.

Let us briefly examine the three arguments. According to the *first argument*, the difference between narrative discourse and scientific discourse consists in this: in the first case we are not interested in the truth, but only in the thought expressed. Thus, to get an idea of what a sentence expressed without assertoric force is, Frege directs us to narrative discourse. This argument is unsatisfactory for multiple reasons, which we cannot dwell on here. The main one, from which all the others follow as corollaries, is that ‘narrative’ discourse presupposes for its intelligibility serious and ordinary discourse, and therefore cannot be used to illustrate what a sentence without assertoric force is to someone who is not already aware of it. Moreover, the actor who plays in the theatre also produces sentences with various illocutionary force: the difference is that the words pronounced on stage commit neither him nor his interlocutors to the consequences they would normally have if pronounced during an ordinary conversation. A second defect of this argument is that, according to the way Frege expresses himself, it seems almost as if to understand narrative discourse one can do without the distinction between *Sinn* and *Bedeutung*. But if this were so, there would have to be an explanation of the *Sinn* of words in these contexts different from that which applies to scientific or ordinary discourse—where the sense of a word is the way that leads to the reference or way in which the object is given. But there is no basis for this differential explanation of senses of single expressions. Perhaps it is desirable to draw finer-grained epistemic distinctions between the situations in which, in good faith, we judge true a singular sentence that is about nothing, perhaps because it contains proper names without a *Bedeutung*, thus unintentionally sliding into poetry, and those situations in which the thought is about a genuine object. Sometimes Frege speaks of ‘pseudo-sentences’, which, as such, would not express thoughts in the proper sense; at other times he says that the sense of the sentence in which a proper name appears remains unchanged when we discover that the bearer of the name does not exist. In short, if it is true that the sense of a sentence does not change when its parts turn out to be without *Bedeutung*, by contraposition one could conclude that concerning the sense of the sentence it does not matter that the world is the way it is, or that there exists something to which we intend to refer. Everything could be different, and the sense remain unchanged. This thought is hard to swallow, even for those who are not too sensitive to sceptical

challenges to the existence of the external world or to its knowability on our part. Only two extreme points of view (that of the idealist and that of the metaphysical realist) can agree on the intelligibility of this separation between the world as it is in itself and the world as it is perceived and thought of by creatures endowed with sensibility and intellect. In conclusion, even leaving aside gnoseological issues, the arguments used by Frege to explain the difference between asserted and merely entertained thought that refer to narrative contexts are not at all persuasive.⁸

The *second argument* is based on the comparison between questions that require a closed answer ('yes' or 'no') and assertions: an assertion and a question can express the same thought, but in the question the thought is without assertoric force, while in the assertion it is presented as true. This argument should at least be enriched by an illustration of what it is appropriate to do with a question and an assertion that we accept as true, respectively. And provided we have succeeded in doing this, we have not yet shown that the (assertoric and interrogative) force (respectively) is a (separable) ingredient of the overall meaning of the sentences produced. Perhaps it is possible to explain the force simply by appealing to the intentions of the speaker and the listener, or to the pragmatic conventions which govern the practice of asserting and asking questions. In either case, the force would not be part of the meaning, and therefore we could not use the contrast between assertion and question to explain what is meant by the content of a sentence without assertoric force.

The *third argument* concerns the occurrence of a subordinate sentence in a complex one. As Dummett has shown on several occasions, the conditional—more precisely, the occurrence of a sentence as antecedent of a conditional—is the most cogent example among those offered by Frege to illustrate the need to distinguish between the possession of a semantic value (truth value) by the sentence, determined by the semantic value of its component expressions, and the assertion of the sentence, the presentation of it as true. Semantic value accrues to the sentence whether we are aware of it or not, and it is in this sense 'objective'. The conditional is to some extent emblematic of other compound sentences because we cannot explain its scope in terms of two separate assertions (of the consequent and the antecedent). When, however, we reach the discovery that the semantic value that belongs to the sentence as a whole is the True, then we are also authorized to formulate our discovery and to communicate it to others with assertoric force. The formulas of BS do not genetically retrace the steps through which we reach this 'recognition', but they show, in the case of logical and mathematical propositions, the sort of justification that we could put forward in support of our assertion. The assertoric force with which we utter the sentence indicates both our striving for truth and our (temporary) conviction that we have achieved our goal; it is an intimation to the listener to take what we are saying as

⁸ One of the first discussions of this theme is in Gabriel (1975).

true. In illustrating even in the most schematic way possible the notion of assertoric force we had to recall the notions of truth, justification, and acceptance of an assertion by the listener. The distinction between ‘constituent sense’ and ‘assertoric content’ is discussed in depth by Dummett (1991) and we can’t examine it here. Illustrating how these notions are interrelated is a task for a theory of meaning: in Frege we can only find hints to that effect.

What then can we say about the controversy between Frege and Peano on the assertion sign? I think we can agree with Peano that the assertion sign can be omitted from the logical notation without harm. But this does not mean that the notion of assertion (together with the notions of truth, justification, and acceptance) is superfluous and should not be counted among the ‘primitive ideas’ if not of logic in the strict sense, at least of the theory of meaning. In this wider sense of ‘primitive idea’, one can agree with Peano’s statement that the identification of the primitives of logic poses more difficult problems than for arithmetic and geometry. Not everything can be formulated explicitly in the ideography: the primitive notions of the ideography can only be clarified through examples, and they remain to some extent inexpressible. This diagnosis is found in both Frege and Peano and, with due distinctions, is comparable to ideas that we find in the *Tractatus Logico-Philosophicus*. Here Wittgenstein does not confine himself to elaborating the theme of ineffability, but gives important indications about what to do in order not to succumb to paralysis: it is to the applications (I am almost tempted to say to the *use*) of signs that one must pay attention in order to understand what is not expressed in signs: ‘Was in den Zeichen nicht zum Ausdruck kommt, das zeigt ihre Anwendung. Was die Zeichen verschlucken, das spricht ihre Anwendung aus’ (TLP 3.262: ‘What does not get expressed in the sign is shown by its application. What the signs conceal, their application declares’).

There is a subtle irony in the observation that Frege and Peano, both passionate devotees of the Leibnizian ideal of a *lingua characteristica*, reached at the end of their investigations: what breathes life into the signs of the ideography is not fully expressible within it. Nevertheless, the search for a perspicuous logical notation is neither futile nor chimerical. Indirectly, the ideography allows us to circumscribe what eludes the true and proper symbolic representation: aiming at the truth to which thought aspires, fixed on the page by a truly *sui generis* sign such as the vertical stroke.

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PART II

FREGE'S PHILOSOPHY
OF LANGUAGE

6

Über Sinn und Bedeutung

An Elementary Exposition

In a series of articles, published between 1891 and 1899, Frege put forward one of the most suggestive philosophical proposals in order to explain the semantic value of a linguistic expression (sentence, proper name, connective, predicate). From a philosophical point of view the most important of these works is the 1892 essay *Über Sinn und Bedeutung* [SuB].¹

Since many of the problems with which philosophers, logicians, and linguists have grappled in the last one hundred years are raised in this work, it is not out of place to subject it to a detailed examination.

1. Language and Logic

The proposals for a semantic analysis formulated by Frege in his famous 1892 essay are closely linked to his logicist and foundationalist ideas. They arise from the reflection on the link between a system of signs and the body of thoughts to which it must give form and expression. Since the system of signs in question is the first example of formal language in the modern sense of the word and the corpus of thoughts to which it must give form and expression is classical logic, it is not surprising that Frege's reflections have general philosophical relevance.

One of the guiding ideas of Frege's semantic construction resides in the thesis that all signs—both categorematic and syncategorematic—that contribute to the formation of a declarative sentence have a sense and a *Bedeutung* and that the semantic value of a sentence, simple and complex, which he identifies with its truth-value (the True or the False), depends systematically on the Meanings of its component parts. This idea would be of little relevance if it were not also

¹ For reasons of uniformity of terminology, I will give here my translation of the quoted passages; page numbers are those of the original edition. To avoid misunderstandings and not to dwell further (cf. my preface to G. Frege, *Scritti postumi*, Naples, Bibliopolis, 1987, pp. 19–21 [SP]) on the pros and cons of translating 'Bedeutung' differently from 'meaning' (e.g. as 'denotation', 'nominatum', 'designatum', 'reference', 'referent'), I will write 'Meaning' with a capital 'M' to indicate the places where this word is used in Frege's sense.

supported by an example of its application and by a semantic theory able to give us a picture, even if schematic, of the connection which holds between linguistic signs, their sense, and what they designate. This clarification is, according to Frege, more urgent in mathematics, where the confusion between sign, sense, and *designatum* seems to be an endemic disease, for which no one cares to find the cure. There are multiple roots for this difficulty. The first one consists in the fact that no constancy in sense and meaning corresponds to graphic constancy: the role that a sign plays in a sentence depends on the overall sense of the latter and therefore it is not an entity that belongs to the sign out of context. The second difficulty derives from the fact that for certain linguistic signs, like for instance predicates and relations, it is not easy to keep apart the Meaning and the sense and to specify the entity that plays the role of Meaning. Finally, for other linguistic signs like connectives, prepositions, digits, and numerals, the three levels mentioned above are even more difficult to keep apart.

Another obstacle that stood in the way of a correct understanding of linguistic meaning, at least until the first half of the 1700s, is a poor semiotic analysis. A symptom of this, for example, is the insistence on the merely conventional character of linguistic signs. Obviously, there is a sense in which it is perfectly legitimate to claim that linguistic signs are 'conventional', that there is no *natural* or *rational* reason for the fact that 'smoke' in English means 'smoke' and not 'fire'. However, there is no reason to get hung up on this fact: what is being claimed is merely that 'smoke' is not to smoke as smoke is to fire. Smoke is a natural sign of fire, whereas the word 'smoke' is not a natural sign of what it signifies. The opposition between the conventional and the natural risks making one lose sight of the crucial fact that linguistic signs are not signs 'of' something, like symptoms of diseases, but rather signs to *communicate something to someone*. The adjective 'conventional' has something forced about it when applied to a natural language as a whole: it suggests a sovereign or legislator who arbitrarily decides that 'cat', say, and not 'catt' is a word of English, that 'cat' and not 'horse' is the word to use to speak of felines instead of equines. The fiction of the legislator, who is the arbiter of sounds and meanings, is misleading: when it comes to language, we are all 'subjects' and 'legislators'. 'Subjects' because in order to make ourselves understood by others we must use words the way in which we were taught to use them and that has proved efficacious to communicate, and 'legislators' because on occasion we may use words to communicate meanings that they don't have literally, while making ourselves understood, nonetheless.

The insistence on the conventionality of signs risks unintentionally transforming an innocuous truism in a misleading image about the way language works. Such sliding is encouraged, on the one hand, by an atomistic prejudice, which leads one to consider words one by one, independently of the context of the sentence and to ask what they mean; and, on the other, by a conviction that

linguistic signs are a mere shorthand of thought.² Thought has already its own order, its own connection, and its own architecture; it is only to communicate to others that we need to cloth it with signs, which, with respect to the ideas that they must signify, are like arbitrary labels. What matters is that there are enough of them and that they be easily distinguishable from one another. Labelling, in and of itself, would not facilitate communication at all if it were not governed by *conventions*. It is a short step from here to conceiving the original convention of sign formation along the lines of a social contract for the regulation of behaviour. A code of behaviour tells us what is subject to sanctions, and, sometimes, what we are obliged to do when certain circumstances occur in order not to incur sanctions. In language, on the other hand, it is difficult to legislate on what makes sense and what doesn't, and on the point beyond which, by moving away from the grammar and the literal meaning of the words, we risk not being understood. This, of course, does not mean that there are not rules that we have learned and that we consciously follow in our linguistic conduct, in addition to rules that we formulate by reflecting on the functioning of language. However, to identify conventions as the precondition for the functioning of language is reductive and potentially misleading.

The image of the functioning of linguistic signs that the pre-Kantian³ tradition gives us is deeply unsatisfactory: it does not explain the centrality of language with respect to all other sign systems, it does not tell us anything about the function of language as a 'formative organ of thought', to use Humboldt's justly famous expression, and it leaves in the shadows the role played by the linguistic community in determining the intersubjective character of meaning. In the nineteenth century, this crude image of language began to be slowly dismantled and, finally, overturned thanks to the joint action of various disciplines with an interest in language, such as linguistics, psychology, and logic.

Frege's work is located in this complex philosophical landscape, in which a thesis diametrically opposed to that of the previous century had become commonplace—the thesis, that is, of the inseparability of language and thought, and, indirectly, of language and knowledge. In Frege's opinion, however, we must be careful not to identify language and thought, as linguists and psychologists tend to do. In fact, if we followed this path, we would be precluded from distancing ourselves from linguistic and graphical forms. This would be detrimental to logic, which should follow step by step the graphical idiosyncrasies of natural languages.

² This is particularly evident in the case of the philosophers of the Cartesian tradition; an excellent example is offered by the authors of Port-Royal's *Logique et Grammaire*, whose logical-semiotic theories I dealt with in 'Notes on Port-Royal's Logic', *Lingua e Stile*, 10 (1976), 3, pp. 337–91.

³ Generally, we tend to think of Kant as a philosopher who completely neglected the philosophical importance of language and of Herder as the one who corrected this oversight. This diagnosis is not exact; for an enlightening discussion of Kant's ideas on language see M. Capozzi, *Kant on Logic, Language and Thought*, in D. Buzzetti and M. Ferriani (eds.), *Speculative Grammar, Universal Grammar and Philosophical Analysis of Language*, Amsterdam, Benjamins, 1987, pp. 97–148.

Of great use to the logician is knowledge of languages, which allows him to train his ear to the element of thought that remains when the linguistic-grammatical form varies.⁴ In an article published in 1882, dedicated to the scientific justification of ideography, Frege observes:

It is impossible, some say, for science to make progress by means of an ideography; in fact, the invention of the latter presupposes the perfection of the former. The same identical difficulty already arises with language. The latter should have made possible the development of reason: but how could man create language without reason? Physical apparatuses are needed to study natural laws. These can be produced by advanced technology, the progress of which rests, in turn, on knowledge of natural laws. In all these cases the circle is resolved in the same way. An advancement in physics has as a consequence an advancement in technology, and this makes possible the construction of instruments by which physics is again advanced. The application of this reasoning to our case is evident.⁵

The construction of the ideography has the aim of contrasting the predominance of grammar with logical reasoning and can be compared to the discovery of the technique of using the wind to sail against the wind. Without signs, mankind would probably not have been able to attain abstract conceptual thought, and, nevertheless, modelling logic on grammar is sterile and misleading.⁶ It is true that natural language is the most comprehensive system of expression that we possess and, as such, we cannot transcend it: however, it is possible to study some specific aspects of it using artificial languages that, although distant from it, allow us to focus on some aspects of the functioning of natural language.

From a chronological point of view, SuB predated by one year the publication of the first volume of *Grundgesetze der Arithmetik* [GGA] and constitutes a sort of philosophical introduction to the main semantic notions employed within it. The main differences between the *Begriffsschrift* [BS] of 1879 and the new formal system of the *Grundgesetze* stem in part from Frege's second 'discovery'—the first one being the invention of a formal system for first (and second) order predicate logic—that is, the construction of a semantics for this language. Frege's specific semantics is contradictory, for it embeds some principles of naive set theory. The

⁴ See, for example, the fragment of the *Logik*, dated between 1879 and 1891, NS [G. Frege, *Nachgelassene Schriften*, H. Hermes, F. Kambartel and F. Kaulbach (eds.), Hamburg, Meiner, 1969, 1983²], pp. 6ff., SP, pp. 72–4: 'To sum up, it is the task of the logician to engage thoroughly in a struggle against psychology and, in part, against language and grammar, insofar as these do not express with purity the logical element.'

⁵ G. Frege, 'Über die wissenschaftliche Berechtigung einer Begriffsschrift', *Zeitschrift für Philosophie und philosophische Kritik*, 81 (1882), pp. 48–56, p. 49, reprinted in *Funktion, Begriff, Bedeutung*, G. Patzig (ed.), pp. 91–7.

⁶ *Ibid.*, p. 55.

paradox discovered by Russell was the occasion for the scientific encounter between these two philosophers. It does not compromise Frege's semantic conception, yet it incontrovertibly shows that naive set theory is not the right semantics for a formal system capable of expressing arithmetic. As an antidote to paradoxes Russell would propose the theory of logical types, which involves a different semantics and a different conception of the relationship between signs and what they indicate or help to denote.⁷

Probably the systematic construction of a semantics for a formal system suggested to Frege a deep reflection on the notion of linguistic sign broadly understood, and on the functions played by signs within a sentence. Frege's attention, understandably, was focused on a specific kind of sentences, namely declarative ones, apt for the assertoric use, and on some specific categories of expressions, that is, proper names, predicative expressions, relational expressions, and logical connectives.

Two regulative principles inform Frege's reflection on language: the *context principle* and the *compositionality principle*. The context principle is stated in the introduction to *Grundlagen der Arithmetik* [GLA] from 1884 and is restated in §§62 and 106. The formulation we find in §106 is as follows:

We have formulated the principle that the meaning of a word should not be explained in isolation, but in the context of a sentence; only by observing this maxim is it possible, in my opinion, to avoid the physical conception of number without falling into the psychological one.

Since Frege in 1884 had not yet introduced the distinction between *Sinn* and *Bedeutung*, it is legitimate to ask whether the context principle concerns the sense of the words, their *Bedeutung*, or both. In my opinion, it concerns both the sense and the reference of words, although in the latter instance its scope may be problematic, at least, in conjunction with other theories advocated by Frege. If we ask what the meaning of the words '4', 'Socrates', 'greater than', 'direction', 'not' is in isolation from the context of the sentence, the answer we can give is, at best, the one afforded by ideational theories of meaning. The answer will be that a written or spoken sign evokes a mental image or an idea. Frege's genius consists in showing not so much the error, but rather the vacuity, of a possible answer that appeals to the mental images evoked by words.⁸ What we must ask ourselves is how the words that compose a sentence contribute to determining as true or false

⁷ We will deal with them in Ch. 6 in this volume. [The volume mentioned here is *La chimica dei concetti*, Bologna, Il Mulino, 1994. Chapter 6 is titled 'Simboli incompleti' and it is based on E. Picardi, 'Alfred North Whitehead/Bertrand Russell, *Principia Mathematica*', in *Interpretationen. Hauptwerke der Philosophie: 20. Jahrhundert*, 1992, Stuttgart, Reclam, pp. 7–42.]

⁸ G. Frege, *Logik* (1897), NS, p. 151, SP pp. 247ff.: "To him who understands the word "horse" the image of a horse will immediately come to mind. But this image must not be mistaken for the sense of

the assertion made: in doing so we determine the thought expressed. In answering this question, we will realize that not all aspects of the meaning of a word are equally relevant to the determination of its semantic value:

We can identify three levels of difference in words, expressions or whole sentences. Either the difference concerns at most the representations, or the sense, or finally also the Meaning. . . . We will not mention representations or images any more in the following: they have been named here so that the representation that a word evokes in a listener is not mistaken for its sense or its Meaning. (SuB, p. 31)

There are two other ingredients of the overall meaning of a sentence that are not mentioned in this passage, namely the stylistic elements, which 'colour' the thought without changing its truth, and the *force* with which words are uttered. The force is what we need to understand to know both the aim that the speaker intends to achieve and the appropriate response on our part. A question and an assertion contain the same thought, but involve different intentions, responses, expectations, and justifications on the part of the speaker and the listener.

Among all the possible uses of a sentence, the assertoric one has a particular importance for Frege, because it foregrounds the privileged relationship between meaning and truth. In making an assertion we do not intend to talk about our imaginings or our epistemic state: the main function of the words in the sentence is to direct the listener to all those elements of extra-linguistic reality that are pertinent to understand the content of our statement. Once the relevant entities have been identified, he will have the elements to judge under what conditions the assertion made is correct. In very general terms we can say that the main function of the words in the sentence is to outline a certain referential path. This path is not the private and incommunicable possession of the person who asserts the sentence, but it is potentially available to the listener: it is, so to speak, immanent in the language. One of the main functions of language is to give form to and make intersubjectively accessible an articulated network of thoughts and knowledge about the world.

The second fundamental principle that informs Fregean semantics is the *compositionality principle*. The compositionality principle, also known as the functionality principle⁹ or 'Frege principle', theorizes a harmony between syntax

the word "horse"; for the word "horse" gives us no hint of the colour of the horse, does not tell us whether it is stationary or in motion or from which side it is seen, etc. If it were possible to project on a screen the representations evoked by the word "horse" in different people, completely different images would appear. Much depends on the context. Compare, for example, the statements "How gladly he rides his valiant horse" and "I have just seen a horse tear through the wet asphalt." We cannot therefore speak of the same representation that would always be associated with the word "horse".

⁹ The compositionality principle is also amenable to two readings, depending on whether it is applied to the sense or to the reference of words. The greatest critic of the 'Frege principle' is

and semantics. The guiding idea of Frege's semantics is that the interpretation to be assigned to a 'part' of a sentence must be unambiguously derived from the *syntactic* category to which the expression belongs and from the particular stipulations about the truth-conditions of the sentences in which it occurs. In *Grundgesetze's* formal language, signs are divided into two major syntactic categories, comprising 'saturated' and 'unsaturated' expressions that correspond, roughly, to terms and functions. Logical connectives and quantifiers are a particular type of function.¹⁰ All complex expressions constructible in the formal language are obtained by completing a functional expression with an argument-expression of the appropriate level. Functional expressions are distinguished according to the number of arguments (into monoargumental and biargumental), and the species of argument with which they can be saturated or completed. The latter distinction gives rise to an organization of functions in levels, depending on whether the arguments with which they can be completed are objects, first-level functions (predicates without quantifiers), or second-level functions (complex predicates). Frege's hierarchy of functions incorporates some natural constraints for the construction of meaningful expressions, dictated by the theory of logical types; for Frege, however, unlike Russell, mathematical reality is not organized in types: all logical objects (the two truth-values, classes, value-ranges, numbers) belong to the same universal domain in which belong individuals as well.

2. Sentences and Truth-Values

The main thesis that Frege advances in SuB as a 'conjecture' which the whole essay is meant to defend is that, from a logical point of view, the semantic value of a declarative sentence consists in its Meaning and not in its sense, and that this Meaning is to be found in one of the two logical objects, the True and the False (SuB, p. 36). The decision to consider sentences as proper names of logical objects, which seems arbitrary at first sight, was probably dictated by the aim to simplify the semantics by admitting only two categories of expressions (terms and functions). The mode of truth-functional composition, familiar to anyone with some inkling of logic, is theorized and exposed for the first time in SuB. The discussion of identity with which the essay opens and closes has a tactical function: on the one hand it prepares the ground for the most difficult case of application of the notion of sense (the case of proper names); on the other hand it introduces the issue of substitutability of identicals, paving the way for the thesis that a

J. Hintikka, the supporters are Richard Montague and the field of linguistic studies that is inspired by Montague's work. See H. J. Hintikka (in collaboration with J. Kulas), *The Game of Language*, Dordrecht, Reidel, 1983.

¹⁰ For a discussion of the functional conception of concepts, cf. M. Dummett, *The Interpretation of Frege's Philosophy*, Cambridge, Mass., Harvard University Press, 1981, chs. 8 and 13.

declarative sentence expressing a thought is nothing but a complex proper name. Since, from the point of view of substitutions, the sentence behaves as a proper name, we can assimilate it directly to a proper name, reifying truth and falsity in the two logical objects, the True and the False. They are characterized as what intersubstitutable sentences *salve veritate* in all (extensional) contexts have in common, according to the familiar scheme of definitions by abstraction.

The main argument of SuB is therefore the demonstration of two theses: i) that a sentence is the proper name of a logical object (truth-value); and ii) that the truth-value of a sentence depends exclusively on the semantic value (*Bedeutung*) of its component parts. In the central part of the essay, Frege tries to isolate the modes of extensional composition (truth-functional and quantificational modes of composition) from other modes of sentential composition. In the final part of the essay Frege sets out to show that the apparent exceptions to the conjecture (the theses i–ii above) are due to the fact that the linguistic form in which sentences occur in natural languages is not transparent with respect to their logical structure. In particular, it is necessary to distinguish between thoughts expressed, thoughts presupposed, and thoughts conversationally or conventionally implied (as we would say today) by the sentences. But how do we make these distinctions? How do we decide whether a thought is explicitly expressed or implied, or presupposed? To answer this question, it is necessary to employ the principles of substitutability of identicals and verify whether the truth-value of the complex sentence remains unchanged when we substitute sentences or parts of sentences with other sentences or parts of sentences with the same Meaning.

In SuB Frege analyses various types of grammatical constructs, arriving at the following classification:

- a) Non-truth-functional modes of composition, which can be traced back to an extensional treatment in a broad sense, once the appropriate Meanings have been identified—the semantic entities, that is, with respect to which the substitution *salva veritate* is legitimate. Frege advances the thesis that in an epistemic or doxastic context the sentence that figures in a subordinate position has an indirect Meaning that coincides with its ordinary sense. The conjecture—the proof of which is left as a difficult exercise for the reader—is that the truth-value of the sentence as a whole will remain unchanged in the substitution of expressions having the same indirect Meaning.
- b) Extensional modes of composition in which however the sentence expresses something less than a thought: sentences with temporal quantifiers, relative sentences, hypothetical nexus of thoughts. This category includes the pre-suppositional analysis of the existential import of definite descriptions.
- c) ‘Mixed’ modes of composition, in which a complex sentence is analysed as containing two occurrences of the same sentence, with different senses and Meanings. For example: ‘The guy is under the illusion that *p*’ is to be

understood as composed of two statements: 'The guy believes that *p* and not-*p*': in the first occurrence '*p*' has its own indirect sense and Meaning, in the second its own ordinary sense and Meaning.

- d) Ways of composition in which the sentence lends itself to different logical analyses. In some of them an unexpressed thought must be recognized as a part of the thought expressed in an incomplete way by the sentence, whereas in other analyses it simply 'accompanies' it, as a *Nebengedanke*, or accessory thought.¹¹ In these cases, even before dealing with the problem of semantic interpretation, it is necessary to understand which and how many thoughts are *expressed* by the sentence, and not merely suggested or presupposed.
- e) Truth-functional and extensional modes of composition.

Once counterexamples have been reviewed, Frege feels authorized to consider the conjecture he has put forward confirmed and states:

Hence it follows with sufficient probability that the cases in which a subordinate sentence is not substitutable with another one with the same truth-value, prove nothing against our claim that the truth-value is the Meaning of a sentence the sense of which is a thought. (SuB, pp. 49–50)

The main objection that can be made to Frege's analysis (and has been made, for example, by Wittgenstein, Russell, and Carnap and, in recent times, by Dummett) is not so much that of reifying the True and the False, but rather that of likening the function of a sentence to that of a proper name. This identification in effect invalidates the privileged position that the context principle gives to sentences, and is probably responsible for the least convincing developments of Fregean semantics, such as, for example, the idea that the mere presence of a vacuous proper name in a sentence radically frustrates our intention to use it to make an assertion and is an insurmountable obstacle to the construction of a correct semantics.

3. Identity

The essay opens by posing a number of questions:

- i) What is the appropriate syntactic and semantic interpretation of the '=' sign? Which linguistic signs can appear alongside the identity sign, and between which entities does identity hold?

¹¹ A paradigmatic case is when the sentence contains words endowed with elements of 'tone' (*Färbung*) the presence of which is not explicable in extensional terms, e.g. the 'because' of a causal link. See the examples of ice floating on water and Napoleon who, having recognized the danger on his right flank, led the attack against the enemy position (SuB, pp. 47–8).

- ii) On what does the greater cognitive value of a true identity statement of the form ' $a = b$ ' depend as compared to an identity statement of the form ' $a = a$ '? Both statements, if true, are true because the proper names designate the same object. And yet what we want to affirm with ' $a = b$ ' is not that an object is identical with itself, no matter how we call it. In fact, self-identity is a logical property enjoyed by all objects, and consequently it will never be worth affirming of any object.
- iii) Since not every true sentence of the form ' $a = b$ ' is justifiable a priori, we must admit that there are true identity sentences that are justifiable only a posteriori. What do we learn when we know that an identity sentence that is not justifiable a priori is true? Is it a knowledge that concerns the linguistic signs used, the sense expressed by them, the object designated by them, or all these things?

Before discussing these questions, it is worth recalling the solutions Frege proposes at the end of the essay:

The fact that in general the cognitive value of ' $a = a$ ' is different from that of ' $a = b$ ' can be explained by reflecting on the fact that, from the point of view of the cognitive value, the sense of the sentence, the thought expressed in it, is no less important than its Meaning, which is its truth-value. If therefore ' $a = b$ ', the Meaning of ' a ' is the same as that of ' b ', and therefore also the truth-value of ' $a = b$ ' is the same as that of ' $a = a$ '. However, the Meaning of ' a ' can differ from the Meaning of ' b ' and therefore also the thought expressed in ' $a = b$ ' is different from that expressed in ' $a = a$ ', therefore the two sentences do not have the same cognitive value. If, as before, by 'judgement' we mean the advancement of the thought to its truth-value, we will also say that the judgements are different.

(SuB, p. 50)

The answer to question (i) does not differ much from the one we find today in modern logic textbooks and anticipates the explanation offered in the first volume of GGA. Here identity is characterized as a relationship between objects, indicated by a first-level functional expression with two arguments, which we can abbreviate as ' lxy '. The semantic evaluation of this function is the following: ' lxy ' is the True if the two argument-places are occupied by names of the same object and the False in all other cases. From the syntactic point of view, identity is a first-level functional expression and can have as argument-expressions only proper names (and individual variables). Such identities as ' $mortal = mortal$ ', ' $quickly = rapidly$ ', far from being examples of the law of identity, are syntactically impermissible. In a natural language that we do not know at all, we cannot formulate or recognize even the most trivial of identities. To this end, we need, at the very least, to know what syntactic aspect a proper name has. Whatever the answer to the question

(ii) about the scope or relevance of identity claims, two points are clear: 1) 'these relations can exist between signs only insofar as they name or designate something' (SuB, p. 25) and, 2) 'sign' or 'name' is to be understood as any linguistic expression 'which takes the place of a proper name, the Meaning of which is a determinate object (in the broadest sense of the term) and therefore neither a concept nor a relation' (SuB, p. 27). In addition, 'places, instants, intervals, are objects, from a logical point of view, and therefore the linguistic designation of a particular place, instant or interval should be conceived as a proper name' (SuB, p. 42).

Clause (2) circumscribes the domain of entities among which the identity relation is defined: among these there are neither concepts nor relations, which Frege conceives as unsaturated entities designated, respectively, by predicative and relational expressions.¹² It includes among 'proper names' also definite descriptions. This, naturally, does not mean that the sense of every proper name must coincide with that of some definite description. Clause (1) excludes from the list of proper names that can occupy the argument-places of the sign of identity 'vacuous' proper names, names, that is, such as 'Pegasus', 'Odysseus', 'the most divergent series', which do not designate anything, even if they can give the impression of doing so. This clause is a particular application of Frege's general doctrine that an assertion containing a vacuous proper name in the referential position, while expressing a thought, is neither true nor false, and the same holds for its negation. This is a controversial aspect of Frege's theory: to avoid the consequences of this position Russell devised the theory of definite descriptions.

The fact that the proper names discussed in the essay are genuine proper names makes it clearer in what sense an identity statement such as ' $a = a$ ' does not expand our knowledge: it holds a priori and, according to Kant, should be called analytic (SuB, p. 25). Indeed, one might object that the truth of ' $a = a$ ' after all communicates knowledge to us, in that it authorizes us to infer the existence of the object designated by the name. In other words, stating ' $a = a$ ' would be a way of saying ' a exists', where 'exists' is meant as a first-level predicate indicating a property of the designated object. By parity of reasoning, we should grant that ' $a = a$ ' could be false, in which case ' $a \neq a$ ' would be true and could be employed to say that the object designated by ' a ' has the property of non-existence. From here it is a short step to conclude, with the introduction of the usual existential quantifier, that everything exists, both what enjoys the property of existence and what enjoys the property of non-existence. The coexistence of the predicate 'to exist' and of the existential quantifier annuls the presumed informative value of the predicate expressing self-identity, and leads us to unreliable hypotheses about the properties

¹² However, in later works Frege will admit that even unsaturated entities can stand among themselves in a relation of identity analogous to that which exists among saturated ones.

with which these mysterious entities, of which, as we know, Meinong's universe is populated, would be endowed.

All these difficulties arise from treating existence as a property of individuals, whereas it is a property of concepts.¹³ Existence is not a property that we can predicate of objects on a par with other properties. Of course, it is perfectly legitimate to infer $(\exists x)(x = a)$ from the truth of $'a = a'$; what is wrong is the way we have formulated the import of the statement. What we are saying is not that 'a has the property of existence', but rather that the concept 'being identical to a' is not empty and that exactly one object falls under it. Thus reformulated, the assertion concerns a concept and not an object.

4. Cognitive Value

According to some philosophers, the function of proper names is exhausted in the function of naming the bearer of the name: if in a language every object had one (and only one) name there would be no need for identity, and there would be no enigma about the cognitive value of identity sentences. According to Frege, in contrast, proper names express a sense, which he describes as a way to reach the object bearing that name: to know the sense of a name means, in a strong sense, to be able to identify its bearer, and, in a weak sense, to have an idea of the appropriate method of identification for the sort of objects to which the bearer of the name in question belongs. Many of the conundrums surrounding identity statements derive from the fact that mastery of the sense of a proper name usually involves a richer knowledge than mere knowledge that the name designates something and a poorer knowledge than full knowledge of the object that the proper name designates.¹⁴ This raises the problem of what an adequate description of the kind of knowledge we intend to communicate by asserting

(1) Hesperus is Phosphorus

is, or, in contrast, of how to account for the cognitive value that (1) may have for a listener who grasps it for the first time and who recognizes it as true.

Let us try to reformulate the cognitive value of (1) as follows:

(2) 'Hesperus' and 'Phosphorus' are names of the same object.

¹³ This problem is discussed by Frege in *Dialog mit Pünjer über die Existenz*, NS, pp. 60–75, *Dialogo con Pünjer sull'esistenza*, SP, pp. 137–55. For a discussion cf. R. De Monticelli, 'Gottlob Frege e la totalità degli enti', *Teoria*, 2 (1982), pp. 105–20.

¹⁴ Regarding this distinction, see M. Dummett, 'Frege's Distinction between Sense and Reference' (1975), reprinted in *Truth and Other Enigmas*, London, Duckworth, 1978, pp. 116–44.

Undoubtedly if all we know and we aim to communicate with (1) is the thought expressed by (2) it would be a limited kind of knowledge. If the listener has no idea of what *sort* of objects are talked about, the only thing he would have learnt is that in a language (in English) an object is being called by two different names. All we would have communicated is that the two signs are syntactically genuine proper names and that they may be used interchangeably to indicate a given object. No difference in what is designated corresponds to the difference of sign. Identity would thus communicate some information about *signs*, but not knowledge about the designated object.¹⁵ The following reformulation is more promising:

(3) Hesperus and Phosphorus are the same *planet*.

This, I think, captures what is at the basis of Frege's theory. The employment of proper names is anchored to the mastery of criteria of identity: in this specific case, to a criterion of identity for 'the same planet'. Criteria of identity concern, in the first place, the kind or *sort* of objects to which the object designated by a given proper name belongs. Identity statements highlight this feature of the use of proper names.

If we were to adopt reading (2) we would be interpreting the import of an identity statement as concerning only the signs employed, and this reading does not do justice to the cognitive contribution that the sense of signs makes to the expression of the thought. Another argument that works in favour of the thesis that proper names have a sense as well as a reference is given by the examination of the semantic behaviour of proper names in belief contexts, of which we will say more later.

Against the hypothesis that identity concerns only the signs that are employed, Frege objects that if a proper names were just an arbitrary label, then by learning that two signs stand for the same object, we would at most learn the result of a personal decision:

Nobody can be forbidden from assuming as a sign of something an object or a process arbitrarily produced. This way, the sentence ' $a = b$ ' would not be about the thing itself, but only about our way of designating it: we would not express in that sentence any genuine knowledge. Yet, this is exactly what we want to do in many cases. (SuB, p. 26)

¹⁵ This is the interpretation of identity that we find in TLP [L. Wittgenstein, *Tractatus Logico-Philosophicus*, London, Routledge, 1922]; Wittgenstein denies that proper names have a sense and excludes from the range of proper names those that do not have a *Bedeutung*. See on this Ch. 7 in the present volume [*La chimica dei concetti*. The chapter is titled 'Ramsey fra Wittgenstein e Russell' and is based on a paper by the same title in R. Simili (ed.), *L'epistemologia di Cambridge 1850-1950*, Bologna, Il Mulino, 1987, pp. 329-55].

The general idea at the basis of Frege's reasoning seems to be this: anyone is free, in principle, to name objects the way they like, and may baptize them as they please; except that the identity statements that can be formulated on the basis of these procedures have no cognitive value, they do not contain knowledge about the object that is the bearer of that name, but they do inform us about certain idiosyncratic preferences of the user of the name. However, it may be objected, if we are not invested with the necessary authority, the decisions we make are devoid of intersubjective validity. If I decide, arbitrarily, to rename Bologna as 'Bloc', the sentence 'Bologna = Bloc' can be understood as follows: what others call 'Bologna' is called 'Bloc' in my idiolect. This kind of stipulation is devoid of intersubjective consequences, since it is not in individuals' power of to rename cities. Sure, cities change their names. However, in these cases, unlike the one in which I decide to call an object by a fancy name or to rename an object by a name usually used for other purposes, we are dealing with linguistic practices immersed in a social dimension (such as toponymy, the compilation of atlases, etc.). Changes in toponymy are a socio-political fact *par excellence*—just think of identities like 'Königsberg = Kaliningrad', 'Turku = Åbo', 'Sterzing = Vipiteno'. This shows how necessary the distinction between arbitrariness and conventionality is: everything that is conventional is arbitrary, but not everything that is arbitrary is conventional.¹⁶ In rejecting the metalinguistic thesis of identity statements' import Frege does not pay sufficient attention to the distinction between 'arbitrary decision' and 'conventional stipulation'.

In the examples just mentioned we have cases in which identity does communicate information, but in which we are still reluctant to say that this information has a cognitive value and concerns the object designated by the name. In cases where identity serves to introduce a linguistic stipulation or abbreviation, we can legitimately say that what we learn is a convention that regulates the use of *signs*. Identities serve to introduce abbreviations that are of this type. For example, 1 kilo = 1,000 grams, 1 metre = 100 centimetres. After all, 'equal by definition' is generally kept distinct from 'equal'. It is from this use of '=' in '= df' that the metalinguistic reading of the import of identity sentences draws its strength.

The question is whether *all* identity statements are of this kind, and the natural answer seems to be no. In many identity statements what we want to say it is not that something or someone *is called* N but that it *is* N. One reason to make this distinction is quite natural: an impostor can take the place of President Yeltsin and thus be called Yeltsin without being Yeltsin. Or one can choose as a stage name a proper name of a person who really existed: for example, we can (indeed, we must) say that the English singer whose name is Engelbert Humperdinck is not

¹⁶ This observation is to be found in D. Davidson, 'Communication and Convention' (1982), reprinted in *Inquiries into Truth and Interpretation*, Oxford, Clarendon, 1984, pp. 24–280; Italian translation by R. Brigati in E. Picardi (ed.), *Verità e interpretazione*, Bologna, Il Mulino, 1994.

Engelbert Humperdinck, the composer pupil of Wagner and author, among other things, of the opera *Hänsel und Gretel*. But let us suppose that all these misunderstandings have been avoided, that what is at issue is not the possibility that the statements ‘such and such is N’ and ‘such and such is called N’ have different truth-values: what is the difference between ‘being Hesperus’ and ‘being called Hesperus’, what is that something more to which the first expression seems to allude?

According to some authors, translation is a useful test to understand what is lost in the metalinguistic reading of the scope of identity sentences. Restating an example from Linsky,¹⁷ we observe that if

(4) Venus is the evening star

is rendered according to the metalinguistic reading as:

(5) ‘Venus’ and ‘the evening star’ designate the same thing (in English)

translating the two sentences into another language alters their informational value. For example, Linsky argues, the translations of (4) and (5) from English to German:

(4’) Venus ist der Abendstern

and

(5’) ‘Venus’ und ‘the evening star’ bedeuten dasselbe (auf English)

can have different informative value: for a German who does not know English (4’) contains relevant astronomical information while (5’) contains information about the English language. Linsky’s assumption is that names enclosed in quotation marks should not be translated but simply quoted. Whatever the merits of this analysis in highlighting the problems of quotation, it captures, in my opinion, only in part the reason that leads Frege to reject this proposal.

What Frege means to question is that all identity statements have their *raison d’être* only in the fact that in the language we have typographically different signs to designate the same thing. If this were the case, it would not be explained either why it would be worth the trouble to formulate them or the kind of knowledge we come into possession in this way:

¹⁷ L. Linsky, *Oblique Contexts*, Chicago, Chicago University Press, 1983, p. 7.

If the sign 'a' differed from the sign 'b' only as an object (in this case by the form) and not as a sign, that is, by the way it designates something, then the cognitive value of $a = a$ would be substantially the same as that of $a = b$, provided that $a = b$ is true. A diversity can therefore be produced only if the difference in sign corresponds to a difference in the way of presenting itself of what is designated. (SuB, p. 26)

Note that Frege does not say that the difference in sign must correspond in all cases to an increase in the cognitive value of identity statements, but only that this is the case when the name contains a way of presenting the object. Although Frege contrasts his present theory with the one expounded in the *Begriffsschrift*, we should not conclude that in 1879 Frege argued that the import of an identity statement pertains only to signs and is, as such, devoid of cognitive value. In *Begriffsschrift*, §8, we find in fact a very similar argument to the one exposed in SuB. The main difference, as far as proper names are concerned, consists in this: that what in 1892 is called *Bedeutung* in 1879 is called 'Inhalt'¹⁸ and that what in 1892 is called *Sinn* in 1879 is referred to as 'Bestimmungsweise' (method of determination) of the content:

The identity of content differs from the conditional and the negation in that it refers to names and not to contents. While signs usually stand for their content, so that each link in which they appear expresses only a relation between their contents, as soon as they are linked through the sign of identity of content they reveal their own identity at once (*ihr eignes Selbst*); in fact, this indicates a circumstance in which the two names have the same content. With the introduction of a sign for the identity of content, a split in the meaning of all signs necessarily occurs, since they stand now for their content, now for themselves. At first, this gives the impression that it is something that concerns only *the expression, not the thought*, and it seems almost as if there is no need for different signs for the same content and therefore not even a sign for the identity of content. To show the absurdity of this supposition I choose from geometry the following example. On a circumference lies a point *A*, around which a radius rotates. When the radius forms a diameter, we denote by *B* the point on the circumference opposite to *A*. We give the name *B* to the point of intersection of the two lines corresponding to the relative positions of the radius, according to the rule that matches the continuous variation of the length of the radius with continuous variations of *B*'s position. Therefore, the name *B* means something indefinite if the relative position of the radius is not indicated. We can now ask:

¹⁸ I have dealt with the distinction between 'content' and 'conceptual content' in 'On Frege's Notion of Inhalt', in *Atti del Convegno internazionale di storia della logica*, San Gimignano, 4–8 dicembre 1982, Bologna, CLUEB, 1983, pp. 307–12.

which point on the circumference corresponds to the position where the radius is perpendicular to the diameter? The answer will be: point A. Therefore, the name B has in this case the same content as the name A; and yet, we could not use only one name from the beginning, since only this answer authorizes us to do so. The same point is determined in two ways:

- 1) immediately through intuition
- 2) as point B, corresponding to the radius perpendicular to the diameter.

Each of these modes of determination corresponds to a specific name. The necessity of a sign for identity rests therefore on this: that the same content can be fully determined in two different ways; that in a specific case the same thing is actually given through *two modes of determination*, it is the content of a *judgment*. . . . It follows from this that different names for the same conceptual content are not always simply an irrelevant question of form, but, where they are connected with different ways of determining the content, they concern the essence of the thing itself. In this case the judgement that has as its object the equality of content is a synthetic judgement in the Kantian sense.¹⁹

In the framework of the BS the scope of an identity statement should be rendered roughly as follows: ‘sign *a* and sign *b* contain two different ways of determining the same content’, and, in this specific sense, an identity statement says something about the signs involved. What in BS was called ‘content’ is in SuB analysed in terms of sense and Meaning: in the case of proper names the Meaning is the designated object. It becomes clear that identity is not about the sense of names: the informative value of the identity is explained based on the sense of proper names and the descriptions that occur in the sentence.

The example used in SuB resembles the one employed in the *Begriffsschrift*, as does the formulation of the problem:

Let *a*, *b*, and *c* be the lines connecting the vertices of a triangle with the midpoint of the opposite side. The point of intersection of *a* and *b* is identical to the point of intersection of *b* and *c*. Thus we have different designations for the same point, and these names (‘point of intersection of *a* and *b*’ and ‘point of intersection of *b* and *c*’) allude at the same time to the way in which the object is given, and thus genuine knowledge is contained in the statement. (SuB, p. 26)

Similar cases, according to Frege, are ‘the morning star is the evening star’, ‘Ateb is Afla’,²⁰ sentences that express, respectively, the recognition that it is the same

¹⁹ BS, pp. 13–14.

²⁰ This example occurs in the draft of an undated letter to Philip E. B. Jourdain in which it is said among other things: ‘An object can be determined in different ways and each of these ways of determination can give rise to a particular name, and these different names therefore have different

planet that we see in different positions in the sky and that it is one and the same mountain that has been designated by different names. As mentioned above, Frege includes among proper names also definite descriptions. That two different definite descriptions of the same object can enrich our knowledge is an incontrovertible fact. What is controversial is how proper names can enjoy this prerogative, without being themselves crypto descriptions. This is, for example, the thesis held by Russell.

In the geometric examples used by Frege we are dealing with non-contingent identity statements. In mathematics, if a sentence is true, it is necessarily so. What Frege takes for granted is that even *a priori justifiable* identity statements, and not only synthetic and contingent ones, expand our knowledge. Frege's comment that 'not all sentences of the form $a = b$ are justifiable *a priori*', which implies that many are, is a detail the reader of SuB might easily miss. In fact, Frege believes that arithmetic propositions, even though they are analytic, broaden our knowledge, thus assigning to conceptual knowledge those prerogatives that Kant had denied to it. One of the most suggestive aspects of Frege's analysis is the attempt to give a uniform explanation for all expressions that can serve as proper names in a language, regardless of whether they designate entities with respect to which the truths we eventually formulate are 'necessary' or 'contingent'. The concept of truth, as Frege understands it, is absolute and not relativized to contexts, worlds, interpretations. Many contemporary disputes over modality and essentialism therefore have no counterpart in Frege's philosophy.

Both in the case of Hesperus and Phosphorus and in the case of the medians of the triangle what seems crucial is that we come to know something that we did not know before and that to express it we use proper names. Kripke²¹ argued that all identity statements, if true, are metaphysically necessary, even if some are knowable only *a posteriori*. This thesis, which by itself does not imply that identity statements have no cognitive value, has nevertheless been used by Kripke and the proponents of direct reference theory to show that proper names perform their semantic function in a 'direct' way, without the mediation of sense. What the direct reference theorist questions is that knowledge of the sense of a proper name is necessary and sufficient for the determination of reference. The causal theory of reference is a hypothesis about the factors which, in a language, contribute to the

senses; that it is the same object that is determined in different ways, is not at all obvious. . . . If the sense of a name were something subjective, then the sense of the sentence in which that name occurs, and therefore the thought, would also be something subjective, and the thought which one person attaches to the sentence would be different from the thought which another attaches to it; a common store of thoughts, a common science would be impossible' (WB [G. Frege, *Wissenschaftlicher Briefwechsel*, G. Gabriel, H. Hermes, F. Kambartel, C. Thiel, and A. Veraart (eds.), Hamburg, Meiner, 1976], p. 128).

²¹ S. Kripke, *Naming and Necessity*, Cambridge, Mass., Harvard University Press, 1980; Italian translation *Nome e necessità*, M. Santambrogio (ed.), Torino, Boringhieri, 1982.

determination of the reference of proper names: among these factors Fregean *Sinne* are missing.²²

5. Identity and Criteria

Frege conceives of identity as a relation defined for all objects belonging to the domain over which one quantifies. Each syntactically well-formed identity statement is either true or false; it possesses a well-determined sense irrespective of whether we are able to ascertain its truth or falsity; however, there must exist objective criteria of identity, capable of fixing its truth-value, at least in principle. For the determinacy of sense of the singular expressions which occur in identity statements, it is enough that such criteria exist, even though it may not be in our power to apply them in each specific case.

In GLA, §62, Frege argues that knowledge of suitable identity criteria is integral to our understanding of the concept of natural number and to justifying the use of expressions such as 'the number of F 's' as proper names of logical objects. He illustrates the context principle by giving an example of its application:

How then is a number given to us, if we can have no image or intuition of it? Only in the context of a sentence do words have meaning. Therefore, we will have to explain the meaning of a sentence in which a numeral appears. This leaves ample room for choice. However, we have already established that with numerals we must mean autonomous objects. Therefore, we have a kind of sentence which must have a sense, a sentence, that is, which expresses a recognition. If sign a is to designate an object, we must have a criterion that decides in all cases whether b is the same as a , even if it is not always in our power to apply it. In our case it is necessary to define the sense of the sentence:

'the number that belongs to concept F is the same as the number that belongs to concept G ';

we must, that is, render the content of this statement in a different way, without using the expression 'the number that belongs to concept F '. In doing so we give a general criterion for the identity of numbers. Once we possess a means of grasping a number and of recognizing it as the same, we can give it the numeral as its proper name.

The gist of the argument is clear: we can use a numeral as the proper name of a number only if we have a criterion to univocally decide the truth-value of all

²² I have extensively dealt with this issue elsewhere. See E. Picardi, *Linguaggio e analisi filosofica: elementi di filosofia del linguaggio*, Bologna, Pàtron, 1992, ch. 6.

identity statements in which this numeral can appear. The criterion indicated by Frege basically corresponds to what is sometimes called 'Hume's law', the law that authorizes the passage from 'There are as many *F*'s as *G*'s' to 'The number of *F*'s is equal to the number of *G*'s' and vice versa. This criterion consists in the existence of a one-one correspondence between classes; it is the basis of cardinality comparisons. Other criteria of identity are, for example, the one that says that two classes are equal if they have the same elements (extensionality principle), or that objects designated by the names 'a' and 'b' are equal if they belong to the same classes (an extensional version of the principle of identity of indiscernibles). Not all criteria of identity correspond to *effective* criteria for decision; for example, the criterion used by Frege is not an effective criterion for deciding, given any two infinite classes, whether the first has a cardinality equal to, greater than, or smaller than the second.

The 'recognition judgements' (*Wiedererkennungsurteile*) Frege speaks about form the background for the use of identity statements and indirectly illuminate the use of proper names in the language. In the case of abstract entities, our recognition judgements must be based on explicit identity criteria (as in the examples mentioned above); in the case of material objects, events, and persons, we generally do not even try to *explicitly* formulate the criteria that inform our recognition judgements, and in many cases it is not even possible to give an explicit formulation. Full mastery of the use of singular terms probably goes hand in hand with language acquisition; proper names are employed initially to direct the listener's attention to discrete medium-sized objects present in the perceptually salient portion of the world. From the perspective of language learning, we learn to recognize aspects and forms of the surrounding world before we are able to reidentify. Quine²³ and Strawson²⁴ have offered, each in their own way, a philosophical reconstruction of how the full mastery of countable terms—terms, that is, that admit an answer to the question 'How many?'—is preceded by the learning of expressions with respect to which the distinction between singular and plural does not arise. This is not the place to discuss the problems surrounding the contribution language makes to the way human beings relate to the social and natural environment in which they are immersed; nevertheless, it is worth remembering that the very semantic notions that seem most elementary to us and hardly in need of explanation, such as the distinction between singular and plural, between mass terms and countable ones, presuppose both a natural set of recognitional, discriminative, and classificatory skills typical of the species, and a fabric of social practices in which the ceremonies of naming and identifying play a role.

In very general terms, it can be said that knowing the sense of a (singular or general) term consists in knowing an *application* criterion and an *identification*

²³ W. V. Quine, *The Roots of Reference*, La Salle, Open Court, 1974.

²⁴ P. F. Strawson, *Individuals*, London, Methuen, 1959.

criterion. If I am told abruptly 'Apparently, eka-aluminum is the same as gallium' and I have no idea what kind of names we are talking about, my natural reaction will be to ask: the same what? When I know that it is the same *chemical element*, depending on my knowledge of chemistry, I can evaluate with interest or indifference this information, but I will at least have an idea of the kind of experimental and theoretical procedures in which that kind of statement is at home. If, as Kripke and Putnam have argued, natural kind terms exhibit a behaviour like the one of proper names, the possession of identity criteria (at least by experts) is key to understand the sense of the sentences which contain them. The fact that the ordinary speaker does not possess these criteria shows that what he knows *individually* is not enough to determine the reference of those terms; but it does *not* show that the existence of such procedures and criteria is irrelevant to the use made of them in the language.

In the case of people's proper names, Geach offered this argument:²⁵

The sense of a proper name certainly does not involve anything about those peculiarities of the individual so named which distinguish him from other individuals of his kind: a child, a youth, an adult may differ from each other to the point of being unrecognizable, and, nevertheless, bear the same name all their lives. But it is meaningless to say without further qualification that the child, the youth, or the adult are 'the same' or 'the same thing' and that this is precisely what authorizes us to use that name. Nor does it depend on our will whether they are or are not considered 'the same'. 'The same' is a fragmentary expression, which has no value unless we say or mean 'the same X', where 'X' stands for a general term. What the use of the term 'the same' does is this. In general, if an individual is presented to me with a proper name, I cannot learn the use of that proper name without being able to apply some criterion of identity, and, since the identity of a thing consists in being the same X, e.g. the same man, one cannot speak simply of 'the same', and my application of the proper name is justified only if its meaning includes being applicable to one man and I continue to apply it to one and the same man.

These application and identity criteria are part of what an adult speaker needs to know in order to be able to use language to communicate (and acquire) knowledge and information. Knowledge of the meaning of the word 'cat' *also* manifests itself in the ability to apply it correctly and is not limited to the mere ability to use the word in conversation.²⁶

²⁵ P. T. Geach, *Mental Acts*, London, Routledge and Kegan Paul, 1971, p. 69.

²⁶ It is not explicit or verbalizable knowledge, but has the features of a practical ability, a *know how* in Gilbert Ryle's sense; cf. 'Knowing How and Knowing That', *Proceedings of the Aristotelian Society*, 46 (1945–6), pp. 1–16 and *The Concept of Mind*, London, Penguin Books, 1970 (1949), ch. 2.

6. The Objectivity of Sense

After illustrating what is meant by 'proper name' Frege observes:

The sense of a proper name is grasped by those who know sufficiently the language or the complex of designations (the terminology or notation) to which it belongs; in this way, however, the Meaning, if there is one, is always illuminated only from one side. For a complete knowledge of the Meaning it would be necessary that for each given sense we could say immediately whether it suits it. We never achieve this. (SuB, p. 27)

Indeed, if we were able to recognize immediately, for any given sense, whether it is a mode of presentation of an object which, say, is already given to us as the Meaning of a proper name or a particular description, and is therefore illuminated from a certain side, we would be omniscient.

To illustrate the objective character of sense, and, at the same time, its 'partiality', Frege employs the simile of the moon and the telescope: the moon would correspond to the *Bedeutung*, while the image of the moon reflected in the objective of the telescope corresponds to the sense that is accessible to anyone who puts his eye near the eyepiece of the telescope. Each person, however, will retain an individual retinal image of the moon, different from the one of other observers. If the sense of the word 'moon' were as subjective as the representation it would be difficult to explain the workings of language. For Frege, representations are radically private, as they are necessarily incommunicable,²⁷ while the sense would be objective, as it is accessible to many. Of course, this does not mean that images and representations cannot be studied and compared in some way, but the object of investigation would no longer be the representation in relation to the object represented, but the representation as such.

But what is the basis for the belief that the sense of a sentence is or can become a 'common possession of many' and does not coincide with the complex of associations and information that the individual speaker attaches to the words, because of his personal experiences and the way he learned the language? As Dummett has observed, Frege's argument in favour of the distinction between sense and reference, so far examined, does not show that the sense of a word is a characteristic of *language*: 'it shows at most that each speaker, in order to associate a reference with

²⁷ For a more detailed discussion of this point cf. M. Dummett, 'The Origins of Analytical Philosophy', *Lingua e Stile*, 23 (1988), pp. 3–49, pp. 171–210; Italian translation *Alle origini della filosofia analitica*, Bologna, Il Mulino, 1990. This is the text of a series of lectures given by Dummett at the Department of Philosophy of the University of Bologna in the spring of 1987. Recently an English edition has appeared, entitled *Origins of Analytical Philosophy*, London, Duckworth, 1993, which differs from the Italian translation in many points and contains an interview with the author by J. Schulte.

a word, must attach to it a particular sense; it does not show that it is necessary for different speakers to attach the same sense to the word, as long as the senses they associate with it determine the same reference. Consequently, it leaves open the possibility that the sense of a word is not part of its meaning at all, provided that the meaning must be something objective and shared by all speakers.²⁸ The argument Dummett alludes to in this passage refers to the conception of sense as the way to reach the Meaning, as the way in which the object is given to us (SuB, p. 26).

The other argument used by Frege to characterize the desired objectivity is more promising. It calls into question the knowledge *communicated* to the listener of the asserted sentence, and more generally, the knowledge acquired by the listener when he accepts a sentence as true. For a sentence to communicate information, it must be possible to understand it before recognizing it as true. In this second interpretation, accepting a sentence as true consists in learning that its truth-conditions are satisfied.²⁹ The explanation we find in GGA I, §32, where the sense of a proper name is defined as the contribution it makes to the thought expressed by the sentence in which it appears, and where the sense of the sentence is defined as the thought that the truth-conditions are satisfied, goes in this direction. There is no doubt that these two ways of characterizing the sense of a proper name are not only different but also potentially in conflict: the former emphasizes the constructive, almost procedural, aspects of Frege's conception, the latter is characteristic of his semantic realism.

We have said that the objectivity of sense consists in its being communicable in principle, unlike the representation. In particular, in the case of proper names, it is necessary to distinguish the representation from unambiguously identifying descriptions that different speakers associate with the same proper name. In footnote 2 on p. 28 of SuB Frege observes:

With an ordinary proper name such as 'Aristotle' opinions can certainly differ. For example, we might assume the following: Plato's pupil and Alexander the Great's teacher. Anybody who does this will attach to the sentence 'Aristotle was a native of Stagira' a different sense from the one who assumes as the sense of this name 'the teacher of Alexander the Great was a native of Stagira'. As long as the Meaning remains the same, these fluctuations of sense are tolerable, although they are to be avoided in the edifice of a demonstrative science and in an ideal language.³⁰

²⁸ Dummett, *Frege's Distinction between Sense and Reference*, p. 130.

²⁹ See the discussion in chapter 1, §§1 and 3 [in *La chimica dei concetti*].

³⁰ In traditional logics, for example in F. Ueberweg's that Frege's knew, these descriptions are called 'equipollent'. In *System der Logik und Geschichte des logischen Denkens* (Bonn, Marcus, 1868²) at §53 Ueberweg gives the following examples of equipollent expressions: 'the teacher of Alexander the Great'/'the founder of scientific logic', and 'perpendicular to a radius in its peripheral points'/'straight line that has a point in common with the periphery of a circle'.

What Frege mentions is information about the bearer of the name 'Aristotle', information that as such is perfectly communicable and therefore not at all private as representations are. What matters, according to Frege, is that all these descriptions converge on the same individual, and function as a way to reach the reference. In this regard Frege comes very close to the idea that will later be supported by Russell, according to which a proper name is a kind of definite description in disguise. Unlike Russell, Frege does not maintain that proper names are eliminable in favour of definite descriptions, or that knowledge of the sense of a proper name must necessarily be articulated in a definite description. He simply observes that sometimes the knowledge of the sense of a proper name is equivalent to the knowledge of a property that applies exclusively to the individual designated by the name.

Not only is being aware of an identifying description to substitute for the proper name unnecessary in order to be able to use it competently, but it is also obvious that not all definite descriptions that we can associate with the name 'Aristotle' are on par. The question arises whether it is possible to single out from the mass of information about the philosopher Aristotle a core of information that is somehow more closely or stably associated with the sense of the word 'Aristotle'. The problem is not whether the sense of the word 'Aristotle' *coincides* with a particular set of information about the philosopher Aristotle; the problem is rather whether there is a set of information halfway between 'dictionary' and 'encyclopaedia'³¹ that is associated in a standard way with the name 'Aristotle'. Undoubtedly, Frege does not pay enough attention to the relationship between what is part of a word's meaning and what is part of an individual speaker's information about the referent of the name. If we *identified* the sense of a proper name with the definite descriptions that each speaker associates with the name 'Aristotle', it would be impossible to characterize the sense of a proper name as something we know as speakers of a language, and not, say, as philosophy scholars. We could never be sure of expressing the same thought using the same sentence at different times, because of the acquired and forgotten information about Aristotle.

In the note cited above, Frege observes that in a formalized language there must be a univocal way of fixing the reference of a term and *therefore a univocal sense* for each term. This implies, among other things, that one must avoid defining the same sign twice and defining the same sign partially. However, he apparently does not believe that such a distinction between '*canonical ways*' of determining the Meaning of a simple or complex expression and information about the bearer of the name is transferable to natural languages. Yet, if we are not to conclude that the *way* in which the name 'Aristotle' fulfils its semantic function depends

³¹ For a discussion of these notions, cf. U. Eco, *Semiotica e filosofia del linguaggio*, Turin, Einaudi, 1984 and D. Marconi, *Dizionari e enciclopedie*, Turin, Giappicchelli, 1981, and 'Two Aspects of Lexical Concepts', *Lingua e Stile*, 22 (1987), pp. 385–95.

exclusively on the information contingently available to individual speakers, we must be able to draw a similar distinction in natural languages as well.

More precisely, it must be possible to account for the intuitive and, in fact, constantly employed distinction between ‘the standard explanation of the reference of a proper name and the standard information about its bearer’.³² The boundary between meaning and information is moveable but it is nevertheless a boundary. If we did not accept this principle, we would have to say that meaning changes all the time (both by experts and by the average speaker) and changes without most speakers realizing that this is happening. To explain what we mean by ‘moveable’ we can take as an example the conception of the Earth in the heliocentric system and in the Ptolemaic system. On the one hand, we want to be able to say that the word ‘Earth’ means ‘the planet we inhabit’ and has the same meaning in the mouths of Dante, Galileo, Avicenna, Aristotle, and us. From the meaning of the word ‘Earth’ we must distinguish both the image that each of us may have of the Earth, and what astronomical, cosmological, or geological theories say about the Earth. On the other hand, we also want to be able to say that it is part of the meaning of the proper name ‘Earth’ in 1994 (and not, for example, in 1300) to be a planet rotating on itself and around the Sun and not an immobile celestial body at the centre of the universe, and we would be surprised if today the average English [Italian, in the original] speaker were not aware of *these* facts—while we would not be surprised if he ignored the length of the diameter, the distance from the moon, the relative mass compared to the other planets in the solar system. What is relevant is that at each linguistic stage it is possible to distinguish between standard explanations of the way in which the reference of the proper name ‘Earth’ is given to us and the encyclopaedic information about the Earth, although there is no single distinction that we can make at all stages in the same way, for all terms. This explanation of how reference is given to us constitutes the *sense* of the word ‘Earth’. According to Frege, it is the ‘store of thoughts common to mankind’ that makes linguistic communication possible. This doctrine requires two clarifications: the first is the recognition that it is a variable ‘store’ of thoughts (new senses become intersubjectively accessible, others fall into oblivion); the second is that what makes the store of thoughts ‘objective’ is that it is ideally accessible, not that each is actually possessed.

7. The Regular Interconnections between Sense and Meaning

So far, we have talked about how the distinction between sense and Meaning is to be drawn in the case of sentences and proper names; it is now time to extend

³² Dummett, *Frege’s Distinction between Sense and Reference*, p. 140.

it to predicates, belief contexts, and deictic expressions. Before proceeding, however, it is worth taking stock of the 'regular interconnections' (*regelmässige Verknüpfungen*) between sense and Meaning: A) The difference in sense of two expressions belonging to the same syntactic category does not entail a difference in Meaning; B) Sense identity entails Meaning identity; C) Difference in Meaning entails difference in sense.

Principle C, despite its intuitive plausibility, hides many pitfalls, which are particularly evident in the treatment of phenomena of contextual dependence (personal and demonstrative pronouns, deictic expressions) and of *oratio obliqua*. There is a fourth principle (principle D) that informs Frege's semantics and which we have already mentioned, that is the thesis that simple and compound sentences which contain an empty singular term in referential position are 'neither true nor false'—where 'neither true nor false' indicates a lack of semantic value and not, for example, an intermediate truth-value or a non-classical semantic evaluation. The sentences in question do express a thought, but they do not designate a truth-value: for this reason, by pronouncing them with assertoric intent, we achieve nothing, or, as Frege says, we inadvertently slip into fiction. According to Frege, principle D entails two corollaries: D1) that the corresponding assertion of the isolated sentence is also vacuous, and D2) that this vacuity is also inherited by the truth-functional and extensional enunciative compounds in which that sentence occurs as an unasserted component.

As we mentioned in section 3, Frege considers the possession of a *Bedeutung* a *presupposition* of the use of both proper names and definite descriptions:

That the name 'Kepler' designates something is rather a presupposition both of the assertion 'Kepler died in misery' and of its negation. . . . The truth of the statement 'there was a man who discovered the elliptical form of the orbit of the planets' depends on the whether the subordinate 'he who discovered the elliptical form of the orbit of the planets' really designates something or only gives the impression to do so, but in fact it is devoid of Meaning. And yet it may seem that the subordinate clause contains as part of its meaning the thought that there existed someone who discovered the elliptical shape of the orbit of the planets. If this were the case then the negation should be, 'He who first discovered the elliptical shape of the orbit of the planets did not die in misery, or there was no one who discovered the elliptical shape of the orbit of the planets.' (SuB, p. 40)

This passage contains *in nuce* the 'presuppositional' analysis of definite descriptions, to which, as is known, in *On Denoting* Russell will counterpose the 'implicative' analysis. The argument used by Frege to determine whether the assertion of existence is to be considered as a constituent of the expressed thought or as its presupposition is based on the test of negation: if it were a genuine constituent, the negation of the statement should reveal it, highlighting two ways in which the

asserted statement can be false: either because of the vacuity of the description used or because the individual described does not fall under the concept indicated by the predicate. But this is not how negation normally works in language, and this indirectly validates the hypothesis that the requirement that singular terms (proper names and descriptions) have a Meaning is a presupposition of the assertoric use of the sentences in which they occur, and not part of what is asserted. If we are aware that the presupposition of existence is not satisfied and we nevertheless use the vacuous proper name to make an assertion about its supposed referent we are deliberately misleading our hearers. But what about the cases in which we, in perfectly good faith and with assertoric intent, utter a sentence containing an improper description or an empty singular term? What justification could we use to describe the assertion made as neither true nor false, vacuous, defective, instead of false or wrong? According to some authors, Frege's proposal, albeit plausible at first sight, is not convincing, if understood as a general explanation of the way language is used in its assertoric function.

The main difference between Frege and Russell regarding definite descriptions (expressions formed according to the scheme 'the so-and-so') lies in the fact that for Russell the assertion of existence is, together with that of uniqueness, an integral part of the meaning of the definite article 'the'. It can be false without the whole assertion being deprived of a truth-value: it is simply false, but by denying it we obtain a true assertion, and in this way we solve the riddle of how to say something true about what does not exist. For Frege 'He who discovered the elliptical form of the orbit of the planets died in misery' and 'He who discovered the elliptical form of the orbit of the planets did not die in misery' both presuppose that there was someone who satisfies the definite description, and if this presupposition is not satisfied, they are *neither true nor false*, and therefore they are not in relation of contradiction (nor of contrariety). Denying a sentence that is neither true nor false we obtain a sentence that is also neither true nor false.

A logic in which the principles of non-contradiction and of the excluded middle do not apply does not appeal to either Frege or Russell, and therefore, in their respective formal systems they both have an *ad hoc* treatment for the descriptions defined as improper. In SuB Frege proposes that one choose an arbitrary object, e.g. 0, as *descriptum* of all vacuous descriptions. In GGA, instead, he introduces a first-level description function ' ξ ' which has objects as arguments and values; intuitively, the description operator extracts from a singleton its only element: where it does not exist or there is more than one the value of the function coincides with the argument. The main difference between Frege's and Russell's treatment is that descriptions in Frege's system have no scope and behave like proper names. If in the case of definite descriptions Frege devises a treatment that, in formal systems, avoids the occurrence of truth-value gaps, in the case of vacuous proper names there are no similar remedies: a vacuous proper name simply cannot occur in the ideography. Avoiding the danger of truth-value gaps is

one of the advantages that Frege claims for formalized languages over natural ones.

Note that the mere presence of an empty proper name may not have undesirable consequences from a logical-semantic point of view. For example, (6) and (7), unlike (8), do not have truth-value gaps:

- (6) Vera says, 'Scylla has six jaws.'
- (7) Vera believes that Scylla has six jaws.
- (8) Scylla has six jaws.³³

Frege sometimes describes sentences like (8) as expressing pseudo-thoughts, but the prevailing tendency in his writings is to conceive of them as having semantic status in their own right. For even if they cannot be employed in an assertion that aspires to truth, they nevertheless serve a semantic function in *oratio obliqua*. (6)–(7), pronounced in normal conditions, have a truth-value: they are true if and only if Vera utters or believes the reported sentences. In accordance with the functionality principle mentioned in section 2, the semantic value of the whole is a function of the semantic value of the parts; it is therefore a matter of identifying the entities that can act as semantic value in these contexts. The theory of *oratio obliqua* is the answer to this problem.

Principle (D) has no exact equivalent in the case of predicative expressions, about which it is time to say something. Until 1906, i.e. until the declaration of bankruptcy of the logicist foundation, Frege was of the opinion that all predicates the sense of which is neither ambiguous nor vague—for which it is possible to decide, in principle, of any object whether it falls under the concept or not, or, according to the substitutional reading, whether the sentences obtained by saturating the predicate with the proper names present in the language are true or false—have a Meaning, which Frege identifies with a concept. The meaning of a predicate is to be conceived as a kind of procedure, knowledge of which allows us to determine for any given object whether it falls or does not fall under the concept. Concepts, in Frege's understanding of the word, are part of the realm of the *Bedeutung* and are neither housed in the minds of speakers nor identified with the sense of words. This is also true for contradictory predicates and for predicates that contingently do not apply to any object ('golden mountain', 'man 4 metres high'). According to Frege, all these predicates designate concepts. The idea of prejudicially excluding from logic concepts under which no object falls would be short-sighted: it often takes a demonstration to know whether at least one object, exactly one object, or no object falls under a concept. Consequently,

³³ The example occurs in NS, p. 141, SP, p. 236.

there is no parallelism between vacuous proper names and ‘vacuous’ predicates. The most appropriate place to discuss these notions is the philosophy of mathematics: in fact, they played a central role in the dispute with the formalists (Hilbert and Thomaë), both with regard to the foundations of geometry and with regard to the notion of mathematical structure.

As we have said, the Meaning of a predicate (*Begriffswort*) is a concept, and not the plurality of objects to which the predicate applies. However, with the introduction of the notion of ‘concept extension’ (*Begriffsumfang*), understood as a logical object taking the place of the concept in appropriate logical contexts, Frege can formulate a principle of extensionality for concepts, which states, roughly, that whenever we are dealing with true predicates of the same objects, we can consider the concept designated by them—and not just the class that is the extension of that concept—as the same. This formulation is deliberately imprecise because, according to Frege, the notion of identity is appropriate only for saturated entities (individuals, classes, truth-values, numbers) but inadequate with respect to ‘unsaturated’ entities *par excellence*, such as concepts and functions. In any case, the search for a criterion for concept identity that is more refined than simply material equivalence is a relatively sophisticated problem compared to the much more serious ones highlighted by the contradiction discovered by Russell. It shows *ad abundantiam* how the passage from a predicate to its *Bedeutung* and from a concept to its class is to be treated with extreme caution (as, moreover, principle A enjoins).

Principle D finds a partial application in the case of vague predicates (such as ‘big’, ‘small’, ‘modern’, etc.), predicates which, however, according to Frege, do not designate any concept in the logical sense, and are also, together with vacuous proper names, responsible for the truth-value gaps that can occur in the sentences in which they appear as components. However, there can be latent forms of vagueness, and therefore the fact that a predicate does not seem vague does not imply that it is not vague, and it would therefore seem illegitimate to conclude, merely based on its sense, that it has a concept as its Meaning.

Principle A applies to all linguistic expressions: they can differ in sense without differing in Meaning. ‘Bertrand Russell’, ‘the author of *On Denoting*’, ‘the winner of the Nobel Prize for Literature in 1950’ are singular terms with different sense, but with the same Meaning. Similarly, predicative expressions such as ‘human being’ and ‘featherless biped’, ‘descendant of the great apes’, ‘homo faber’, ‘animal ridens’ express different senses, but can be used to designate the same *concept*, insofar as they are extensionally equivalent. Both Frege’s proposals—that the Meaning of a predicate is to be found in the concept and that the extension of the concept is to be understood as a logical object and not as a collection or aggregate consisting of the individuals that contingently fall under the concept—seem to have solid intuitive justifications, even independently of the main argument offered by Frege for this identification. This argument consists, as we know,

in the observation that both normal quantifiers (one, all, some) and numerical quantifiers (exactly 1, exactly 2, etc.) take concepts as arguments and not objects that fall under concepts. They are collectively referred to by Frege as 'second-level concepts' which, when complemented by first-level concepts, give rise to sentences, i.e. names of truth-values.

As in the case of proper names, it is certainly possible to grasp the meaning of two predicates without realizing that they stand for the same concept. To use less trite examples than 'man' and 'featherless biped' think of the notions 'lambda-computable function' and 'Turing-computable function', or 'tautology of classical propositional calculus' and 'theorem of classical propositional calculus'. We need no argument to realize that we can *understand the sense* of the words 'theorem' and 'tautology', in the strong sense of knowing how to recognize a theorem and a tautology of classical propositional calculus, and yet be surprised by the discovery that the two notions have the same extension. Metatheoretical results broaden our knowledge, and identities such as 'the class of tautologies of the classical propositional calculus = the class of theorems of the classical propositional calculus' certainly have 'cognitive value'. Consequently, the problem of whether there are informative and true identity statements seems to have an affirmative answer in the framework of the Fregean theory of concepts—on the understanding that the notion of 'identity' is not applicable in a strict sense to unsaturated entities such as concepts and functions. If in the case of proper names the burden of proof is to show that they have a sense as well as a Meaning, in the case of predicates and sentences, the difficulty is to show that in addition to the sense they also have a Meaning and to propose candidates to serve as their semantic value. Frege's arguments for the choice of the *concept* as the appropriate semantic entity are based, like in the case of proper names, on considerations of substitutability, that is of invariance with respect to the truth-value of the simple and complex sentences in which a predicate can appear as a component.

From the truth of 'Phosphorus = Hesperus' and 'Phosphorus is a planet of the solar system' we can infer not only the truth of 'Hesperus is a planet of the solar system' but also the truth of sentences obtained by substituting 'a planet of the solar system' with a predicate designating the same concept. The question arises whether it is possible to specify criteria that would allow *systematic* substitution of predicates for predicates *salva veritate* and the reasoning follows the same line of argument that applies to proper names. The criterion is provided by the extensional equivalence of two predicates. Two predicates that are *extensionally equivalent*, that is, true of the same objects, designate, according to Frege, the same concept, which as such constitutes the 'semantic value' of the predicate, what all co-extensional predicates to a given predicate have in common. Consequently, equivalent predicates can be substituted *salva veritate* in all 'ordinary' contexts. Standard examples of extensionally equivalent predicates are 'rational animal' and 'featherless biped', 'creature with kidneys' and 'creature with a heart';

other predicates with the same extension are synonymous predicates, e.g. 'round'/'circular', 'child'/'kid', 'docent'/'teacher', 'pleasant'/'agreeable', etc.

Before the discovery of the contradiction, Frege uses two different formulations to state the conditions of substitutability of two predicates, namely: 1) two predicates are substitutable with each other *salva veritate* if they have the same (truth) value for all arguments; or 2) two predicates are substitutable with each other *salva veritate* if the extensions associated with the respective concepts are the same.³⁴ After the discovery of the contradiction, the first formulation will remain the only possible one for Frege.

8. *Oratio Obliqua*

We have seen above that in normal conditions sentences like (6) and (7) have a truth-value: they are true if and only if Vera says the sentence enclosed in quotation marks or believes the sentence introduced by 'that', meant as a sentence of the English language. The latter qualification is necessary since, as Frege and other philosophers have observed, the same succession of sounds or signs in different languages can have different senses and express a sentence which is true in one language and false in another.³⁵ According to Frege, the words that make up the sentence do not contribute to the determination of the semantic value of (7) through the *ordinary Meaning* they have in (8), but through their *indirect Meaning*. Frege, as we have seen, characterizes the Meaning or semantic value of a linguistic expression as that which is common to expressions that can be substituted for one another *without altering the truth-value of the sentence as a whole*. If, irrespective of vacuous proper names, we consider sentences such as:

- (9) Vera believes that Hesperus is a planet
- (10) Vera believes that Phosphorus is a planet,

we see that their truth-value does not truth-functionally depend on the subordinate sentences; in fact (9) and (10) are true or false according to what Vera believes, and not according to the truth-value of the subordinate sentences, which in our example is the same. Vera may not know that Hesperus is Phosphorus, and she

³⁴ Cf. *Ausführungen über Sinn und Bedeutung (Reflections on Sense and Meaning)*, NS, pp. 132ff., SP pp. 226ff.

³⁵ SP, p. 235: 'It is clear that it is not to the sequence of sounds in which the sentence is presented but to its sense that we ascribe the truth; in fact, on the one hand the truth continues to exist when we translate a sentence into another language and, on the other hand, it is at least conceivable that the same sequence of sounds has a true sense in one language and a false sense in another.' Cf. A. Church, 'On Carnap's Analysis of Statements of Assertion and Belief', *Analysis*, 10 (1950), pp. 97–9, reprinted in L. Linsky (ed.), *Reference and Modality*, Oxford, Oxford University Press, 1971, pp. 168–70.

may believe that Hesperus is a planet without believing the same about Phosphorus. Frege notes that the value of such compound sentences will remain unchanged if we substitute for the subordinate sentence another one believed by that specific speaker—independently of the question of whether it is true or false (SuB, p. 37). Believing a sentence does not systematically depend on its truth or falsity, and believing true a sentence that is true and believing true a sentence that is false does not commit us to believing any true sentence or any false one. It is not surprising, then, that the application of the principle of substitutability *salva veritate* of co-designating expressions on the basis of the true identity

(11) Phosphorus is Hesperus

may not preserve the truth-value of (9).

Since in *oratio obliqua* the substitution of expressions with the same ordinary Meaning may not preserve the truth-value of the sentence, Frege proposes to recognize in the ordinary sense of words their semantic value. In a simple *oratio obliqua*, therefore, the legitimate substitutions are those of expressions designating the same (ordinary) sense. Frege appeals in SuB to an intuitive notion of synonymy or 'sense identity' which, however, he does not illustrate at all:

In all these cases it is not permissible to replace in the dependent sentence an expression with another that has the same ordinary Meaning, but with one that has the same indirect Meaning, i.e. the same ordinary sense. (SuB, p. 36)³⁶

Quine³⁷ described as 'referentially opaque' the occurrences of singular terms that are not susceptible to substitution *salva veritate* (and quantification). The opacity seems to be due to the fact that in these contexts the singular term 'is not used in the sentence purely to specify its object'.³⁸ One might say that this is so because an object can be determined in a wide variety of ways, some of which are unknown to the individual speaker. As Frege notes in a posthumously published paper, it may have been precisely the non-substitutability of co-designative terms in oblique contexts that hinders the recognition that what is designated in different ways is one and the same object.³⁹ The possibility that there are true and informative

³⁶ Note that Frege considers substitutable *salva veritate* expressions designating the same indirect Meaning; whether expressions having the same indirect Meaning should express the same indirect sense is a matter of dispute. For a discussion of this point see M. Dummett, *The Interpretation of Frege's Philosophy*, Cambridge, Harvard University Press, 1981, ch. 6; T. Parsons, 'Frege's Hierarchies of Indirect Senses and the Paradox of Analysis', *Midwest Studies in Philosophy*, 6, Minneapolis, University of Minnesota Press, 1981, pp. 37–57; Linsky, *Oblique Contexts*, ch. 3: *Indirect Sense and Reference*.

³⁷ For example, W. V. Quine in *Word and Object*, Cambridge, MIT, 1960, §30; Italian translation *Parola e oggetto*, Milan, Il Saggiatore, 1970.

³⁸ *Ibid.*, p. 142.

³⁹ NS, pp. 43ff., SP, p. 361.

identity statements seems to depend on these considerations, transferred, so to speak, from the level of the individual speaker to that of the linguistic community as a whole. The consequence that Quine draws from the inapplicability of the principle of substitutability of identicals is quite different from the one drawn by Frege in his 1892 essay. Quine not only distrusts the shifts of reference (and sense) that Frege's theory involves but has serious doubts about the usefulness of conceiving senses as 'objects' in the absence of clear criteria for sense identity.⁴⁰

As we have seen, Frege characterizes the Meaning or semantic value of a linguistic expression as what is common to several expressions that can take the place of one another *without altering the truth-value of the sentence*. When we utter with assertoric force the words 'The earth is moving' we express a thought and show that we recognize it as true; to know whether the sentence is true we must know whether the object designated by the proper name falls under the concept designated by the predicate. Also, in saying 'Vera believes that the earth is moving' we express a thought and we show the recognition of its truth: to know if the sentence is true we need to know if Vera believes the *thought that the earth is moving*, that is, if between Vera and the object designated by the subordinate sentence introduced by 'that' there is the relation indicated by the doxastic verb. To be fair, Frege does not say that 'believe' or 'know' are relational predicates, and the fact that they can occur iterated brings them closer to operators rather than predicates.

Subordinate sentences have not only an indirect Meaning, but also an indirect sense:

In indirect speech, words are used indirectly, that is, they have their own indirect Meaning. We therefore distinguish the ordinary Meaning of a word from its indirect Meaning, and its ordinary sense from its indirect sense. (SuB, p. 29)

The reason why in a simple *oratio obliqua* the subordinate sentence, in addition to designating an indirect Meaning, must also express an 'indirect' sense different from the ordinary one, depends both on the need to preserve the distinction between sense and Meaning in all contexts, and on the need to maintain the distinction between 'expressing' and 'designating' even when the designated object is a sense. If the ordinary sense has to function as the Meaning, it will be necessary to find a name to designate it: the complex name thus formed will not be able to express the ordinary sense it has when the words it contains are used as a vehicle of truth or falsity, but it will have an indirect sense, by means of which it directs the listener to the choice of the semantic value appropriate to the context, that is, to the ordinary sense of the words, which functions here as an indirect Meaning.

⁴⁰ Quine, *Word and Object*, p. 151.

The intuition underlying Frege's reasoning is that in a sentence such as 'A says (feels, means, is persuaded, concludes) that *p*' the conjunction 'that' signals that in the subordinate sentence the words have their indirect Meaning, and do not designate a truth-value; their sense is that of the words 'the thought that...' (SuB, p. 37). The expressions '2 is a prime number' and 'The thought that 2 is a prime number' have different meanings: the first designates the Truth, the second a thought. According to principle C, the sense given by the two expressions is also different. What Frege calls the indirect sense of the words is therefore the sense expressed by the name 'the thought that 2 is a prime number'. Similarly, when we want to talk about the sense of 'A' we must use the formulation 'the sense of the expression "A"'; obviously, the sense of 'A' is different from the sense of 'the sense of "A"'. To talk about a sense, I must be able to name it; *the most direct way* to name a sense is certainly the one proposed by Frege. However, it should be possible to name a sense without exactly repeating the original linguistic formulation, but rather by using a different language, or a different grammatical construct. Frege does not tire of repeating that the same sense must be able to be given in different ways, because, if this were not so, we would not be able to recognize the same thought under different guises: the very possibility of definitions would be jeopardized.

Therefore, it must also be possible to name a sense in ways other than the 'most direct' one. Assuming that different sentences can *express* the same thought, in the way just described we can form names with different indirect senses, which serve as ways of presenting the same thought in contexts of *oratio obliqua*. This proposal, plausible at first glance, becomes cumbersome, however, when applied to a double or multiple *oratio obliqua*. Indeed, simple indirect senses can in turn serve as doubly indirect Meanings of a double *oratio obliqua*, to designate which names having doubly indirect senses are needed, which in turn can serve as indirect Meanings in a triple *oratio obliqua*, to designate which names expressing triply indirect senses are needed, and so on, indefinitely.

One perverse aspect of this proliferation of indirect senses was noted by Russell: he observed that in the picture described by Frege we first have the *Bedeutung* (the ordinary sense) of an expression, and from there we derive 'the (indirect) sense of that *Bedeutung*'.⁴¹ But there is no road leading from denotation to sense, and, for that matter, there is no need for it, for if we have denotation we have already secured what we needed. One can reply to Russell by observing that Frege's proposal is safe from this objection: the construction introduced by 'that', thanks precisely to its indirect sense, directs us to the type of *Bedeutung* appropriate to the context, in harmony with the principle that sense determines Meaning. However,

⁴¹ B. Russell, 'On Denoting' (1905), in D. Lackey (ed.), *Essays in Analysis*, London, Allen and Unwin, 1973, pp. 103–19, especially pp. 110–11; Italian translation in A. Bonomi (ed.), *La struttura logica del linguaggio*, Milan, Bompiani, 1973.

this reply is of no help in clarifying the relationship between the more direct way of designating a sense and the less direct ways of designating it, nor does it account for the relationship between the senses that the same expression can acquire in all its possible occurrences in sentences of arbitrary complexity.⁴² Another somewhat perverse aspect of Frege's proposal is that to designate a sense we necessarily have to use a name that expresses a different sense from the one to which we want to refer, and we always have the impression that we are speaking of something other than that of which we want to speak.⁴³

Contrary to what is generally thought, Frege did not believe that the problems surrounding the constructs he collectively referred to as *oratio obliqua* arise only for the semantics of natural languages. In fact, Frege returns to indirect contexts in two letters addressed to Russell (dated, respectively, 28 December 1902 and 21 May 1903). Here he proposes, as a way out of the semantic version of the contradiction discovered by Russell, an 'intensional' solution, which exploits the difference between the indirect and the ordinary Meaning of words. Let *M* and *N* be two modes of determination of the same class: it is possible to substitute them reciprocally *salva veritate* in all ordinary contexts but not in contexts of *oratio obliqua*. In fact, Frege argues, in the sentence 'The thought that all thoughts belonging to the class *M* are true' the subordinate clause introduced by 'that' has a different Meaning than the subordinate clause introduced by 'that' in the sentence 'The thought that all thoughts belonging to the class *N* are true', because whoever did not know that *M* and *N* coincide could consider the former false and the latter true. When we affirm: 'The thought that all thoughts belonging to the class *M* are true, belongs to the class *M'*', the former occurrence of *M* is in *oratio obliqua* and the latter is not; consequently the latter occurrence of *M*, but not the former, can be replaced by *N*.⁴⁴ This difference of Meaning should also be signalled typographically, for example: 'The thought, that *the thought that all thoughts belonging to the class M are true*, does not belong to the class *M*.' Frege proposes to signal the difference indicated here by italics and bold font by means of a simple and a double underlining, respectively: the simple underlining indicates the first-order indirect Meaning and the double underlining the second-order indirect Meaning.

Unlike in SuB, in the letters addressed to Russell Frege does not mention the 'indirect meaning' that words would have in these contexts and this leaves open the possibility of the following amendment: to confine the diversity of sense of the expressions in *oratio obliqua* to the number of iterations of the operator 'the

⁴² On this issue, beside Dummett, *The Interpretation of Frege's Philosophy*, see also H. Heidelberger, 'Dummett on Frege's Philosophy of Language', *Metaphilosophy*, 6, 1 (1975), pp. 35–43; A. Code and S. Blackburn, 'The Power of Russell's Criticism of Frege', *Analysis*, 38 (1978), pp. 65–77.

⁴³ Cf. J. Searle, *Russell's Objections to Frege's Theory of Sense and Reference* (1957), reprinted in G. Klemke (ed.), *Essays on Frege*, Urbana, University of Illinois Press, 1968, pp. 337–45.

⁴⁴ WB, p. 236; Italian translation, *Alle origini della nuova logica*, p. 209.

thought that...'. The typographical differences (and similarities) would serve to point out this fact, showing—at the same time—that it is a chain of symbols originating from the same sentence. The disappearance of indirect senses suggests a simplification of the hierarchy of indirect Meanings: a subordinate sentence would have only one indirect Meaning, equivalent to the ordinary sense, but a multiplicity of names, corresponding to its degree of iteration. This interpretation, however, clashes with Frege's motivation for introducing typographical differences, namely to restrict the range of permissible *substitutions* to the range of expressions with an indirect Meaning of appropriate degree. This, however, means that the plethora of indirect Meanings remains intact, and the only variation is that instead of the hierarchy of indirect senses there are many different names, according to the degree of iteration, with as many new senses. This is, in essence, the solution adopted by Church.⁴⁵

The advantages of the disappearance of indirect senses are therefore more apparent than real: as various authors have observed, the 'typographical' treatment proposed by Frege is equivalent to the introduction of new semantic primitives, that is, new names typographically different from each other. Thus, the situation is not changed with respect to the one described in SuB: while in SuB we have the problem of finding conditions of identity that specify when two expressions designate the same indirect sense of degree $n - 1$ when it becomes the indirect Meaning of degree n , the proposal contained in the correspondence with Russell poses the problem of how to formulate the conditions of substitutability *salva veritate* for the new names introduced at each iteration.

According to Davidson, the infinite hierarchy of 'indirect senses' sketched by Frege in SuB and elaborated by Alonzo Church implies a language with an infinite number of *semantic primitives*: a language with this characteristic is not learnable.⁴⁶

9. Synonymity

But are there general conditions for determining whether in an *oratio obliqua* two typographically different linguistic expressions *designate* the same sense, have, that is, the same *indirect Meaning* of first or second (or any) degree?

⁴⁵ In 'A Formulation of the Logic of Sense and Denotation', in P. Henle et. al. (eds.), *Structure, Method and Meaning*, New York, The Liberal Arts Press, 1951, pp. 3–24.

⁴⁶ Davidson's proposal, which we cannot go into here, is to separate the problem of treating the conjunction 'that', which introduces *oratio obliqua*, from the treatment of the problem of sense identity. See Davidson, *Inquiries into Truth and Interpretation*. For a discussion of this issue see R. Riemma, 'Una difesa della teoria fregeana dell'*oratio obliqua*', *Annali del Dipartimento di Filosofia dell'università di Firenze* (1986), pp. 185–234; P. Leonardi, 'Crede che', *Lingua e Stile*, 23, 2 (1988), pp. 271–99.

The treatment of this topic in SuB is very brief; Frege appeals to an intuitive notion of synonymy but does not specify conditions of identity for ordinary senses and this brings with it a series of questions concerning indirect senses, the most delicate of which is the following: are expressions designating the same indirect Meaning (= ordinary sense) synonymous or not? Are the indirect senses of such expressions different or coincident? This is relevant for predicting the behaviour of the expression having the indirect sense in question when it will in turn become the doubly indirect Meaning of a double *oratio obliqua*. Let us illustrate the situation with an example. Based on the synonymy of 'circles' and 'circumferences', we can pass *salva veritate* from

(12) Copernicus believed that the orbits of the planets were circles

to:

(13) Copernicus believed that the orbits of the planets were circumferences.

However, one can, indeed must, agree, if one accepts B. Mates's paradox of synonymy,⁴⁷ that the indirect sense of the expression 'that the orbits of the planets were circles' differs from the indirect sense of the expression 'that the orbits of the planets were circumferences'.

In fact:

(14) No one doubts that whoever believes that the orbits of the planets are circles, believes that the orbits of the planets are circles.

and

(15) Someone doubts that anyone who believes that the orbits of the planets are circles believes that the orbits of the planets are circumferences.

If these sentences can both be true, then the words 'circle' and 'circumference' are not synonymous, otherwise (14) and (15) would be contradictory. Therefore, the indirect sense expressed by 'the thought that the orbits of the planets were circles' differs from that which is expressed by 'the thought that the orbits of the planets were circumferences'.⁴⁸

⁴⁷ B. Mates, 'Synonymy' (1950), reprinted in L. Linsky, *Semantics and the Philosophy of Language*, Urbana, University of Illinois Press, 1952; Italian transl. *Semantica e filosofia del linguaggio*, Milan, Il Saggiatore, 1969.

⁴⁸ H. Putnam, 'Synonymy and the Analysis of Belief Sentences', *Analysis* (1954), pp. 114–22, reprinted in N. Salmon and S. Soames (eds.), *Propositions and Attitudes*, Oxford, Oxford University Press, 1988, pp. 149–58, especially pp. 155–6.

However, from the point of view of Frege's doctrine, up to this point there would be nothing strange: in a double *oratio obliqua* words have their doubly indirect Meaning, which corresponds to their simple indirect sense. Mates's paradox highlights the fact that in a double *oratio obliqua* it is possible to substitute only expressions that designate the same indirect sense and not the same ordinary sense; apparently the indirect senses of the expressions in question are different. Therefore, there is no guarantee that in the transition from

(16) Kepler knew that Copernicus believed that the orbits of the planets were circles

to

(17) Kepler knew that Copernicus believed that the orbits of the planets were circumferences

the truth-value remains unchanged. For this to happen, the sentence following the second 'that' in (16) must be replaced with one designating the same doubly indirect Meaning (= the indirect sense corresponding to the first-degree indirect Meaning). It is therefore not surprising that substitutions of expressions with different doubly indirect Meanings can alter the truth-value. This simply shows that the (indirect) sense expressed by 'the thought that the orbits of the planets were circles' is different from the (indirect) sense expressed by 'the thought that the orbits of the planets were circumferences': this difference comes to light when the expressions in question function as (doubly) indirect Meaning. Mates's 'paradox', therefore, far from being a paradox, would be an indirect confirmation of the soundness of the Fregean doctrine that postulates a whole procession of different senses according to the degree of subordination. But how is it possible to argue that 'circle' and 'circumference' are synonyms in (12) and (13) and yet cease to be so in (16) and (17)? What nuance of sense could escape Kepler (or anyone else) to make it at least conceivable that (16) and (17) could differ in truth-value? The only possible alternative seems to be, as Dummett has suggested,⁴⁹ that Kepler, while not knowing the sense of the expression 'circumference', knows the sense of the expression 'the sense of "circumference"': therefore, what he ignores is the reference of 'circumference' in (13). This situation is strongly counterintuitive.

For those who accept this result, the question of what substitutions are legitimate in a double *oratio obliqua* arises with urgency. The *prima facie* natural answer is that in a double *oratio obliqua* we can no longer make any substitutions since, literally, we do not know when two expressions endowed with doubly

⁴⁹ Dummett, *The Interpretation of Frege's Philosophy*, p. 93.

indirect senses designate the same indirect (simple) sense. Coming to this conclusion is tantamount to admitting that there are no conditions of identity for indirect senses and if we accept this conclusion it is hard to see why we have accepted the presupposition of Frege's argument, namely that there are genuinely synonymous expressions, expressions, that is, that despite being linguistically different can be used to designate the same sense. Avoiding Mates's paradox is not a merit of Frege's proposal, but it highlights the difficulties inherent in conceiving of sense as an object. Indeed, it is not clear how an indirect sense can serve as a referent if it is impossible to say whether or not two names present the same indirect sense and thus whether they are mutually substitutable *salva veritate*. If the indirect sense cannot become in turn the semantic value, the bricks with which to build the hierarchy of indirect senses and Meanings are missing. A way out of these difficulties is to give up the *hierarchy* of indirect senses for the *same* expression, without however abandoning the idea that the same ordinary sense can serve as a reference of expressions that are linguistically different but equivalent, in the way that we will illustrate below. This solution assumes—which is far from clear—that it is possible to apply to the sense, understood as the object of reference, the same principles that apply to the *Bedeutung* of words in ordinary contexts. If one accepts this solution, one must, however, look for an explanation of Mates's 'paradox'.

Mates's paradox seems to be caused by the neglect of some important distinctions about what understanding the meaning of a word entails. The problem of synonymy is not, in my view, about the breadth of vocabulary or knowledge of this or that individual speaker. Returning to example (7), what is at issue is not the breadth of Vera's vocabulary, say, compared to that of Thomas who is unaware that 'jaw' means 'throat' in this context, so that replacing 'jaw' with 'throat' would preserve the truth-value of (7), while replacing 'throat' with 'jaw' would not preserve the truth-value of (18):

(18) Thomas believes that Scylla has six throats.

Because of considerations such as these we conclude that in a natural language there are no synonymous expressions, since it is always possible to find some speaker who is unaware or distracted. Yet there is a difference between being conceptually confused and having a limited lexicon, a difference that examples like these tend to obscure.

The issue of synonymy is, in some respects, complementary to that concerning the cognitive value of identity statements. The discussion of contingent identity statements shows that it is possible to know the sense of two proper names without knowing that their reference is the same. But is it possible to fully grasp the sense of two words without realizing that the sense is the same? What is at issue is whether a speaker who fully grasps the sense of two words can say

whether they have the same sense or not. From the point of view of the context principle the question is ill-posed, since it refers to occurrences of isolated words, and presupposes an atomistic conception of the sense of words, which for many reasons is not sustainable. The context principle enjoins, in fact, not to investigate the sense of words divorced from the context of the sentence in which they occur and this must also apply to questions concerning the identity and difference of sense. In fact, the most treatable case of synonymy is that which concerns sentences and not isolated words. Such a criterion of cognitive synonymy is found in one of Frege's posthumous papers:

Two sentences A and B can be in such a reciprocal relation that who recognizes as true the content of A, must at the same time recognize as true the content of B and that, conversely, the one who recognizes as true the content of B, must immediately recognize as true the content of A (equipollence); we assume here that there is no difficulty in understanding the content of A and B.⁵⁰

Frege believes that two sentences can be said to be 'equipollent' if a speaker who fully understands their sense and recognizes one of them as true must *immediately* recognize the other as true. Equivalence of sense (as far as sentences are concerned) should therefore be intuitively decidable for someone who has no difficulty in understanding the sense of the words used: if two sentences are synonymous and a speaker assents to one but not to the other, we must conclude that he has not grasped the sense of one of these sentences.

A further useful distinction in this context concerns the intensional and extensional way of 'understanding the sense of a word' suggested, for example, by M. Dummett,⁵¹ and, in substance, already present in Langford's⁵² and Scheffler's⁵³ essays: in the intensional way, to understand the sense of a word implies recognizing the word as having that sense; in the extensional one, to understand the sense of a word implies, at least, to have mastered the concept expressed by the word (in the non-Fregean sense of 'concept'), even if one is not familiar with the specific word. In the previous example, we can say that Thomas

⁵⁰ NS, p. 213, SP, p. 325. In my opinion Frege here means to characterize a relation between thoughts stronger than the mere material and logical equivalence. For a discussion, see 'A Note on Dummett and Frege on Sense-Identity', *The European Journal of Philosophy*, 1 (1993), pp. 69–80. For a discussion of this issue, see also D. Bell, 'Thoughts', *Notre Dame Journal of Formal Logic*, 28 (1987), pp. 36–50.

⁵¹ Dummett, *Alle origini della filosofia analitica*; another important distinction drawn by Dummett in this context is the one between the dispositional and the occurrent sense of understanding: the former concerns the isolated word and refers to the speaker's ability to use it in the sentences in which it occurs; the latter concerns the actual comprehension that the speaker has of a word's sense in a given context.

⁵² C. H. Langford, 'Moore's Paradox of Analysis', in A. Schilpp (ed.), *The Philosophy of G. E. Moore*, Evanston, Northwestern University Press, 1942, pp. 321–42.

⁵³ I. Scheffler, 'On Synonymy and Indirect Discourse', *Philosophy of Science*, 22 (1955), pp. 39–44.

believes (in the extensional sense) that Scylla has six jaws, if he is able to correctly apply the concept expressed by the word, if, for example, he knows how to distinguish this anatomical part from the other ones, has an idea of its function and location. Let us say that he knows the sense of the word 'jaw' (in the intensional sense of the term) if he recognizes the word as having the sense it actually has. The same is true for 'circles' and 'circumferences': Vera knows that Thomas has drawn a circle if she is able, for example, to indicate the kind of figure Thomas has drawn, even if she is not able to express her belief by using the word 'circumference' but only the word 'circle' and even if she would hesitate when asked if Thomas has drawn a circle.

In *Meaning and Necessity* Carnap proposes to distinguish logically equivalent sentences and isomorphically equivalent sentences to address the problem of synonymity posed by belief contexts. For example, 'The concept Brother is identical with the concept Male Sibling' and 'The concept Brother is identical with the concept Brother' are L-equivalent sentences, but they do not have the same intensional structure, not having been constructed in the same way with the same bricks.⁵⁴ The requirement of *intensional isomorphism* imposed by Carnap prescribes, roughly, that there is a point-by-point correspondence in both lexical and logical complexity of the expressions that can be considered intersubstitutable in belief contexts. This proposal is open to the objection that both the notion of lexical complexity and that of logical complexity are relative to a particular semantic 'reconstruction' of the language and to the specific semantic rules adopted. Putnam's solution to Mates's paradox is in the vein of Carnap's semantics. He observes that in Mates's examples the logical form of the subordinate sentences is different (in one case the subordinate clause is of the form 'all A's are A', in the other 'all A's are B'), and that an appropriate definition of synonymity must also take into account the overall context in which the pairs of synonyms appear.

According to Church,⁵⁵ Putnam's proposal is based on an incorrect assessment of the scope of Mates's paradox. What Church questions is the genuineness of the doubt hypothesized in Mates's examples. The error consists in not sufficiently distinguishing between two forms of ignorance: ignorance of the sense expressed by a word and ignorance that a given word has that sense. Even without going into the details of Church's criticism or the proposed amendments to the Carnapian notion of intensional isomorphism, it is worth recalling the (dis)solution of Mates's paradox, while mentioning the 'paradox of analysis'. In the paradox of analysis, unlike in the examples discussed by Mates, there are not simple

⁵⁴ R. Carnap, *Meaning and Necessity*, Chicago, Chicago University Press, 1947; Italian translation *Significato e necessità*, Florence, La Nuova Italia, 1976.

⁵⁵ A. Church, 'Isomorphism and Identity of Belief', *Philosophical Studies*, 5 (1954), pp. 65–73, reprinted in Salmon and Soames (eds.), *Propositions and Attitudes*, pp. 159–68.

synonyms, but expressions that should serve to *analyse* or make explicit the content of a word. The paradox consists in the apparent impossibility of reconciling the fruitfulness of analysis with its correctness: if the proposed analysis captures exactly the content of the notion to be analysed, it is vacuous; if it appears informative, it is incorrect. As Bealer has pointed out, the paradox of analysis is not to be confused with Mates's paradox; the former highlights the problematic character of definitions and shows the necessity of distinguishing mastery of a concept from knowledge of a particular 'analysis' of that concept as given in a definition. Let us consider sentences (19)–(21) by adapting an example used by Bealer:⁵⁶

- (19) Vera knows that everything that is a circumference is a circumference.
- (20) Vera does not know that the circumference is the geometric locus of the points of the plane equidistant from the same given point.
- (21) Circumference = the geometric locus of the points of the plane equidistant from the same given point.

The paradox in Bealer's formulation is as follows: (19) and (20) are simultaneously satisfiable, (21) is a true identity, but (20) in conjunction with (21) and (19) results in a contradiction. Church gets around this formulation of the paradox by denying that expressions that do not satisfy the requirement of synonymous isomorphism designate the same concept. However, the example chosen by Church in the 1954 essay 'fortnight = a period of fourteen days' or 'lustrum = a period of five years' satisfies the conditions imposed by Church without, however, avoiding Mates's paradox. Assuming in fact that it is true that 'fortnight = a period of fourteen days' or 'lustrum = a period of five years', we have:

- (22) Anyone who believes that Napoleon's empire lasted less than a lustrum believes that Napoleon's empire lasted less than a lustrum.
- (23) Anyone who believes that Napoleon's empire lasted less than a lustrum believes that Napoleon's empire lasted less than five years.

According to Mates (24) is true while (25) is false:

- (24) No one doubts that (22).
- (25) No one doubts that (23).

To avoid the paradox, we have only to admit that it is impossible to have doubts about (23) without at the same time doubting (22). Anyone who believes that Napoleon's empire lasted less than a period of five years believes that it lasted less

⁵⁶ G. Bealer, *Quality and Concept*, Oxford, Oxford University Press, 1985, pp. 69–74.

than a lustrum: what he ignores is a fact about the English language. What (23) actually says:

(23') Whoever satisfies in English the enunciative matrix 'X believes that Napoleon's empire lasted less than a lustrum' satisfies in English the enunciative matrix 'X believes that Napoleon's empire lasted less than five years'.

But (23') is not synonymously isomorphic to (22'):

(22') Whoever satisfies in English the enunciative matrix 'X believes that Napoleon's empire lasted less than a lustrum' satisfies in English the enunciative matrix 'X believes that Napoleon's empire lasted less than a lustrum'.

Thus, Mates's paradox does not constitute a counterexample to synonymous isomorphism. According to Church, a further merit of his analysis consists in showing that it is impossible to translate the English example into Italian or German without running into an obvious contradiction, since in Italian and German it is impossible to render 'fortnight' otherwise than with 'period lasting fourteen days' or 'period lasting two weeks'. If with examples like this Church's solution seems plausible, it nevertheless has counterintuitive consequences when applied to the case of definitions. In fact, as Bealer observes, in the example of the definition of circumference, Church's way out consists in denying that (19) and (20) are jointly satisfiable: the only alternative is to suppose that (20) should be reformulated in metalinguistic terms: what Vera does not know is that everything that satisfies the English enunciative matrix 'X is a circumference' satisfies the English enunciative matrix 'X is the geometric locus of points in the plane equidistant from a given point'. According to Bealer, this, by contraposition, would be equivalent to claiming that if I say of a person performing a certain geometric construction that he knows that circles are the geometric places, etc., all that I have attributed to him is linguistic knowledge and not actual knowledge. What the person in question knows is that two matrices are satisfied by the same objects. This makes it impossible to explain the fruitfulness and cognitive value of a good definition. A tentative conclusion that can be drawn from this example is that definitions, precisely because they contain a conceptual analysis, are not examples of sentences that express an identity of sense, and that definitions do not preserve the sense of the expressions they were meant to analyse. It is precisely because of this (moderate) infidelity that they are fruitful.⁵⁷

⁵⁷ A solution along these lines is proposed by Dummett in 'Frege and the Paradox of Analysis' (1987), in M. Dummett, *Frege and Other Philosophers*, Oxford, Oxford University Press, 1991. For a broader discussion, see, by the same author, *Frege: Philosophy of Mathematics*, London, Duckworth, 1991, chs. 12–14.

In many of his writings Frege gives as examples of sentences having the same content or expressing the same thought the various functional decompositions that are possible for a given sentence. The question arises whether these different decompositions could not be seen as ways of designating the ordinary sense in constructs of *oratio obliqua*. Is it possible to apply the functional conception of thought composition to explain how different indirect senses can designate the same ordinary sense without running into the paradox of analysis or Mates's paradox?

At first sight it seems possible: Frege himself cites as examples of sentences expressing the same thought the different functional decompositions of which a sentence is possible. Let us consider as an example the sentence:

(26) 2^4 is equal to 16.

According to Frege we have various options as to how we might render the scope of this sentence, i.e. we can see it as the exemplification of different functions. We can say that 2 is a fourth root of 16, but also that 4 is the logarithm of 16 in base 2. This example is trivial, but it is a first step in dealing with the stumbling blocks hidden in the locution 'the same thought'. Suppose I say to Vera, 'Calculate the result of 2 to the power of 4,' and Vera replies, ' $2^4 = 16$.' 'Well then,' I might reply, 'I suppose you will also be able to tell me immediately what the positive fourth root of 16 is and also what the logarithm of 16 in base 2 is.' Obviously this is absurd: it is not enough to know how to decompose the statement in various ways to answer the previous questions: you must also have learned the concept of logarithm, of root extraction, and to learn these concepts it is not enough to have an idea of the notion of raising a number to power. The calculation techniques are different in the two cases. Thus, it would be wrong to say that we can go *salva veritate* from:

(27) Vera believes that 2^4 is equal to 16

to

(28) Vera believes that 4 is the logarithm of 16 in base 2.

Yet, this should be possible if we take literally Frege's conception that the same thought can be decomposed in different ways; the example given is in fact an application of an example used by Frege himself.⁵⁸

⁵⁸ NS, p. 18, SP, pp. 88ff.

The problem that examples like this involve should not be confused with others that could be found in a natural language by reflecting on the active–passive transformation of sentences without quantifiers, or on the fact that although every relation has a converse, there is not always a term in the language to indicate the converse of the relation and speakers are not always aware of it. Thus, a child or a foreigner who has not mastered English might not recognize that the sentences ‘Carla is Vera’s grandmother’ and ‘Vera is Carla’s granddaughter’ express ‘the same thought’. Assuming, however, that the child or the foreigner has no difficulty in understanding the kind of kinship indicated by the word ‘grandmother’, there is no further concept that he has failed to master the lack of which prevents him from grasping the second statement: what he is ignorant of is the English word for the converse of the relation. In cases like these, unlike in the case of definitions, Church’s solution seems acceptable.

The previous considerations suggest rather liberal criteria for the substitutability of ‘synonymous’ expressions in *oratio obliqua*; however, not so liberal as to admit as ways of designating the same thought different ‘decompositions’ of the same sentence, to understand which it is necessary to appeal to further or different concepts from those required for the understanding of the sentence. In Frege’s writings there is an oscillation between strict criteria of synonymy, according to which, for example, ‘if p then q ’ expresses a different thought than ‘not (p and not q)’, and lax criteria, according to which the statements ‘line a and line b are parallel’ and ‘the direction of line a is the same as line b ’s’ would express essentially the same content. The first choice is certainly wise: in fact, not observing it would lead us to conclude that all logically equivalent propositions are synonymous with one another and this is obviously unsatisfactory. The second choice, however, is questionable and highlights some inherent difficulties that the notion of ‘analysis’ encounters in the context of definitions. Using the distinction between intensional and extensional meaning we can modify, following Dummett’s suggestion, the criterion of synonymy indicated by Frege by adding the clause that for the recognition of the synonymy of two expressions the mastery of further concepts should not be necessary.⁵⁹ This restriction is necessary to put constraints on the decompositions of a sentence that can be considered as expressions of the ‘same’ thought.

The ‘functional’ model applied to the composition of the sentence suggests, roughly, to conceive the thought expressed by ‘Rab’ as the value of the (sense) function ‘Ray’ for the sense of ‘b’ as argument; the same can be replicated for ‘Rxb’ and ‘Rxy’. We could compare this kind of composition of the sense of the sentence to a kind of ‘seeing as’—in the sense proposed by Wittgenstein in section XI of

⁵⁹ Dummett, *Alle origini della filosofia analitica*, ch. 10.

Part Two of the *Philosophische Untersuchungen*. Furthermore, we could assimilate to a 'seeing' the grasping of the sense expressed by a sentence along the lines of its linguistic constitution, without paying attention to the possible ways in which it might be decomposable. Just as 'seeing' has conceptual priority over 'seeing as', similarly, grasping the thought expressed by the sentence on the basis of its component parts has priority over 'seeing it as' the value of different sense functions. These different ways of conceiving of the same thought can be understood as the indirect senses of the expressions formed according to the pattern 'the thought that...'. The functional model does not replace but relies for its intelligibility on the intuition that sense is the way the reference is given to us, and thus, in the first instance, a medium and not an endpoint.

10. Sense and Extra-Linguistic Context

One class of sentences with respect to which the application of principle C, stated above, raises perplexity is that of sentences containing personal pronouns, such as 'I', 'you' and temporal and locative adverbs, such as 'now', 'yesterday', 'here', and so on.

The perplexity has an explanation opposite to that of the previous case: if in the *oratio obliqua* what is perplexing is whether and to what extent the characteristics of the indirect Meaning of an expression *must* or *can* be reflected by the indirect sense of the nouns employed to refer to it, here what is problematic is how such a difference *can* at all be reflected in a difference in the *sense* of the linguistic expressions. In fact, if we stick to principle C we would have to say that the first person-pronoun 'I' expresses different *senses* depending on the identity of the person using it. If this were so, we would have to know the *Bedeutung*, the bearer of the name first, and from knowledge of the one who uttered the sentence in which the word 'I' figures we would then trace our understanding of the thought expressed by the sentence on that particular occasion.

The problem of personal pronouns is not discussed in SuB. In that essay, as well as in writings prior to 1892, one finds interesting insights about the mechanism by which the external circumstances of the context in which a sentence is uttered contribute to the determination of the thought expressed. In SuB, Frege's attention is focused on the different ways in which the indication of time contributes to determining the sense of sentences that superficially have the same grammatical form. In a sentence such as

(29) If now the sun has already set, the sky is densely covered with clouds,

the indication of the present time is explicit and determinate, and both the antecedent and consequent of the conditional express *complete* thoughts that

designate truth-values. Thus, (29) can be rendered: 'Either the sun has not yet set or the sky is now covered with clouds.' In the sentence

(30) If the sun is in the Tropic of Cancer, in the northern hemisphere we have the longest day,

the *tempus praesens* does not play the same role as in (29): the antecedent and the consequent, taken separately, express *incomplete* thoughts, since they contain time as a common indeterminate part. The sense of (30) could be rendered more appropriately with a quantification on times: 'Every time the sun is in the Tropic of Cancer, in the northern hemisphere we have the longest day.' Another kind of 'incompleteness' is that signalled by the anaphoric link between relative pronouns or pronouns such as 'as many ... as'. For example, the relative pronoun that occurs in the example 'he who discovered the elliptical form of the orbit of the planets died in misery' is treated by Frege as one of the ways that the natural language uses to signal the relations of binding and co-reference that exist between the main sentence and the relative sentence, relations that in logic are indicated by means of bound variables. Frege considers both the subordinate and the main sentence as expressing, each taken separately, an 'incomplete' thought, because of the variable contained therein. This way of conceiving of sentences containing relative pronouns and generality indices reflects Frege's conviction that a quantified sentence such as 'all men are mortal' is obtained by subsuming within the second-level concept 'all' the first-level concept 'what is a man is mortal': from this point of view, it becomes clear in what sense 'what is a man' and '(it) is mortal' separately express only incomplete parts of thought united in a nexus of subordination. These incomplete parts are, in turn, obtained from molecular sentences, e.g. 'If Socrates is a man, Socrates is mortal', by omitting both occurrences of the proper name 'Socrates'.

While what unites these might at first sight seem to be different kinds of 'incompleteness', it is in fact a result of the latent quantificational structure: it is responsible for the element of indefiniteness that can be felt in (30) and in the sentences with relative pronouns. By contrast, the sentence which appears as the antecedent of (29) does not present any incompleteness. In these cases Frege seems to believe that the possible indication of place and time has the function of *making explicit* something implicit and not that of *completing* something indeterminate.

A good place to start when trying to understand this argumentative strategy is the account of the sense and Meaning of predicates that was mentioned above (section 7). Frege locates the Meaning of a predicate in the concept, and not in the collection of objects that satisfy the predicate. Regardless of the weaknesses of the Fregean conception of logical objects, if we reason in terms of variable domains or possible worlds, the distinction between the concept designated by a predicate and

the collection of objects that contingently satisfy the predicate in a given world can be useful. For example, if instead of thinking in terms of a universe of discourse that contains the largest possible number of objects—barring contradictions—we think in terms of worlds with a different number of individuals from the actual world, it may well be possible that some predicates that are not co-extensional in a given world will become so in a world with a smaller number of individuals. However, this difference in the number of objects that contingently satisfy a predicate in a world or at a certain time should not affect the concept (the Meaning of the predicate) and consequently, according to principle C, not even the sense that the two predicates have in different worlds or in successive time stages. At least, this is a possible line of argument given the distinctions proposed by Frege.

Frege, however, when considering examples from natural language, follows another path. Consider a very simple example: the concept 'living human being' is assigned different numbers in 1983 and 1883—that is: the number of living human beings in 1883 is different from the number of living human beings in 1983, yet the sense of the predicate 'living human being' is the same in 1983 and 1883. Frege proposes instead to incorporate the spatio-temporal determination into the sense of the predicate. This problem is discussed both in GLA, in the *First Logical Investigation*, and in letters and fragments found in posthumously published writings. In GLA, §46, we read:

The concept 'belonging to the German state' contains time as a variable constituent part, or, to put it in mathematical terms, is a function of time. Instead of 'a is a member of the German state' we can say: 'a belongs to the German state' and this refers to the present point in time. So there is also something fluid in the concept. On the other hand, the concept 'belonging to the German state at the beginning of the year 1883 according to Berlin time' has the same number forever.⁶⁰

The predicate as presented here must be completed by indicating spatio-temporal coordinates. In our example the relevant predicates would be 'human being living in 1883' and 'human being living in 1983'; they have different senses and have different concepts as *Bedeutung*. From a semantic point of view this choice is short-sighted, since it leads to an unjustified proliferation of predicates. Instead of incorporating the explicit indication of spatio-temporal determinations in the sense of the predicate, it is better to introduce times and places as parameters

⁶⁰ Not by chance Quine, *Word and Object*, calls 'eternal sentences' those sentences the truth-value of which is fixed once and for all, no matter whether they are true independently of the time and place in which they are used, or whether the spatio-temporal indication is explicitly supplied in the sentence.

that refer to the circumstances in which the sentences are used and uttered. An argument similar to the one just discussed is also found in an unpublished piece written many years later:

The expression 'the number of inhabitants of Jena changes' is strictly speaking incorrect. It certainly does not mean that the same number assumes different properties in the course of time, but that in the course of time other numbers rise to the rank of 'the number of inhabitants of the city of Jena'. It is like saying that in countries like England and Holland the monarch is variable in terms of sex, and from this to conclude that there are human beings who are now male, now female. The misunderstanding arises because the expression 'the king of England' is considered as a proper name. However, it is not a proper name, but can only be a proper name if it is completed by the time.⁶¹

Therefore, sentences containing an implicit temporal parameter would not be true, but they would resemble open formulas, of which it is pointless to ask if they are true or false *tout court*: what is true or false in this way are the sentences completed with the explicit indication of the relevant spatio-temporal coordinates. The argument that in GLA had been applied to predicates is here applied to singular terms. The idea of seeing the contextual coordinates not as external indices of the sentence, but as an integral part of the sense expressed by the same sentence on the various occasions in which it is used, remains constant in Frege's theory. In this view, a sentence is not true or false at a certain index, but true or false absolutely: the same sentence can be used to express different thoughts, which will be (immutably) true or false according to the different modes of 'completion' inferred from the extralinguistic context. This shows, according to Frege, that what should be labelled as true is the thought and not the sentence. This conclusion is not cogent, however, since with equal (or even greater) reason it could be argued that it is the assertion made using the sentence that is true or false.

In the *Logik* of 1897, while discussing personal pronouns Frege observes that 'I am cold' does not express a complete thought until we know something more about who pronounced this sentence:

... this sentence, uttered by different people, expresses different thoughts. Simple words do not contain the entire sense, but the person who utters them must be taken into account. Thus, in many cases, the spoken language requires the accompaniment of gestures, facial expression, and incidental circumstances. The word 'I' designates precisely different persons in sentences uttered by

⁶¹ NS, p. 174, SP, p. 274.

different people. It is not necessary that the thought that one is cold be uttered by the one who is cold. The thought may therefore have as its clothing a sentence more suitable to show its independence from the thinking subject. It is by virtue of this possibility that the thought differs from a state of mind that can be externalized by an interjection. Words such as 'here' and 'now' are conferred a complete sense always and only by the circumstances in which they are employed.⁶²

Frege here suggests in fact that instead of 'I' one could substitute the proper name or a description of the person who has uttered the words 'I am cold' and thus obtain a sentence more suitable for the expression of thought; he does not say whether for every sentence containing a personal pronoun one can find an equivalent sentence that does not contain demonstrative and deictic expressions, nor does he go into the merits of this alleged equivalence. The general tendency of his argumentation suggests, however, that at that time he thought it possible to completely eliminate the deictic element. At the basis of this proposal is the analogy between personal pronouns and free variables. And, in fact, we can see some analogies between a sentence containing a personal pronoun and a sentence with a free variable, both in the superficial sense that a sentence containing a free variable is not true or false, and in the sense that for an open sentence like ' x is a book' we cannot formulate the truth-conditions in a direct way by saying "' x is a book" is true if and only if x is a book' just as we cannot say that "'I am sick" is true if and only if I am sick': the relations of co-reference and designation are not as transparent as in the case of sentences containing proper names and bound variables.

In the *First Logical Investigation*, *Der Gedanke*, Frege no longer proposes to render the sense of 'I' by substituting the pronoun with a singular term (proper name) indicating the one who speaks, but by means of the description '*the one who is speaking to you at this moment*': this would be the constant and communicable sense of 'I', that is, the conventional meaning of the word. Obviously, this proposal, far from eliminating the deictic element, enriches it by introducing the mention of the listener. Frege also argues that to grasp *the* full sense of the sentence, the listener must also know something about the identity of the speaker. However, this knowledge is in no way contained in the conventional sense of the pronoun 'I' but has to be inferred from the information we can draw from the context. Furthermore, there is also a part of the sense of the word 'I' that remains inaccessible to the listener. The non-communicability we encounter here is not to be confused with that of representations or mental images: it concerns the *thought* expressed by a sentence such as 'I have been wounded' uttered by the person

⁶² NS, p. 146, SP, p. 242.

concerned. Each of us has *thoughts* about himself that are true or false, indeed, atemporally true or false—according to the Fregean conception—but essentially inaccessible to others. To explain this strange situation, one could say, using a piece of Russellian terminology, that the one who uses the word ‘I’ identifies the object that is ‘I’ directly or by *acquaintance*, whereas others can do so only by description, noting some property of which the one who said ‘I’ is the sole bearer. When I say ‘I am happy’ I identify myself as the object of my assertion in a different way from someone who would say ‘Eva Picardi is happy.’⁶³ Presumably, for the person who says ‘I have been wounded’ there is nothing to complete or add: the sentence already expresses for him a complete thought, which only he can grasp in that particular way; instead, the listeners have to ‘complete’ the meaning of the sentence by resorting to information inferable from the context and, of course, they will never be able to complete it in the same way as the person who used the sentence in the first person. On the contrary, each one will complete it in his own way and it may happen that he will not recognize *the same fact* under different descriptions or in sentences with different proper names. Such a situation is described by Frege in the *First Logical Investigation*.⁶⁴

Whatever the right way to approach the philosophical problems of the first person may be, one thing is certain: personal pronouns do *not* work like proper names. To be fair, Frege never says that personal pronouns work semantically in the same way as proper names: however, the general tendency of his thought is to conceive of a sentence containing a personal pronoun as expressing a thought in need of ‘completion’, which once completed expresses a thought that is atemporally true or false: sometimes completion can occur by *linguistically* explicating the relevant information by means of dates, proper names, and geographical indications, while in other cases the ‘completion’ is entrusted to the context. The problem is whether what is obtained by ‘completing’ the sense of sentences containing pronouns and deictic expressions through information deduced from the extralinguistic context is a thought, and, above all, *which* thought it is.⁶⁵

Let us try to outline, with the help of a very simple example, some difficulties inherent in the semantics of demonstrative and deictic expressions. Let’s suppose that we have a piece of paper in our hands with the following written on it:

⁶³ Another more plausible reading is that others understand the content of my assertion by analogy with their personal case. Another person understands the kind of thought I express by saying ‘I have been wounded’ because he presumes that it is the same kind of thought he would express by uttering the first-person sentence.

⁶⁴ Frege, *Der Gedanke*, pp. 54ff.

⁶⁵ For a broader discussion of this theme, cf. J. Perry, ‘Frege on Demonstratives’, *Philosophical Review*, 86 (1977), pp. 474–97, and, by the same author, ‘The Problem of the Essential Indexical’, *Notus*, 13 (1979), pp. 3–21; G. Evans, ‘Understanding Demonstratives’ (1981), reprinted in *Collected Papers*, Oxford, Oxford University Press, 1985; Dummett, *The Interpretation of Frege’s Philosophy*, ch. 6.

(31) The lecture you gave today is more boring and cumbersome than the one he gave here yesterday.

The first idea that comes to mind is that if, instead of personal pronouns and adverbs of time and place, we put proper names of people and places and dates, we would restore the information necessary to understand the content that the person who wrote (31) was trying to communicate to the recipient. For example:

(32) The lecture that Neri gave on 27 April 1988 is more boring and cumbersome than the one that Bianchi gave in Bologna on 26 April 1988.

This description may be misleading. We need to keep two types of information distinct: the perceptual information (visual as well as acoustic) that we could have obtained from being present as witnesses to the episode described in (31), and the semantic information that proper names, geographical places, and dates provide. Obviously, we could have witnessed the situation and heard the words written in the mysterious note pronounced without knowing the names of those present and without knowing what day it was. Consequently, based on the information inferred from the context, we might not be able to formulate sentence (32). The information that can be obtained from an assertion of (31) in salient circumstances does *not* coincide with the semantic information provided by sentence (32).⁶⁶ All we can expect is that (31) and (32) uttered as assertions—provided no misidentification has been made—have the same *content*. However, this content is presented *in different ways*. We can expect the truth-value of (31) and (32) to coincide, but not that the *thought expressed* by (32) coincides with the information imparted by an assertion of (31) under appropriate circumstances. Whether and how much of the overall information inferred from the circumstances in which the words are uttered can be used as *semantic information*, and thus as part of the 'sense' of the sentence, is a question that has no uniform solution. It is hard to see how, for example, perceptual information can act as a *completion* of the linguistic sense expressed by sentences with demonstrative and deictic expressions, and consequently it is very doubtful that the sense expressed by an assertion of (31) in the relevant context is a thought in the Fregean sense. In conclusion, in the case of personal pronouns and deictic expressions, it is essential to distinguish the conventional sense they express from the *information* derived from the circumstances in which they are used. The problems posed by these expressions are not tractable using just the tools afforded by Frege's semantics.

⁶⁶ For a discussion of the different semantic behaviour of proper names and personal pronouns see, for instance, H. N. Castañeda, 'On the Logic of Attributions of Self-Knowledge to Others', *Journal of Philosophy*, 65 (1968), pp. 439–56; R. Chisholm, *The First Person*, Minneapolis, University of Minnesota Press, 1981; D. Lewis, 'Attitudes De Dicto and De Se', *Philosophical Review*, 87 (1979), pp. 513–43.

The Chemistry of Concepts

0. Introduction

While the role played by biological metaphors and models in the writings of the early comparativists has been widely acknowledged, the occurrence of terms reminiscent of chemical theories in the writings of nineteenth-century and contemporary philosophers and logicians has not, for all I know, been the object of much attention. And yet, widespread use has, for example, been made by logicians and linguists of the notion of valency to describe the syntactical behaviour of predicates in sentence formation. This conceptual borrowing should have aroused people's curiosity and led to enquiries into its origins and justification. My impression is that the employment of chemical models and metaphors is by no means confined to the province of logic but extends to linguistics as well. I am not aware, however, of any detailed study comparing, for instance, Saussure's notions of linguistic structure and value with Kekulé's or Butlerov's notions of chemical structure and value.¹ Yet to be sure in the nineteenth century a chemical molecule was regarded as a prime example of a non-mechanical whole in which 'tout se tient'. A very intuitive illustration of what is meant by saying that the value of a linguistic element is a function of the place it occupies in a structure is offered by Butlerov's explanation of isomerism which, roughly speaking, is based on the insight that the four bonds of the atom of carbon, though equipollent, may play different roles in different compounds: the 'value' of each of the four bonds is a function of the position it occupies in a molecule vis-à-vis the other ingredient elements.² Since a chain is as strong as its weakest link, one may predict that a

¹ A relative of Ferdinand de Saussure was the Nicolas-Théodore de Saussure (1767–1845) who wrote the important book *Recherches chimiques sur la végétation*, Paris, 1804 (repr. in the *Ostwalds Klassiker*, 1890). He taught geology and mineralogy at the university of Geneva; in honour of his father Horace-Bénédict de Saussure (1740–99)—also an eminent chemist and geologist—he bestowed the name 'saussurite' on a mineral whose structure he had successfully analysed. In 1875 Ferdinand de Saussure took courses in chemistry and geology at the University of Geneva. For further details see T. De Mauro, *Notizie biografiche e critiche su F. de Saussure*, in F. de Saussure, *Corso di linguistica generale*, 2nd Italian ed. 1972, Bari, Laterza. August Schleicher employed a notation strongly reminiscent of the chemical one in order to represent morphological types in grammar; for a detailed study see C. Vallini, 'Speculazioni e modelli nell'etimologia della grammatica', *Aion* 9 (1987), pp. 18–81.

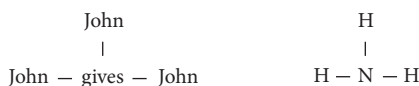
² My examples and information are drawn from the following books: J. R. Partington, *A History of Chemistry*, 4 vols., London, 1964; I. A. Asimov, *A Short History of Chemistry*, Garden City, 1965; G. Bugge, *Das Buch der grossen Chemiker*, 2 vols., Weinheim, Verlag Chemie, 1929, repr. 1984; Irene Strube, Ruediger Stolz, Horst Remane, *Geschichte der Chemie*, 2nd ed., Berlin, 1988; Jurij Solov'ev, *L'evoluzione del pensiero chimico dal '600 ai nostri giorni*. Mondadori, 1976.

chemical transformation is most likely to occur where the bonds of an atom of carbon are weakest: a very neat instance of how a synchronic description can be used as a clue to a diachronic explanation.

These, however, are only asides. In what follows I shall confine myself to the province I know best, i.e. that of the philosophy of language.

Perhaps the most thorough employment of *chemical models* in the logical literature occurs in the work of the German mathematician and philosopher Gottlob Frege (1848–1925), whereas the most extensive use of *chemical similes* occurs in the writings of the American philosopher and logician Charles Sanders Peirce (1838–1919).³ But, as we shall see, both Frege and Peirce were preceded by the German philosopher Arthur Schopenhauer (1788–1860). There is no evidence that Frege and Peirce, though almost contemporaries and with largely overlapping interests, were acquainted with each other's works. This is the more surprising in the case of Peirce, also in view of the fact that he was very well read in German logical literature; what he knew best, however, were the works of authors within the algebraic tradition (i.e. the tradition initiated by Boole, Venn, and De Morgan and continued in Germany by Ernst Schröder), and Frege did not belong to that tradition. That Frege did not take notice of Peirce's work is readily intelligible: in his lifetime Peirce published little on logic, and only two of his papers (of 1892 and 1897) make full use of the analogy between chemical compounds and sentential compounds.

According to Peirce, a complete proposition is like a chemical molecule without free bonds and both can be regarded as 'medads'. Monadic relatives (e.g. to run, to play)—which are called 'non-relative rhemes' in Peirce (1892)—correspond to the chemical elements H, Li, Na, K, Rb etc. belonging to group I of the periodic system; dyadic relatives (e.g. to love, to eat) correspond to the elements B, Al, etc. of group III. The non-monadic relatives are called 'relative rhemes' in Peirce (1892). Thus there is an exact correspondence between the proposition 'John gives John to John' and a molecule of ammonia (N_3H), as Figure 7.1 shows.



³ The relevant papers by Peirce are 'The Critic of Arguments', published in 1892 in the journal *The Open Court*, and 'The Logic of Relatives', published in 1897 in *The Monist*. In the latter paper Peirce mentions a memory of the mathematician A. B. Kempe, 'Theory of Mathematical Form', published in 1886 in the *Philosophical Transactions*. The analogy between chemical diagrams and topological trees occurs for the first time in the writings of the mathematicians Arthur Cayley and J. J. Sylvester, to whose writings Peirce alludes in his papers. Cf. Murray G. Murphy, *The Development of Peirce's Philosophy*, Harvard, 1960, chs. 8–10.

The advantage of this new way of looking at things is obvious: predicates can be viewed as a special case of relations and this in turn undermines the relevance of the distinction between subject and predicate.⁴ As Frege had been urging since 1879, the distinction between subject and predicate ought to be dismissed from logic altogether and replaced by the distinction between function and argument.

It is from Peirce, and not from Frege, that the now widespread terminology of monadic, dyadic, triadic predicates stems; it has gained currency through the influential textbooks of the American logician W. V. Quine.⁵ The conclusions which Peirce himself draws from that analogy do not seem to be very interesting: in Peirce the chemical analogy remains a mere simile which never grows into a model. Peirce himself regards his diagrams as a way of depicting the arrangement of concepts in a sentence. But he concedes that their use for the purpose of deductive calculation is minimal, even though at a later stage he tries to provide algorithmic techniques for extracting from such diagrams⁶ information which could be used for the resolution of algebraic equations.⁷

Unlike Peirce's iconic diagrams or graphs, Frege's concept-script or *Begriffsschrift* was intended to supply a representation of those ingredients of the conceptual content of a sentence which are relevant from the point of view of deductive inference. According to Frege, arithmetical, geometrical, and chemical notations were partial realizations of Leibniz's ideal of a *characteristica universalis*. He saw his own task in the project of inventing a notation suitable for representing the import of a sentence in such a way as to make its conceptual content readily amenable to deductive treatment. To him the applicability of the usual diagrams employed in the algebraic tradition seemed very limited. These diagrams chiefly contributed to fostering the (mistaken) doctrine that a concept is a sum of marks and that new concepts are obtained by means of addition, multiplication, and complementation from concepts already given. But this, Frege argues, is not a way of forming interesting mathematical concepts.

A logical notation is adequate if the logical constitution of concepts is represented by the same logical signs which are used in logical calculation. In this case

⁴ Peirce did not fully appreciate this consequence and preferred to talk instead of a predicate with a 'collective subject'.

⁵ W. V. Quine, *Methods of Logic*, 3rd ed., New York, 1974 (1st ed. 1950).

⁶ This is precisely the reverse of the use to which chemists like Charles Frédéric Gerhardt (1816–56) and August Kekulé (1829–96) thought that a structure-formula should be put; structure-formulae were not meant to represent the actual position of the atoms in a molecule, but rather as a tool for illustrating the kind of bonds which atoms and radicals display within the several families of organic compounds and for getting a perspicuous representation of the relationships (homologies) in which such families stand. Cf. Bugge, *Das Buch der grossen Chemiker*, vol. 2, pp. 209ff.

⁷ On Peirce's entitative and existential graphs see: D. D. Roberts, *The Existential Graphs of Charles Sanders Peirce*, The Hague, 1973; J. J. Zeman, 'Peirce's Graphs: The Continuity Interpretation', *Transactions of the Charles Sanders Peirce Society* 4 (1968), pp. 144–54; P. Thibaud, *La logique de Charles Sanders Peirce: de l'algebre aux graphes*, Aix en Provence, 1975; M. Ferriani, 'Peirce e la logica deduttiva: gli anni giovanili', *Annali di discipline filosofiche dell'Università di Bologna* (1986–7), pp. 5–88.

the uncovering of conceptual content goes hand in hand with our understanding of the deductive transformations to which the content is amenable. Peirce's diagrams are quite unsuitable for this task. Paraphrasing Hertz's dictum we might say that a picture is pointless unless the consequences of the picture are to some extent pictures of the consequences. Peirce's iconic diagrams fall short of this basic requirement. Thus, even though the terminology nowadays used in logic textbooks actually derives from Peirce, the soil in which Peirce's terminological seed has taken roots was not prepared by Peirce himself but by Frege.

Frege appeals to the image of unsaturatedness (*Ungesätttigkeit, Ergänzungsbedürftigkeit*) both to describe a predicate-expression as that part of a sentence which is left over when one or more occurrences of a proper name are dropped and to characterize a peculiar feature of its sense (and reference) which he calls 'Unselbständigkeit'. In Frege's paper *On Concept and Object* (1892) we read:

For not all parts of thought can be complete; at least one must be 'unsaturated' or predicative; otherwise they would not hold together. For example the sense of the phrase 'the number 2' does not hold together with that of the expression 'the concept *prime number*' without a link. We apply such a link in the sentence 'the number 2 falls under the concept *prime number*'; it is contained in the words 'fall under' which need to be completed in two ways . . . only because their sense is thus unsaturated are they capable of serving as a link.⁸

To signal the empty places left by removing proper names from sentences Frege employs small Greek letters, or pairs of brackets containing blanks, or just gaps (as in Peirce).

As is shown by Frege's long and during his lifetime unpublished essay *Booles rechnende Logik und die Begriffsschrift*⁹ and by a letter to Anton Marty,¹⁰ his use of the notion of unsaturatedness goes back to 1880. This notion occurs throughout Frege's work from 1880 to 1919 and is used to fulfil a number of different but interconnected tasks.

In order for the deductive role of unsaturated entities to come to the fore it is essential that the conceptual priority of the molecule over its ingredients, and of a sentence over its component parts, be firmly established. It is no chance that authors like Schopenhauer, Frege, and Wundt, who are particularly fond of

⁸ G. Frege, *Über Begriff und Gegenstand*, 1892. Repr. in I. Angelelli (ed.), *Kleine Schriften*, Darmstadt, 1967, pp. 167–78. Engl. transl., B. McGuinness (ed.), *Collected Papers on Mathematics, Logic and Philosophy*, Oxford, 1984, p. 193.

⁹ G. Frege, *Booles rechnende Logik und die Begriffsschrift*, 1883. In *Nachgelassene Schriften*, ed. by Hans Hermes, Friedrich Kambartel, Friedrich Kaulbach, Hamburg. Engl. transl. by P. Long and R. White, Oxford, 1979.

¹⁰ G. Frege, *Wissenschaftlicher Briefwechsel*, ed. by G. Gabriel, H. Hermes, F. Kambartel, C. Thiel, A. Veraart, Hamburg, 1976, p. 164.

chemical metaphors, emphatically insist on the priority of the whole over its parts. In what follows I shall confine myself to the role played by unsaturated entities in deduction, for this is the context where the use of the chemical model is likely to prove most rewarding. I shall argue (a) that the mechanism of a chemical reaction offers a good model for understanding the cognitive gain obtained by means of deductive inference; and (b) that in deduction Frege's unsaturated entities play a conceptual role similar to that played by the radicals of organic chemistry.

1. The Cognitive Value of Deduction

In the second edition (1844) of *Die Welt als Wille und Vorstellung*¹¹ Arthur Schopenhauer develops in full detail an interesting parallel between a chemical 'transformation' and a deductive 'transformation'. Not only does he offer a fitting analogy to indicate where the cognitive gain of deductive inference is to be sought, but he also locates the fault of the syllogistic presentation of inferential processes in an insufficient understanding of the mode of containment of the concept expressed by the middle term in the major and minor premises.

The ancient issue concerning the value of syllogism can be put roughly as follows: it is a necessary condition of the validity of a deductive argument that the conclusion should not go beyond the premises (i.e. the conclusion must not be ampliative); yet, in order to be fruitful or worth our while, a deductive argument must tell us something which we did not know in advance. Thus it looks as if a deductive argument could be useful only at the cost of being invalid. According to Schopenhauer the cognitive gain of a deductive inference consists in this: that in a sense we come to know what we knew already, that we arrive at explicit knowledge of what used to be latent or implicit knowledge. The task of deduction is to set free 'latent' or 'bound' knowledge.

He who has salt [sodium chlorate] also has chlorine, but it is as if he did not have it, for only when chemically freed can it act as chlorine, that is, only then has he really got it. It is the same with the gain delivered by a mere inference from known premises: knowledge which used to be bound or latent is thereby set free. Now, these comparisons might look somewhat exaggerated but probably they are not... thus it seems that no premises of possible inferences will remain unused for any length of time but that for all premises within our ken we have got the conclusions ready at hand. But this is not always the case: on the contrary, two

¹¹ The third edition was published in 1858 and contains new material for a total of 136 pages; no addition, however, was made to chs. 9 and 10 of the First Book.

premises may for a long time dwell in the same head but in isolation from each other until at last they are prompted to meet, so that a conclusion comes forth.¹²

Schopenhauer is here thinking of uncomplicated cases in which one simply fails to draw a conclusion. Thus we may, although aware that all diamonds are stones and that all diamonds are inflammable, fail to realize that some stones are inflammable. The value of the comparison does not reside, however, in the psychological considerations mentioned by Schopenhauer but in his drawing attention to the parallel between the chemical mode of containment of elements in a chemical compound and the logical mode of containment of a conclusion in its premises.

Frege, in describing the cognitive significance of arithmetical propositions, says that the latter are enshrined in the logical axioms like plants in their seeds and not like bricks in a house. This he says in his *Grundlagen der Arithmetik* of 1884, and the same simile recurs in his later work, *Die Grundgesetze der Arithmetik* (vol. II, sec. 147). In his early *Begriffsschrift* he had compared the 'surprise' ensuing for example from the discovery that the proposition expressing the law of trichotomy can be deduced from purely logical axioms to the surprise of a child who discovers that air is not 'nothing' when noticing that if subjected to pressure it condenses to little drops of water. The elements of which water is constituted must be among the elements of which air is constituted: in order to understand this kind of containment we must try to grasp the process of qualitative transformation which takes place in a chemical reaction. The gist of Frege's comparison is that in order to explain the informative character of arithmetical propositions we need not appeal to any peculiar intuition of time, as Kant had urged: what we need is a method of unfolding the conceptual potential implicit in the logical notions which are already within our ken.¹³

Now the question arises, which means are there at the logician's disposal of uncovering the constitution of concepts which occur in sentences featuring as premises or conclusions of deductive arguments?

In the second edition of his main work Schopenhauer devotes two entirely new chapters (chs. 9 and 10) to what may be called a chemical account of the syllogism. Perhaps unsurprisingly ch. 10 closes with a comparison between the syllogism and a voltaic pile. I shall here have to skip many interesting details and confine myself to the bare outline of Schopenhauer's argument. He suggests that the syllogism should be viewed as a machine for comparing concepts. In a syllogism of the third figure, what is to be investigated is the relation between two concepts in predicate position, and the concepts which are to be compared are like two substances up

¹² *Die Welt als Wille und Vorstellung*, 1844, vol. II, Frankfurt, 1960, p. 142.

¹³ G. Frege, *Begriffsschrift: Eine der arithmetischen nachgebildeten Formelsprache des reinen Denkens*, Halle, 1879. Repr. in *Begriffsschrift und andere Aufsätze*, ed. by I. Angelelli, Hildesheim, 1971.

for chemical testing. The function of the medius is akin to that of a chemical reagent: from the way the two concepts react to it information can be gathered about how they will behave in connection with one another. Accordingly, the ingenuity of the logician will consist in finding substances appropriate to build a bridge between premises and conclusion. And in so doing, we may add, it is possible that he will learn if they could be obtained from one another by means of a series of transformations; and this might then lead to investigations as to whether they have a common ancestor or are formed in conformity with a common matrix.

Schopenhauer's suggestion is open to the objection that it locates the value of the syllogism wholly outside it, i.e. in the discovery of the relevant middle terms. However, the point Schopenhauer tries to make in the following passage is not trivial. He there says that syllogistic logic makes a mistake similar to that made in organic chemistry

when in the analysis of plants these were immediately dissolved into their *ultimate* constituents, and here it was found that all plants contain carbon, hydrogen and oxygen; the specific differences however were lost, and in order to get hold of those we should stop at their proximate constituents, the so-called alkaloids, and refrain from decomposing them any further.¹⁴

The logician, like the chemist, is confronted with 'wholes': compound substances and sentences (or judgements). The elements which occur in the conclusion are contained in the premises as the elements of a chemical compound are contained in the original substances. Now very few substances occur as isolated elements, most of them occur in compounds: therefore we have no choice but try to uncover the constituent elements, starting from the compounds (or whole judgements)¹⁵ and subjecting them to a number of tests which may yield information as to their internal constitution.

¹⁴ According to Kant our understanding of organized bodies is guided by the analogical use of the notion of *Zweckmässigkeit* or purposiveness; he denies that mechanical laws suffice for explaining the behaviour of *organized* bodies. Schopenhauer goes even further and denies that this holds true only of organized bodies: mechanical explanations are, in his opinion, insufficient for explaining the phenomena of 'Chemismus', electricity, magnetism, and crystallization. (Cf. the Appendix to vol. I. of *Die Welt als Wille und Vorstellung*, pp. 714ff.)

¹⁵ When coming across a self-contained utterance, which, as far we can make out, consists of just one word, we have no alternative but to regard it 'holophrastically' as the expression of an entire sentence. It is however unwarranted to conclude that there is no such thing as the original constituents of the sentence, as some linguists have argued in the case of linguistic roots. In a footnote to *Booles rechnende Logik* Frege draws attention to the notion of 'sentence-word' employed by the linguist A. H. Sayce in his *Introduction to the Science of Language* of 1880, reviewed by A. Fick in the *Göttingische gelehrte Anzeigen*, 6 (April 1881). Sluga (in his book *Gottlob Frege*, London, 1980) and Dummett (in his book *The Interpretation of Frege's Philosophy*, Harvard, 1981) read this passage differently: Sluga thinks that Sayce is quoted approvingly by Frege, while Dummett thinks that the opposite is true. Dummett's reading seems to me the correct one.

If we are to appreciate Schopenhauer's point it will be helpful to make a short detour into organic chemistry. The problem confronting the organic chemist was that of singling out a certain number of 'Gruppenmerkmale' useful for grouping substances into families. In order to discover the relevant characteristic marks it was necessary to study the chemical compounds under many different chemical conditions and keep track of their transformations under these conditions. It was soon found that certain substances provoke very specific modifications in organic compounds resulting in typical new compounds. For instance, organic compounds treated with chlorine, bromine, or iodine would give rise to halogenous compounds. The term 'alkaloid' used by Schopenhauer was introduced in 1818 by the chemist Karl Friedrich Wilhelm Meissner (1792–1855) for grouping substances like morphine, caffeine, etc., whose main characteristic mark is their basic character. The chemical structure of alkaloids, however, was only understood many years later.

Now, according to Schopenhauer, the syllogism aims at reducing judgements to their ultimate components (concepts), regardless of their quantity, quality, and modality, and in so doing it loses sight of the 'Bindungsmittel' of concepts,¹⁶ that is, it loses sight of the *intermediate level of organization*, which lies between the compounds and their ultimate elements.

To push analysis until it reaches ultimate elements does not belong to the main aim of the chemist, for knowledge of the properties of each individual element will yield only rough indications of the way they will react together and of the chemical properties of their possible compounds. Indeed, the interest of a chemical reaction resides precisely in this, that, as Mill put it, the law of the composition of causes does not directly apply to it. This point is put very well in his *System of Logic, Ratiocinative and Inductive*:

The different actions of a chemical compound will never, undoubtedly, be found to be the sums of the actions of its separate elements; but there may exist, between the properties of the compound and those of its elements, some constant relation, which, if discoverable by a sufficient induction, will enable us to foresee the sort of compound which will result from a new combination before we have actually tried it, and to judge of what sort of elements some new substance is compounded before we have analysed it... in chemistry, these undiscovered laws of the dependence of the properties of the compound on the properties of its elements, may, together with the laws of the elements themselves, furnish the

¹⁶ The syllogism, as is well known, contains three predicates, variously arranged in four figures, for a total of 256 possible syllogisms. It turns out that there are only twenty-four valid argument patterns. In the conclusion the order of the terms is fixed, i.e. the minor term occurs in subject-position and the major in predicate-position; the medius does not appear in the conclusion.

premises by which the science is perhaps destined one day to be rendered deductive.^{17, 18}

The chemical combination of two substances produces a third substance with properties different from either of the two substances taken separately. Moreover, the reaction of electrolytic separation discovered by Faraday showed that the ions thereby obtained have chemical properties different from those of the original reagents. Thus from the mere knowledge of the physical properties of the single elements (atoms) we are, according to Mill, not in a position deductively to infer the chemical properties of the resulting compounds or those of the intermediate 'components' which may develop between the recognized stages of chemical transformations or reactions.

The logician, like the chemist, is confronted with wholes; the chemist with substances, the logician with sentences. He has no alternative but to work back from the wholes to their parts. Here the thought irresistibly presents itself that that which analysis uncovers are the building blocks out of which molecules are constructed. Ideally, analysis and synthesis are two sides of the same coin.

This picture is quite adequate for the simple binary molecules of inorganic compounds. Berzelius strove to cast the ternary molecules of inorganic chemistry and also the large molecules of organic compounds into the mould of binary composition. In taking this step, however, the chemist should be aware of the difference between discerning a pattern in a family of molecules and discerning the actual constituents out of which an individual molecule is built. Thus, for example, the individuation of the hydroxyl group is very useful for bringing ternary formulas into binary form: OH behaves, as a matter of fact, like a single atom and also has specific properties of its own. Yet, its status is problematic: it is a theoretical construct to a higher degree than the atoms themselves. Besides, this way of considering 'radicals', though very useful for bringing order into large molecules and into families of molecules of organic compounds, is, in the long run, an obstacle to a correct understanding of the atom of carbon itself as occurring outside the context of, say, CH, CN, and other instructive radicals. The objection that he had come to regard radicals as having real independent existence was levelled against Berzelius by the new generation of chemists who laid the foundations of organic chemistry.

These considerations bring to the fore an ambiguity in the notion of an *element* which, in chemistry, was successfully dispelled; the same cannot be said of the cognate logical notions. The most intuitive characterization of an element is also

¹⁷ John Stuart Mill, *System of Logic Ratiocinative and Inductive*, 8th ed., London, Book IV, ch. VI, §2: 245, 246 (1st ed. 1843).

¹⁸ This is why, according to Mill, mechanics is a deductive science while chemistry is not; the law of composition of causes is not applicable to biology and chemistry, for these are governed by what he calls 'heteropathic' laws.

historically the first (cf. Boyle, *The Sceptical Chemist*, 1661); this is a functional notion: a simple element is a substance which cannot be decomposed any further by the methods available.¹⁹ This characterization must be sharply distinguished from the following one: an element is a substance which need not be decomposed any further for the purpose at hand although such a decomposition would be possible. As is well known, in chemistry the functional characterization was superseded, at the end of the nineteenth century, by an absolute characterization of an element: around 1870 chemists and physicists arrived at a finite, albeit incomplete, list of the ultimate elements out of which all organic and inorganic stuff is made. The functional characterization was replaced by a structural characterization: an element is a substance constituted by atoms having the same unique and characteristic atomic number. No such list of primitive concepts is available for a natural language, though both logicians and linguists start from the assumption that in order to be learnable a language must have only a finite number of primitives and only a finite number of syntactic devices of sentence formation. Even if there were such a finite list, an attempt to represent its items in the form of a periodic table would, in my opinion, not merely be futile but intrinsically incoherent. In the limited province of logic, however, the quest for a small number of primitive notions is not only legitimate, but, in circumscribed areas, has actually turned out to be quite successful.²⁰

The discovery of a finite list of elements reinforced the need for seeking, at a more sophisticated level, an explanation of the chemical behaviour of organic compounds. This level is the structural one connected with the spatial arrangement of atoms in a molecule.

2. Logical Form

The interesting aspect of Schopenhauer's simile resides in the insight that in order to fill the gap between premises and conclusion we have to advert to an intermediate level of form or organization (similar to the 'alkaloids' of the vegetable kingdom) and try to arrive at a pattern which may be *visible* neither in the

¹⁹ Cf. Frege's *Über Begriff und Gegenstand* (*On Concept and Object*): 'Man kann auch nicht verlangen, dass alles definiert werde, wie man auch vom Chemiker nicht verlangen kann, dass er alle Stoffe zerlege. Was einfach ist, kann nicht zerlegt werden... [Voraussetzung ist in den beiden Fällen die logische Zusammengesetztheit.] Das Logischeinfache ist nun ebensowenig wie die meisten chemischen Elemente von vornherein gegeben, sondern wird erst durch wissenschaftliche Arbeit gewonnen. Wenn nun etwas gefunden ist, was einfach ist oder wenigstens bis auf Weiteres als einfach gelten muss, so wird eine Benennung dafür zu prägen sein, da die Sprache einen genau entsprechenden Ausdruck ursprünglich nicht haben wird.' The sentence in square brackets occurs in Frege's draft of the paper, printed in *Nachgelassene Schriften*, pp. 97–8.

²⁰ The comparison between the ideal of a universal language in chemistry and in the logical foundations occurs in an (unpublished) paper by G. Kreisel, 'A Parallel between Wittgenstein's Ways and Works' (1985).

conclusion nor in the premises but needs to be postulated if we are to explain both the *mechanism* of a deduction and its *cognitive significance*. Plainly, we cannot hope to appreciate the latter unless we have understood the former. In short, what is needed is a new conception of logical form, and that is precisely the notion which Frege undertakes to provide.

What Schopenhauer had overlooked is the fact that inferential patterns themselves could play the same role which heat, water, or electricity play in chemical reactions: if compounds react in very different ways under similar circumstances, this may be taken as evidence for thinking that they differ in their ultimate elements or in the way these elements are arranged in the molecules of which the two compounds are made, or that they differ in both respects. Thus a moment's reflection will make us see that the sentences 'There are five apples' and 'There are ripe apples', though grammatically similar, have different internal constitutions: 'five' does not distribute over each apple, whereas 'ripe' does. The paraphrase 'There is something which is an apple and ripe', is alright, while 'There is something which is an apple and five' is nonsense. The concepts 'being five' and 'being an apple' do not belong to the same level, for if they did they would react in similar ways when subjected to the same inferential tests. As Frege put it, a concept like 'being five' cannot be saturated by objects but only by concepts. Therefore to the expressions 'ripe' and 'five' there correspond different classes of substituends: the word 'five' may be substituted by expressions like 'many', 'some', 'a few', while the word 'ripe' may be replaced by words like 'red', 'tasty', 'expensive', and so forth. The grammatical criterion which provides us with a substitution class *salva congruitate* does not suffice for yielding a substitution class *salva logica congruitate*. On the other hand, substitution of a relational expression for the word 'ripe' would be illegitimate, for it would lead to results that are obviously ungrammatical, such as 'there are larger than apples', 'there are heavier than apples', etc. Only concepts with the same valency can be substituted for one another. Thus the 'valency' of a concept-word is characterized both by the number of its argument-places and by the logical type of its appropriate substituends.

In order to explain the syllogistic mode of inference the equivalents of the empirical formulae of inorganic chemistry will do. But when it comes to dealing with trickier modes of containment syllogistic logic is plainly insufficient, as is shown by the incapacity of the traditional theory to provide an acceptable treatment of such entailments as 'All circles are figures, therefore all who draw circles draw figures', or 'All horses are animals, therefore all horses' heads are animals' heads', or 'If anyone killed Brutus, he was an honourable man; Brutus killed Brutus, ergo Brutus was an honourable man.'

It is here that we can finally cash in on the advantages of Frege's doctrine of concepts as unsaturated entities. Intimately related to the notion of unsaturatedness is the insight that one and the same sentence can be decomposed in many different ways. And what such different ways of decomposition can serve to bring out are

different logical patterns which we are required to discern in the relevant premises and conclusion if we are to account for the mechanism of deductive inference.²¹

This point is clearly stated in the following passage from Frege's paper on *Booles rechnende Logik* which, because of its relevance to the point I want to make, I shall quote in full:

We may now express ' $2^4 = 16$ ' by the sentence '2 is a fourth root of 16'... But we may also just as well say '4 is a logarithm of 16 to the base 2'. Here the 4 is being treated as replaceable and so we get the concept 'logarithm of 16 to the base 2': ' $2^x = 16$ '. The x indicates here the place to be occupied by the sign for the individual falling under the concept. We may now regard the 16 in ' $x^4 = 16$ ' as replaceable in its turn, which may be represented, say, by ' $x^4 = y$ '. In this way we arrive at the concept of a relation. And so instead of putting a judgement together out of an individual as a subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgement. Of course, if the expression of the content of possible judgement is to be analysable in this way, it must already be itself articulated. We may infer from this that at least the properties and relations which are not further analysable have their own simple designations.... Hence in the concept script their designations never occur on their own, but always in combinations which express contents of possible judgement. I could compare this with the behaviour of the atom: we suppose an atom never to be found on its own but be combined with others, moving out of one combination only in order to enter immediately into another.²²

It is strictly speaking only radicals or ions which can *never* occur on their own; the atoms of *some* elements are in nature never to be found on their own (e.g. fluorine) while the atoms of certain other elements (e.g. the inert gases like argon, krypton, neon, etc.) are in nature never to be found in compounds. Thus the more likely candidates to play the role assigned to the 'atoms'—that is, to the concepts extracted from sentences—are either the radicals of organic chemistry or the ions which are freed in an electrolytic reaction (for instance the hydroxile ion OH^- mentioned above). For reasons at which I can only hint here I am inclined to think that in order to understand the role of unsaturated entities an interpretation in terms of radicals is more helpful than an interpretation in terms of ions, for the opposition of plus and minus, which is essential for the notion of an ion, has no role to play in logic.

²¹ For a fuller discussion of this issue cf. Dummett, *The Interpretation of Frege's Philosophy*, ch. 15 ('Alternative Analyses').

²² Frege, *Booles rechnende Logik*, in *Nachgelassene Schriften*, Engl. transl. pp. 16–17.

In a later paper Frege himself warns us not to assimilate affirming and denying to redox processes (that is, processes of reduction and oxidation)²³ for the opposition of plus and minus has no role to play in accounting for the unity of a proposition or thought.²⁴ The model which uses the notion of unsaturatedness brings to the fore the categorial difference between objects and concepts, whereas the electric model suggests a picture of two self-contained entities of the same kind waiting to be joined.²⁵

Let us use some very simple examples to illustrate the kind of job such ‘concept-radicals’ are supposed to perform:

[1]	[2]
Jill loves herself	Jill does not love herself
Jill loves someone who loves her	Someone is not loved by everybody

The question which arises with respect to such simple arguments does not concern the possibility of convincing oneself of their soundness but the possibility of grasping that which accounts for their soundness. And in order to grasp this it will not be enough to see that the concept of love is somehow involved in the premise as well as in the conclusion. What we have to understand is how simple sentences like ‘Jill loves Jill’ or ‘Jill does not love Jill’ are related to the relevant quantified sentences. Frege’s insight consisted in this: that quantified sentences should be conceived as gradually constructed out of simple sentences, which could then be viewed as their generating instances. The process which makes this type of construction possible is one of removing one or more occurrence of the same proper name at a time and binding the resulting ‘free radical’ by means of a quantifier. Let us consider example [1] and some simple variations of it:

²³ Cf. Frege, *Die Verneinung* (1919), repr. in *Kleine Schriften*: ‘So erscheinen Urteilen und Verneinen als ein Paar entgegengesetzte Pole, die eben als Paar gleichen Ranges sind, vergleichbar etwa mit dem Oxydieren und Reduzieren der Chemie. Wenn man aber eingesehen hat, dass durch das Urteilen kein Zusammenhang gestiftet wird, sondern dass die Ordnung der Teile des Gedankens schon vor dem Urteile bestanden hat, erscheint alles in andern Licht.’ Cf. also W. Wundt, *Völkerpsychologie*, vol. II, 1897, p. 448 (3rd ed., Leipzig, 1912): ‘Daraus geht hervor, dass der Ausdruck, der Satz sei eine “Verbindung von Vorstellungen” ebenso wie die andere, er sei “eine Verbindung von Wörtern”, psychologisch unhaltbar sind. Vielmehr ist er die Zerlegung eines im Bewusstsein vorhandenen Ganzen in seine Teile.’ For a discussion of Frege’s reading of Wundt see my ‘The Logics of Frege’s Contemporaries or “der verderbliche Einbruch der Psychologie in die Logik”’, in D. Buzzetti, M. Feriani (eds.), *Speculative Grammar: Universal Grammar and Philosophical Analysis of Language*, Amsterdam, 1971, pp. 173–204.

²⁴ The notion of polarity which was to play such an important role in the genesis of Wittgenstein’s *Tractatus logico-philosophicus* applies primarily to truth and falsity and not to assertion and denial.

²⁵ Incidentally, Peirce, unlike Frege, makes a lot of electric polarity; the parallels which he draws in this context, however, strike me as at best far-fetched.

[1]

Laa

Laa & Laa

Laa

Laa

Laa

*Lxa & Lax**Lax**Lxa**Lxa*

(Ex) (Lxa & Lax)

(Ex) (Lax)

(Ex) (Lxa)

(Ex) (Lxa)

(Ex) (Lxy)

(Ey) (Ex) (Lxy)

The formulae written in italics indicate unsaturated concepts which do not appear, neither in the conclusion nor in the premises: they are part of a transformational history of the sentence which we tell ourselves to account for the transition from complete (or saturated) sentences to different complete (or saturated) sentences. Let us have a look at another set of simple sentences:

Lab

Lac

Lab

Lac

Gb

Gc

Fa

Ga

*(X)b**(X)c**F(x)**G(x)*

A moment's reflection will show us that both patterns of logical form which we have imposed on 'Lab' and 'Lac' are quite legitimate: it all depends on where we want to arrive and which inferences we want to account for, that is, it depends on whether what we want to infer is that *b* and *c* enjoy the same property (viz. being loved by Jill) or whether we want to be able to say that *a* has two different properties: viz. being a lover of *c* and being a lover of *b*. The next example illustrates also the inverse process, i.e. that of getting from quantified sentences to quantifier-free sentences via universal instantiation:

[2]

 $\neg \text{Laa} \rightarrow (\text{Ey}) \neg (x)(\text{Lxy})$ 1. $\neg \text{Laa}$ 2. $(x) (\text{Lxa})$ 3 *Laa*4. $\text{Laa} \ \& \ \neg \text{Laa}$ 5. $\neg (x)(\text{Lxa})$ 6. $\neg (x)(\text{Lxy})$ 7. $(\text{Ey}) \neg (x) (\text{Lxy})$ 8. $\neg \text{Laa} \rightarrow (\text{Ey}) \neg (x)(\text{Lxy})$

Obviously in the schemes sketched above also the rules governing the behaviour of the expressions of generality (the two quantifiers) play a role. They appear, as it were, twice: once in the construction of radicals (concepts) and another time in the 'calculation'. And this is precisely as it should be, for, as Frege remarks in *Booles rechnende Logik* (p. 35):

It is true that the syllogism can be cast in the form of a computation . . . Still the fact that it follows a few fixed and perspicuous forms gives it a high degree of certainty. But we only derive any real benefit from doing this, if the content is not just indicated but is constructed out of its constituents by means of the same logical signs as are used in the computation. In that case, the computation must quickly bring to light any flaw in the concept formations.

Similarly, in order to appreciate the complex structure of the large molecules of organic compounds and group them into families, the chemists had to get rid of Berzelius's dogma that all chemical compounds are, at bottom, binary compounds, i.e. result from the union of two atoms or groups of atoms (that is, radicals) which are held together by electrical affinity (or polarity). The simple picture of electric affinity postulated by Berzelius proved an obstacle in the way of accounting for the complex architecture of organic molecules. One of the crucial notions which were to undergo change and refinement between 1825 and 1870 was, accordingly, the notion of a radical. Laurent in 1836 first urged that radicals should not be viewed, as Berzelius had done, as groups of atoms which could be shifted from one molecule to another one and which in effect behave like atoms. Thus Auguste Laurent (1807–53), elaborating insights of Jean Baptiste André Dumas (1800–84), replaced the notion of a radical with that of a nucleus (or type) out of which different families or organic molecules were formed by means of the substitution of the atoms of hydrogen.²⁶ Laurent's theory of 'chemical types' or 'chemical kernels' was not without faults; indeed, within a few years it was supplemented by the theory of 'rests' or 'residues' elaborated by Charles Frédéric Gerhardt (1816–56) and Laurent himself, but surely it meant progress. After all, it prepared the background of Kekulé's and Butlerov's conceptions of chemical structure²⁷ and valency and made an indirect contribution to the development of that important branch of chemistry called 'stereochemistry' (in 1888 by Victor Meyer (1848–97)).

3. Conclusion

Always when one comes across extensive terminological loans one is led to wonder how relevant, revealing, or instructive such phenomena may be. Often such borrowings occur in the infancy of a new discipline; subsequently they either disappear or linger as pedagogical devices to help the freshman to form a rough picture of the problems at stake. Probably this applies to the biological models used by the early comparativists—a subject on which I have said something

²⁶ For a discussion of the theory of chemical types see Partington, J. R., *A History of Chemistry*, ch. 14; and also M. Bloch, in Bugge, G. (ed.), *Das Buch der grossen Chemiker*, in particular pp. 102–12. The title of Gerhardt's main work is *Introduction à l'étude de la chimie par le système unitaire* (1848).

²⁷ A. Kekulé, 'Über die sogenannten gepaarten Verbindungen und die Theorie der mehratomigen Radikalen', *Ann. Chemie und Pharm.* (1857), p. 104.

elsewhere.²⁸ The chemical model, on the other hand, seems to me to be still at work in contemporary philosophy of logic and language. In consequence it is much more difficult to view it impartially, from the outside, as it were. As a matter of fact, I do not think that for the time being there is a better model available which could compete with the chemical paradigm.

²⁸ Cf. E. Picardi, 'Some Problems of Classification in Linguistics and Biology: 1800–1830', *Historiographia Linguistica* 4, 1 (1977), pp. 31–57. For criticisms and suggestions I have to thank M. Capozzi, R. Casati, and C. Thiel.

Assertion and Assertion Sign

In spite of the fact that a number of semantic notions currently used in modal logic go back to the work of Gottlob Frege as it was interpreted by Carnap in 1947,¹ Frege's rare remarks on the subject of modality show that he considered modal distinctions of little relevance to logic. And this may strike one as rather odd: for was it not one of Frege's aims to show that arithmetical propositions are analytic, if they are derivable as theorems in a sufficiently strong logic, on the basis of impeccable definitions and purely logical axioms? Moreover—the objector may continue—analytical propositions are knowable a priori (actually, necessarily so) and whatever is knowable a priori is a necessary truth (though, perhaps, not vice versa). Since in *Die Grundlagen der Arithmetik*² Frege did appeal to some of these notions, the question arises how we are to interpret his statements on the subject of modality. This may at first seem a rather roundabout way of approaching the subject of assertion and assertion sign, and perhaps it is. But if we are to appreciate the profoundly innovative character of Frege's doctrine of truth and assertion it may prove a good strategy to touch briefly on the way in which Frege addressed the subject of analyticity and modality. Accordingly, after my having advanced some tentative suggestions as to the grounds of Frege's disparaging remarks about modality, I shall concentrate on the issue of Frege's recasting certain traditional distinctions in the theory of judgement, epitomized as it were in the introduction of a new sign into his logical notation—a truly 'momentous event' (to borrow a phrase of Wittgenstein's (TLP 5.452)), who, however, emphatically denied that this was a case in point (TLP 4.442).³ I shall also mention a number of reinterpretations of Frege's judgement stroke proposed by later authors, so that the peculiarity of Frege's symbolic notation can be better appreciated.

In §4 of his *Begriffsschrift*⁴ Frege briefly mentions certain distinctions customarily drawn in logic books concerning the quantity, quality, relation, and modality

¹ R. Carnap, *Meaning and Necessity*, Chicago: The University of Chicago Press, 1947 (2nd ed. 1956).

² G. Frege, *Die Grundlagen der Arithmetik: Eine logisch mathematische Untersuchung über den Begriff der Zahl*, 1884, Breslau: W. Koebner, 1884, *Centenarerausgabe*, ed. C. Thiel, Hamburg: F. Meiner, 1986, Stuttgart: Reclam (= GLA). Engl. transl. by J. L. Austin, 1950, 2nd rev. ed. Oxford: Blackwell, 1953.

³ L. Wittgenstein, *Logisch-philosophische Abhandlung* (1921) = *Tractatus Logico-Philosophicus* (1922). Engl. transl. by D. Pears and B. McGuinness, London: Routledge & Kegan Paul, 1961 (= TLP).

⁴ G. Frege, *Begriffsschrift: Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*, Halle: L. Nebert, 1879, Hildesheim: Olms, 1964 (= BS).

of judgements and then comes to the conclusion that, while the first two distinctions really concern the contents of judgements, the distinction between categoric, hypothetic, and disjunctive judgements has but grammatical significance. He then goes on to say that the modal distinction between apodictic and assertoric judgements is based on this difference: that apodictic judgements suggest that there are grounds on which a speaker's judgement rests, that he is aware of general judgements from which a given proposition may be inferred, whereas in the case of assertoric judgements there is no such suggestion (which, however, does not mean that the speaker is not aware of any judgements of that kind). Correspondingly, when a proposition is advanced as possible, this may mean one of two things: either the speaker knows of no laws from which the negation of that proposition follows, as for example in the case of 'It is possible that the earth will sometime collide with some heavenly body' (which, again, does not mean that there are no such laws);⁵ or else it is tantamount to a particular affirmative judgement such as 'A cold can result in death.'

As these examples suggest, Frege is here thinking of the use of modal words as a way of disclosing (in the case of apodictic judgements) or of qualifying (in the case of ascriptions of possibility) the speaker's entitlement to advancing his claims, relative to his present motives, his state of information, or his knowledge.

The only other place I know where Frege says something which has a bearing on the topic of modality is his (posthumously published) *Introduction to Logic* of 1906. In defending his construal of the conditional as material conditional he there says:

People probably feel the lack of an inner connection between the thoughts; they find it hard to accept that it is only the truth or falsity of the thoughts that is to be taken into account, that their content does not really come into it all. This is connected with what I realized about Sense and Reference. Well, just let someone try to give an account in which the thought itself plays a bigger role and it will probably turn out that what has been added from the thought is at bottom quite superfluous, and that one has only succeeded in complicating the issue.⁶

One may speculate as to which authors, if any, Frege is here alluding to. The only philosopher who comes to my mind is Edmund Husserl, who in 1891 had

⁵ This sense of 'possible' is, if I am not mistaken, one which Bolzano, for instance, considered a spurious or derivative sense of possibility. In his *Wissenschaftslehre* II, *Elementarlehre*, §182, he uses the expression 'möglich scheinend' and calls this a third sense of 'possibility' which is said to apply sometimes also to Begriffswahrheiten.

⁶ G. Frege, *Nachgelassene Schriften*, H. Hermes, F. Kambartel, F. Kaulbach (eds.), Hamburg: F. Meiner, 1969 (2nd. ed. 1983), pp. 202f. (= NS); Engl. transl., *Posthumous Writings*, Oxford: Blackwell, 1979, p. 187 (= PW).

published a paper entitled ‘Der Folgerungscalcul und die Inhaltslogik’.⁷ In this article he outlined an intensional reinterpretation of Schröder’s calculus of classes in order to let the thought-content ‘play a bigger role’ (to use Frege’s words). It is quite plausible to think of Husserl in this context, if one recalls that in the controversy between the ‘Logiker des Inhalts’ and the ‘Logiker des Umfangs’ Frege had sided with the latter because in his opinion they came closer to the insight that what matters in logic are truth-values, extensions, and that the specific sense of a sentence is irrelevant to understanding the logical role it plays in a deduction.

However, apart from the piece from the Nachlaß just mentioned, §4 of Frege’s BS contains his only explicit statement on the subject of modality. Let us go back to that passage and scrutinize it more closely.

The terminology there employed by Frege is strongly reminiscent of Kant’s table of the forms of judgement. While the distinctions between the affirmative and the negative and between the universal and the particular are recast by Frege as distinctions pertaining to the content of a judgement, the distinctions concerning relation are discarded altogether for the reason that by means of negation and material conditional all the remaining types of compound thought can be expressed. As regards modality, Frege repeats almost verbatim Kant’s statement that ‘the distinctive trait of modal words is that they contribute nothing to the content of a judgement but only concern the value (Wert) of the copula in relation to thinking’ (KrV, B99/100; A74/75). In Kant’s *Logik*—a work which Frege quotes in GLA—similar considerations are put forward, but this time no special role of the copula is mentioned. It is clear enough that Kant’s understanding of modal distinctions is—at least in the writings just mentioned—epistemic in the strict sense that he conceives of them as correlative to the speaker’s psychological attitudes of doubt, belief, and knowledge. For this very reason Frege, while endorsing Kant’s position in broad outline, considered the whole issue logically irrelevant and, because of its psychologicistic overtones, possibly misleading.

I think that Frege’s unmitigated antipsychologism may have played a role in his distrust of modal notions. Lest we succumb to psychologism, the notion of subjective entitlement alluded to above must be sharply distinguished from that of objective justification, the latter being crucial for Frege’s whole foundational enterprise. It was imperative therefore that these two notions should have been kept apart.⁸

In a well-known passage of GLA, where Frege declares that the Kantian distinctions between the a priori and the a posteriori, between the synthetic and

⁷ Cf. E. Husserl, ‘Der Folgerungscalcul und die Inhaltslogik’, *Vierteljahrsschrift für wissenschaftliche Philosophie* (1891), pp. 169–356.

⁸ See my ‘The Logics of Frege’s Contemporaries, or “der verderbliche Einbruch der Psychologie in die Logik”’, in D. Buzzetti, M. Ferriani (eds.), *Speculative Grammar, Universal Grammar and Philosophical Analysis of Language*, Amsterdam: Benjamins, 1987, pp. 173–204.

the analytic, do not concern the content of a judgement but our justification for making it, he qualifies his statement thus:

When a proposition is called a posteriori or analytic in my sense, this is not a judgement about the conditions, psychological, physiological and physical, which have made it possible to form the content of the proposition in our consciousness; nor is it a judgement about the way in which some other man has come, perhaps erroneously, to believe it true; rather, it is a judgement about the ultimate ground upon which rests the justification for holding it to be true. (GLA, §3)

Now, if a proof of the analytical (i.e. purely conceptual) nature of arithmetical propositions could be secured, Kant's appeal to an a priori intuition to justify their necessity could be dropped as redundant. Frege shares Kant's opinion that an appeal to intuition is indispensable in the case of geometrical propositions, on the ground that the denial of geometrical axioms (unlike that of analytical propositions) does not lead to inconsistency (GLA, §14). Whereas the acceptance of specific geometrical axioms is forced on us by the peculiarity of our pure intuition of space, no appeal to intuition is required in the case of arithmetical propositions. Having discarded Kant's account of necessity,⁹ Frege sets himself two formidable tasks: (a) to defend the thesis that pure conceptual thought, issuing in analytical propositions, can extend our knowledge, and (b) to show that logic, far from being barren, can beget peculiar 'objects', i.e. numbers. Having rejected Kant's thesis that without sensibility no objects are given to us, he squarely faces the question: 'How then are numbers given to us, if we cannot have any idea or intuition of them?' (GLA, §62). The surprising, indeed revolutionary, answer he gives to this question is that we should rather concentrate our attention on the assertions in the context of which number-words occur and investigate the grounds on which the justification of their truth rests. This way of recasting epistemological issues in linguistic

⁹ Contrast the above-quoted statement from GLA with Kant's characterization of necessity in KrV (Einleitung, B17): 'Was uns hier [in der Mathematik] gemeinlich glauben macht, als läge das Prädikat solcher apodiktischen Urteile schon in unserm Begriffe, und das Urteil sei also analytisch, ist bloß die Zweideutigkeit des Ausdrucks. Wir sollen nämlich zu einem gegebenen Begriffe ein gewisses Prädikat hinzudenken, und diese Notwendigkeit haftet schon an den Begriffen. Aber die Frage ist nicht, was wir zu dem gegebenen Begriffe hinzu denken sollen, sondern was wir wirklich in ihm, obzwar nur dunkel, denken, und da zeigt sich, daß das Prädikat jenen Begriffen zwar notwendig, aber nicht als im Begriffe selbst gedacht, sondern vermittels einer Anschauung, die zu dem Begriffe hinzukommen muß, anhängt.' It seems to me that here Kant wants to draw attention to the actual processes whereby we apprehend the content of synthetic (mathematical) judgements: the relevant question to be asked in this context is not, according to Kant, one concerning what we ought to add in thought to a given concept, but rather what we really think into it, albeit obscurely, for then it becomes apparent that the necessity of a certain predicate inhering in a given concept is brought about by an intuition which must be added to that concept. Frege's notion of objective justification, on the other hand, relates to the logical form of the sentences expressing arithmetical propositions, and thus, in a sense, to that which we 'ought to think'; the further question of whether and how the content of such logically analysed sentences is related to that which we really think is discarded by him as logically irrelevant.

terms (i.e. as questions concerning meaning, reference, and conditions of justification) constitutes, in my opinion, the background against which Frege's doctrine of assertion is to be understood.

By reformulating the distinction between the analytic and the synthetic as a distinction, not between types of propositional content, but between types of justification Frege got rid of a host of traditional modal distinctions. As a consequence, the issue of analyticity came to fall squarely within the philosophy of mathematics, on account of Frege's identification of the analytic sentences with those that can be derived from purely logical axioms and definitions. This solution, though appealing, raises a number of questions.

No doubt, once we know how to derive an arithmetical proposition from logical axioms or know that such a derivation is available, we have gained some kind of knowledge: for instance, we have a better insight into the interconnections of truths. This cognitive gain, however, cannot, according to Frege, be imported as it were into the content of the proposition itself. Content is to be spelled out in terms of truth conditions and not in terms of conditions of justification. Yet, some perplexities remain. Thus one might argue that, if we have really succeeded in proving a proposition's analyticity in Frege's sense, then it must be a feature of its content that it is capable of being proved to be analytic. Analogously one might argue that, if a proposition is known a priori, then it must be a feature of its content that it is knowable a priori.

To lessen the tension I suggest that we try the following formulation: analyticity is not a feature of the content (which is what Frege will later call a 'thought') but a feature of the logical form¹⁰ of the sentence which in a given formal system goes proxy for that content or thought. Bearing in mind that analyticity in Frege's sense is analyticity relative to a formal system, viz. derivability from logical axioms and definitions, and further bearing in mind that different reconstructions are possible (a fact of which both in BS and GLA Frege shows himself to be fully aware), we can see the point of Frege's claim that questions of justification, and hence of derivability, do not concern content as such.¹¹

I am well aware that by stressing the epistemological aspect of Frege's foundational approach during the years from 1879 to 1884 I am widening the gap between Frege's early philosophy and his later philosophy: for it is by no means obvious that in uncovering the deeper grounds on which the justification of the assertion of an arithmetical proposition rests (to use the terminology of GLA) we at the same time uncover the laws of being true (*Wahrsein*) which reign in the

¹⁰ I understand the notion of 'logical form' in the broadest sense, so as to comprise the contribution to the analysis of a sentence given by analytical and constructive definitions of terms occurring in it.

¹¹ When viewed from this angle, Frege's doctrines can be seen to have a lot of modal bite in them. However, it is not my present task to explore the connections between provability, derivability, and modality.

realm of logical objects (to use the terminology of *Grundgesetze*¹²). At any rate we shall need an argument to show that this is a possible route.¹³

Now, the notion of truth is, unlike those of analyticity and derivability, an absolute one.¹⁴ And the fact that it plays this privileged role both in Frege's conception of logic and in his conception of meaning explains why he does not consider the assertoric modality as being on a par with the other alethic modalities: the act of recognizing as true the thought expressed by a given sentence and that of making public the recognition of its truth by means of a linguistic act of assertion are primitive activities which we may endeavour to elucidate but not to reduce or to explain away. But judging and asserting are not like twin brothers, the former shy and aloof and the latter forthcoming and chatty. It took Frege's ingenuity to see that pride of place was due to the *linguistic* and public aspect, and it was from this point of view that he readdressed an issue which had exercised philosophers since the time of Aristotle. What I mean is the issue of the *logos apophanticos*, to which I shall now turn.

The Judgement Stroke

Frege's negative valuation of what traditional theories of judgement had accomplished created an obligation on his part to offer a more adequate account of the content of a sentence and of the significance of putting forward a sentence as true in an act of assertion. Unlike his contemporaries, Frege did not believe that such an answer could be found on the lines of a traditional theory of judgement, i.e. that part of the theory of knowledge which deals with the mental act of judgement. In Frege's writings we should look in vain for a theory of judgement of the kind put forward by Mill, Brentano, Sigwart, or Lotze. He never discusses such questions as, e.g., What distinguishes an act of judgement from an act of having a representation? Is the relevant distinction to be sought in the contents of these mental acts or in our ways of looking at them (in our being in different intentional states, as a follower of Brentano might say)? How do representations combine with each other and under which circumstances can we say that their combination gives rise to a belief which we put forward as true in an act of judgement? Frege was no

¹² G. Frege, *Die Grundgesetze der Arithmetik*, vol. I 1893, vol. II 1903, Jena: Pohle, Hildesheim: Olms, 1966 (= GGA I, GGA II).

¹³ Indeed, I believe that Frege's conception of logic underwent significant changes in the period between BS and GGA; cf. Picardi, *Premessa* to G. Frege, *Scritti postumi*, Naples: Bibliopolis, 1987, and Picardi, 'Frege on Definition and Logical Proof', in C. Cellucci, G. Sambin (eds.), *Atti del convegno della SILFS su 'Temi e prospettive della logica e della filosofia della scienza contemporanee'* (Cesena, 7-10 gennaio 1987), Bologna: CLUEB, 1988.

¹⁴ Also in the specific sense of not being relativized to a 'world' or to a context of utterance. This consideration, together with Frege's conception of logic as having a fixed (universal) domain, affords a further ground for understanding his neglect of modal notions in the modern sense. I think, however, that it would be anachronistic to attribute this motive to Frege.

doubt aware of the fact that such questions had been asked, but he thought that before we can give an answer to them we first have to get clear on the more basic issue of providing an account of the meaning and structure of the sentences by means of which we formulate our judgements. He believed that in order to grasp the content of a sentence we have to ask ourselves how it is determined as true or false in virtue of its component words and the way they are put together, and in order to do this we must find a way of representing its content in such a fashion that the subjective and the objective elements are sharply distinguished. But this is not the only task; in addition we must find a means of representing content in a way which makes its composition clear and renders it amenable to deductive treatment.

Now I do not propose to review Frege's logical achievements in their entirety. I merely want to point out that in order to drive home the required distinctions between the content of a judgement, the mental act of judgement, and the public act of assertion, a number of traditional doctrines had to be overturned or even dismissed altogether. The first step in this direction was Frege's confutation of a theory according to which the copula fulfils the functions both of a sign of second intention signifying affirmation and of conjoining subject term and predicate term in a unit, thus functioning as a sign for the logical relation of predication. Here I can only go into the former of these two issues. As to the latter, I shall content myself with mentioning that Frege developed an entire battery of arguments, culminating in the replacement of the subject/predicate distinction with the argument/function distinction and in dismissing the conception of predication as a (logical) relation over and above that of functional application which already is enshrined, as it were, in the concepts themselves. What is particularly relevant in this to the theory of judgement is Frege's implicit criticism of the conception that judgement is the result of comparing ideas and finding out whether they fit each other (that is, whether the predicate is contained in the subject, if one prefers the intensional idiom). Frege clearly saw—and in this Mill may have preceded him—that, if we are to advance from the thought to the truth-value (which was Frege's quasi-definition of the act of judgement), we must not confine ourselves to the level of sense or ideas but must systematically connect linguistic expressions and their referents. And this is a purpose the doctrine of sense and reference is to serve.

So much for predication. Now I shall turn to the other alleged function of the copula, i.e. as a sign signifying affirmation.

In Frege's early writings it is the 'Vergötterung der Kopula' where he locates one of the mainsprings of philosophical confusion (NS, p. 71; PW, p. 64); and in his later writings he often speaks of the necessity of separating assertoric force from the predicate, where linguistic usage misleadingly tends to place it. Natural language, however, has no sign for assertoric force, a lacuna which can be filled in in a formal language. Here we must be wary of inadvertently following linguistic

usage and rendering the sense of the (vertical part) of the judgement stroke by means of phrases such as 'it is a fact that' or, worse still, by means of the predicate 'it is true that'—a temptation to which young Frege had succumbed,¹⁵ as old Frege keenly realized. In fact, when the purpose of making an assertion is frustrated by certain features of the sentence used (e.g. the occurrence of an empty name in referential position) or by certain features of the context of utterance, such as its being uttered in the course of a stage performance or a reading of poetry, it cannot be restored by adding words like 'it is true that' or, for that matter, 'it is a fact that'.

From a linguistic point of view the vertical stroke appears to have the status of a primary performative (to borrow Austin's terminology¹⁶), that is, a sign which effects that which it signifies. Here, however, we are chiefly interested in the role this sign plays in Frege's logical systems, viz. BS and GGA.

The judgement stroke (*Urteilsstrich*) which Frege introduces in §2 of his BS is a composite sign, made up from a horizontal stroke called the 'content stroke' (*Inhaltsstrich*) and a vertical stroke called the 'judgement stroke' (*Urteilsstrich*). Frege often refers to the whole sign as '*Urteilsstrich*'.¹⁷

According to Frege's explanation, that which follows the content stroke must be a judgeable content, that is, a well-formed sentence fulfilling a number of syntactic and semantic requirements. It is a peculiarity of Frege's syntactic requirements that open sentences with free variables are not to count as judgeable contents. The semantic requirements rule out the occurrence of vague predicates and empty proper names. The vertical stroke has the office of conveying that the content in question is recognized or accepted as true.¹⁸

In GGA the relevant terminology as well as the conception of the judgement stroke itself are modified. This is a consequence of the 'discovery' of the sense/reference distinction and the assimilation of sentences to proper names of truth-values. This change is described by Jourdain when he says that 'the hypothesis as to the judicability of contents was dropped. . . the relation of implication between p and q may hold when they are not propositions at all' (WB, p. 299). These are in fact consequences of Frege's reinterpretation of the two parts from which the judgement stroke is made up. What used to be the content stroke is now called the

¹⁵ In his BS (§3) Frege had suggested considering the expression 'is a fact' as a common predicate of all judgements.

¹⁶ Cf. J. L. Austin. *How to Do Things with Words*, Oxford: OUP, 1962, p. 69. For a discussion, cf. M. Dummett, *Frege: Philosophy of Language*, London: Duckworth, 1973, ch. 10, pp. 335ff.

¹⁷ Russell in his *Principles of Mathematics* (1903), London: Allen & Unwin, 1972, renders the German word by means of the expression 'judgement sign' (Appendix A, §§478f.). The translation of '*Urteilsstrich*' as 'judgement stroke' is Jourdain's and occurs in a paper of his devoted to the exposition of Frege's work (published in 1912 in the *Quarterly Journal of Pure and Applied Mathematics*, reprinted in WB).

¹⁸ In his BS Frege employs a psychologistic terminology and talks of '*Vorstellungsverbindung*' where in his later writings he will use the word 'thought'. The earlier terminology is explicitly censured in his correspondence with Jourdain (cf. G Frege, *Wissenschaftlicher Briefwechsel*, G. Gabriel, H. Hermes, F. Kambartel, C. Thiel, A. Veraart (eds.), Hamburg: F. Weiner, 1976, p. 280 (= WB)).

'horizontal' and has acquired the status of a sign for a first-level function from objects to truth-values, i.e. a sign for what in Frege's new terminology is called a 'concept'. The semantic interpretation of this function, like that of the other eight primitive functions of GGA, is laid down by stipulating which values this function takes for appropriate arguments. The stipulation is as follows: '—x' is the True if the argument place is filled with a name of the True, in all other cases it is the False. Falsity is defined in terms of negation and the horizontal function as the concept under which all objects with the exception of the True fall. According to the new formulation, not only, say, '2 is an even number' but also '2' can fill the argument place of the horizontal; the resulting expression is a complex name of the False. Expressions of that type stand for one of the two logical objects, no matter whether we shall ever come to know which one. Frege assumes without argument the equivalence of 'it is not true that p' and 'it is true that not p',¹⁹ which is hardly surprising on account of his firm adherence to bivalence. We cannot here go into all the relevant questions concerning the connections between denial, negation, and falsity. What it is important for our present discussion to bear in mind is that Frege argues at length against the alleged need of recognizing denial (*Verwerfung*) as an act of judgement distinct from acceptance (*Anerkennung*). To repudiate a thought as false and to acknowledge its negation as true are, in his opinion, one and the same act of judgement.²⁰

Of the many criticisms which have been levelled against Frege's assimilation of sentences to names I shall mention only one, which I think may help us to get clear on the reason why Russell's assertion sign bears only a typographical resemblance to Frege's judgement sign. The criticism is this: that the horizontal stroke is really a universal truth predicate which offends against the vicious-circle principle and threatens to lead to semantic paradoxes. The theory of orders and the introduction of typical ambiguity into the notion of truth are the antidotes Whitehead and Russell adopt in *Principia* against semantic paradoxes.²¹ The criticism in question is expressed concisely by Wittgenstein in TLP (4.442) where, after having argued that the judgement stroke is meaningless, he goes on to say that 'It is quite impossible for a proposition to state that it itself is true'—which simply is an application of the general principle expressed in 3.332 according to which 'No proposition can make a statement about itself, because a propositional sign cannot be contained in itself (that is the whole of the "theory of types")'.²²

¹⁹ GGA I, §6.

²⁰ Cf. NS, p. 201; PW, p. 186 and 'Die Verneinung: Eine logische Untersuchung', *Beiträge zur Philosophie des deutschen Idealismus*, 1 (1918–19), pp. 143–57, p. 154.

²¹ Cf. *Principia Mathematica*, vol. I (1910; 2nd ed. 1925), Cambridge: Cambridge University Press, 1962, pp. 41f.

²² This formulation of Wittgenstein's may seem to suggest that self-application suffices to cause paradoxes, which is not the case. Both Russell and Wittgenstein felt uneasy about Frege's treatment of

It is not quite clear which of the two parts of the judgement stroke Wittgenstein is here alluding to; but be that as it may, I have chosen this quotation because it conveys as concisely as possible the reason why Russell's assertion sign is merely related, but by no means identical,²³ to Frege's judgement stroke. Whitehead and Russell's assertion sign is not a compound sign; in fact, it corresponds to one half of Frege's sign, viz. the vertical stroke. In *Principia Mathematica* (Introduction, p. 8) they say:

The sign '⊢', called the 'assertion sign', means that what follows is asserted. It is required for distinguishing a complete proposition, which we assert, from any subordinate propositions contained in it but not asserted. ... Also a proposition stated in symbols without this sign '⊢' prefixed is not asserted, and is merely put forward for consideration, or as a subordinate part of an asserted proposition.

Thus Whitehead and Russell fully endorse Frege's distinction without committing themselves to Frege's conception of truth or his assimilation of sentences to names.

We can try to capture from a modern standpoint the difference between Frege's judgement stroke and Russell's assertion sign in sharper terms by appealing to the distinction between object language and metalanguage. Thus, for instance, some authors, e.g. H. Weyl in his paper 'The Ghost of Modality',²⁴ maintain that the non-embeddability of the assertion sign is due to its metalinguistic status.²⁵ This may be true of Russell's interpretation of Frege's sign but it does not apply to

the truth-predicate for reasons having to do with the semantical paradoxes. They did not, however, locate the danger in the horizontal stroke, since neither of them paid sufficient attention to its composite structure. I am here following Peter Aczel's diagnosis of (one of) the sources of (a family of) paradoxes in GGA, as put forward in his 'Frege Structures and the Notions of Proposition, Truth and Set', in J. Barwise, H. J. Keisler, K. Kunen (eds.), *The Kleene Symposium*, Amsterdam: North-Holland, 1980, pp. 31–59.

²³ Contrary to what they themselves state: "The sign "⊢" is called the assertion sign; it may be read "it is true that" (although philosophically this is not exactly what it means)", to which they append the footnote: 'We have adopted both the idea and the symbol from Frege'. PM, Part I, Section A.

²⁴ Cf. H. Weyl, 'The Ghost of Modality', in M. Faber (ed.), *Philosophical Essays in Memory of E. Husserl*, Harvard: Harvard University Press, 1940, pp. 278–303. In stressing that in GGA the truth-predicate belongs to the object language I am by no means suggesting or implying that it is in itself a bad thing to have the truth-predicate in the object language; it is a feature, however, which, together with the assumption of the law of bivalence, will straightaway lead to paradoxes. Recently non-Tarskian approaches to the concept of truth have been discussed in the literature, cf. the papers in Robert L. Martin (ed.), *Recent Essays on Truth and the Liar Paradox*, Oxford: OUP, 1984.

²⁵ In *Principia mathematica* Frege's assertion sign is used *also* to mark the metalinguistic relation of logical consequence, and when so used it may give the impression of occurring as an embedded part of the sentence in question; in reality it relates a set of sentences (premises) to a certain conclusion. This further application of Frege's sign is spelled out in greater detail in §38 of *The Principles of Mathematics*. In this work Russell interprets Frege's assertion sign as going proxy for the word 'therefore' of natural language, which marks the transition from the asserted premises to the (asserted) conclusion of an argument. Contrary to what is generally claimed, Russell was well aware of the distinction between use and mention, between material implication and entailment; admittedly, however, he did not always heed it.

Frege's own use of it. In the case of Frege's sign the requirement of non-embeddability does not hold for the horizontal, which, like all other function signs, can be reiterated, combined, or 'fused' with other signs to form compound expressions. Besides, a sign made up from two parts, one of which belonging to the object language and the other one to the metalanguage, would be a sign exceedingly *sui generis* indeed. In fact, it is precisely because the judgement sign occurs in the object language, on a par with the other signs of functions but not on a par with, say, Greek consonants,²⁶ that Frege can claim that 'p' and 'p is true' say the same thing and that, unless they are issued with assertoric force, no judgement is conveyed.²⁷

Let us pause briefly on this point. It would be a mistake to construe Frege's repeated claim that 'p' and 'p is true' have the same sense as a precursor of the redundancy theory advocated e.g. by Ramsey, and it would be so for two reasons. The first and more obvious one is that in Frege's formal language the notion of truth plays precisely that fundamental role which the redundancy theory denies to it; the second, somewhat obscured by Frege's own claims, is that in the language of GGA the word 'true' cannot be eliminated from all contexts, and makes, after all, a contribution to the expression of the thought. If we pay close attention to §6 of GGA, where the equivalence is stated between Γ and $\neg\Gamma$ for the case in which Γ is a name of a truth-value (but *not* for the case in which ' Γ ' is a name of a different object), we spot a (mild) incongruence in Frege's own explanation. For at the end of §6 he says:

According to our stipulation $\top 2^2 = 5$ is the True; therefore

$\vdash \top 2^2 = 5$

In words: $2^2 = 5$ is not the True; or: the square of 2 is not 5.

The same holds for $\vdash \top 2$.

However, if we try to express in words the thought expressed by the latter formula we realize at once that all we can say is '2 is not the True', and that the 'or' of the disjunctive clause is left dangling. This clearly shows that the word 'true' (actually 'not true'), far from being eliminable, does the duty of the main predicate of the sentence.

²⁶ These symbols belong to the 'Darlegung' of the BS (GGA, §1 n.)—what Frege will later call the 'Darlegungssprache' in contrast with the 'Hilfssprache'; cf. NS, p. 280; PW p. 260.

²⁷ Apparently Frege did not envisage the possibility that in 'it is true that p' 'p' can be construed as occurring within an oblique context, for, if this were the case, 'p' would have its indirect sense and indirect reference, and, on the basis of Frege's own doctrines, it would then be wrong to say that in asserting 'p' and in asserting 'it is true that p' we were expressing the same thought. Or, to put the matter more accurately: Frege nowhere discusses the connection between "'p" is the True' and 'it is true that p'. I am inclined to think that he would have considered the difference in meaning between those two sentences of merely stylistic significance.

I believe that Wittgenstein was the first to draw attention to this feature of Frege's symbolic language; at least, this is the way I interpret TLP 4.063–4.064, where he says: 'the verb of a proposition is not "is true" or "is false" as Frege thought: rather that which "is true" must already contain the verb'.

The Frege Point

The necessity of separating assertoric force from content arises most clearly when we consider compound sentences. In Frege's own words:

Fiction apart, it seems that it is only in subordinate clauses that we can express thoughts without asserting them. One should not allow oneself to be misled by this peculiarity of language and confuse grasping a thought and making a judgement.²⁸

The clearest case exemplifying the relevant distinction is the conditional construction. When I say, 'If Geach is right, then Russell is wrong', I am asserting neither the antecedent nor the consequent; that is, both these sentences occur unasserted. In both of them, however, there is a predication; in Frege's terminology, they are cases of subsumption of an object under a concept. Moreover, if assertoric force were part of the content, it would turn out that the most basic form of inference, detachment, rested on equivocation; because then it would not be a matter of recognizing the same proposition occurring once as a premise and once as the antecedent of a conditional. The premises and conclusions of an inference are asserted propositions, while that on which the inference rules operate, that is, that which the logical particles connect, are unasserted propositions.

This distinction is particularly easy to grasp in Frege's notation as it is explained in GGA or 'Funktion und Begriff', where the expression obtained by filling in the argument place of the horizontal stroke (which signifies a first-level function from objects to truth-values) with a sentence stands for a truth-value, whereas the whole sentence prefixed by a vertical stroke does not have the function of designating anything, let alone the truth-value True. The vertical stroke has the peculiarity that

... it cannot be used to construct a functional expression; for it does not serve, in conjunction with other signs, to designate an object. '<⊢2+3=5>' does not designate anything; it asserts something.²⁹

²⁸ PW, p. 198; NS, p. 214.

²⁹ G. Frege, *Funktion und Begriff*, Jena: Pohle, 1891, p. 23 n. (= FuB).

The same point is made in GGA as well as in Frege's correspondence with Jourdain, where he says that 'of a judgement one can neither say that it refers to something nor that it is expressed' (WB, pp. 119f.). In other words, the vertical stroke does not fulfil either of the two functions: referring (*bezeichnen*) or expressing (*ausdrücken*), which according to the doctrine advanced in 'Über Sinn und Bedeutung'³⁰ characterize the semantic behaviour of all signs. For this reason it is said to be a sign *sui generis*.

Surely in this context it would have been better to use two different expressions, e.g. correctness in the case of an assertion, on the one hand, and truth (or falsity) in the case of a thought, on the other. This terminological distinction is something that we can add without becoming unfaithful to Frege's meaning. The ingenuity of Frege's formulation lies, among other things, in its being generalizable—a potentiality which remains hidden in the classical two-valued system. For with the distinction between semantic value (or truth-value) and correctness at our disposal we realize at once that, e.g., the explanation of the notion of assertability given by Lewis and Langford is in harmony with Frege's reading of the two parts from which the assertion sign is made up. In chapter 7 of their *Symbolic Logic*, where they give a survey of truth-value systems and go on to discuss the distinction between designated and undesignated values, they say:

The property represented by any 'designated' truth-value must be such that whatever proposition has this property will be validly assertable. It should be noted here that 'assertability' is a general character which is common to a considerable variety of more specific properties—e.g., truth, universal truth, probability-value = 1, logical necessity.³¹

Rephrasing all this in Fregean terminology we may say that only propositions which take the truth-value true are assertable, which means that the True is the only designated value. If one allows for more than one designated value, one will appreciate the importance of distinguishing between semantic value and correctness (the latter can be taken to be indicated by the vertical stroke).

But the relevance of Frege's dictum that it must be possible to express a thought without at the same time recognizing it as true can be appreciated also from a different perspective. For the record, what we are here dealing with is what Geach likes to call 'the Frege point', and he puts it the following way: 'A sentence still has the same sense and truth-value even when it is not asserted.'³² Now, this point, once made, may seem trivial but it constitutes one of the most important assumptions that we make in logic. The argument we have used with respect to the

³⁰ G. Frege, 'Über Sinn und Bedeutung', *Zeitschrift für Philosophie und philosophische Kritik*, 100 (1892), pp. 25–50 (= SuB).

³¹ Cf. C. I. Lewis, C. H. Langford, *Symbolic Logic*, 1932, repr. New York: Dover Publications, 1951, p. 232.

³² 'Critical Notice' of Dummett's *Frege: Philosophy of Language*, *Mind*, 85 (1976), pp. 436–49.

material conditional can be extended so as to apply to all other logical particles, and then it amounts to the thesis that the logical particles do not connect assertions; that is, what they connect are, not sentences prefixed by a force indicator, but unasserted propositions. According to Frege, that which occurs on the right side of the vertical stroke is without assertoric force; in other words, no embedding of verticals is admissible, nor, for that matter, of any other force indicator. Plainly, in natural language we do employ conjunction and disjunction to connect sentences prefixed with different force indicators (for instance, an assertion and a question, or an assertion and a command), but I am not aware of any special method of handling this type of occurrence of the logical particles, nor is it clear to me whether one is needed.

If the logical particles joined, not unasserted propositions, but assertions, it would be hard to explain the point of the rules of introduction and elimination for each logical constant. By the way, it is lamentable that the assertion sign was not employed by Gerhard Gentzen, to whom we owe the clearest account of what it is for a sentence containing a certain logical constant as its main operator to be justifiably assertable.³³ Indeed, except for Russell, Kolmogorov, and Heyting, I know of no other logician of the 1920s who made use of the assertion sign. Kolmogorov and Heyting, however, did not take it directly from Frege but from Russell.

The mention of Heyting brings us to the question whether it is only in classical logic, where the semantic value of a sentence is exhausted by its having a certain truth-value, that Frege's distinction is to be observed. Here I can fortunately rely on the philological excavations contained in Troelstra's paper 'Logic in the Writings of Brouwer and Heyting' (1982).³⁴ As an appendix to his paper Troelstra prints a few letters Heyting and Freudenthal exchanged in 1930, and this correspondence is extremely interesting and revealing. The point at issue is the legitimacy of making a distinction between a proposition as simply entertained, a proposition as proved, and a proposition conceived as provable. For Heyting, as for Frege, acknowledging the distinction between regarding a proposition as proved and regarding it as provable becomes crucial because of the need to give a sufficiently comprehensive explanation of the conditional (in Heyting's case, the intuitionistic conditional). For surely, Heyting argues, 'If A then B' cannot possibly mean, as Freudenthal suggests, that if A is proved then B is

³³ Dana Scott rightly misses the assertion sign in Gentzen and turns the arrow of the calculus of sequents into a metalinguistic sign for the relation of conditional assertion, i.e. entailment. This is a very interesting reading of the assertion sign but not faithful to Frege's intentions (and, according to some authors, it is not even faithful to Gentzen's intentions) (cf. 'On Engendering an Illusion of Understanding', *The Journal of Philosophy*, 1971, pp. 787–807). Alonzo Church, in his *Introduction to Mathematical Logic*, vol. I, Princeton University Press, 1956, advisedly chooses two slightly different signs: one for Frege's assertion sign and another one, the turnstile, which no doubt functions as a metalinguistic sign, for the relation of derivability or theoremhood (cf. n. 65, p. 24).

³⁴ Cf. A. Troelstra, 'Logic in the Writings of Brouwer and Heyting', in V. M. Abrusci, E. Casari, M. Mugnai (eds.), *Atti del convegno internazionale di storia della logica. San Gimignano 4–8 dicembre 1982*, Bologna: CLUEB, 1983, pp. 193–210.

proved: according to Heyting it must mean something like 'I am in possession of a construction by means of which a proof of B can be derived from any proof of A' ('Ich besitze eine Konstruktion, welche aus jedem Beweis für A einen Beweis für B ableitet'). If this explanation is accepted, Heyting argues, then the distinction between 'A' and 'A is proved' (that is, 'A' prefixed by an assertion sign) must be acknowledged:

If I understand you correctly, you want to make do without the distinction between A and $\langle \vdash \rangle A$; for me it is indispensable if in 'If a then b' I wish to give b a sense of its own, and this is necessary because the above interpretation mentions a proof of b even if b has not yet been proved. Thus a is simply to mean a proposition put forward as a problem; $\langle \vdash \rangle A$ is to be the assertion 'A is proved'.³⁵

Now I do not want to embark on a discussion of the intuitionistic interpretation of the meaning of implication, nor do I wish to do further exegetical work. All I want to do is to call your attention to a certain parallel that can be drawn between classical logic and intuitionistic logic; and I should like you to reconsider in the light of that parallel something I said in the first section of my paper, where I stated quite dogmatically that content in Frege is given, not by means of specifying justification conditions, but through truth conditions. Now we are in a position to reformulate that claim in the following way: understanding the meaning of a sentence amounts to understanding its contribution to the truth-value of a conditional in which it occurs as the antecedent, that is, as an unasserted proposition. For, as Frege observes, it is only in subordinate clauses that sentences occur unasserted, and this is the very use of a sentence from which we draw our justification of a realistic conception of truth, uncontaminated by considerations of assertibility or justifiability. The parallel in intuitionistic logic is that to grasp the meaning of a sentence is to have a conception of what it is for it to be provable or to become justifiably assertable, no matter whether we are at present in a position to do so.

There are many difficulties in the picture sketched above. First of all, Frege seems to take it for granted that from our understanding of the meaning of a sentence, spelled out in terms of truth conditions, we can automatically derive a grasp of the conditions under which it would be correctly assertable. This, in turn, implies that we have a conception of truth prior to the practice of making assertions. However, as soon as we press the question of how this notion of truth is to be understood, what it is to advance from the thought to its truth-value, we receive no answer. This is by no means due to an oversight on Frege's part; it is a consequence of a conviction of his which he stuck to throughout his life

³⁵ Ibid., p. 207.

and which he expressed by saying that truth is undefinable and that all attempts at defining it (such as, for instance, the correspondence theory) presuppose the very notion which they are meant to explain.³⁶

Now, if we confine our attention to simple (atomic) sentences, we can interpret the undefinability thesis as amounting to the requirement that it be intuitively decidable whether a given situation satisfies the truth conditions of a sentence or whether it does not (unless these truth conditions are recognition transcendent—but in that case we should have no relevant situation to begin with). (The parallel with intuitionistic logic is obvious.) Under this construal I see no objection to Frege's thesis that truth is undefinable; it is hard to see how it could be otherwise. However, there is no denying that this reading of the undefinability thesis does not chime in with Frege's Platonistic outlook. Let us try the following reformulation: the notion of truth which informs our judgements and their contents manifests itself in our choice of those propositions which we consider neither capable nor in need of justification (and choose as axioms of a logical system) as well as in our choice of principles guaranteeing soundness, i.e. governing the transition from certain asserted sentences to certain assertable sentences. That is, our conception of truth shows itself in our logic and cannot be further explained.³⁷ Is this reformulation any help? At first sight, we seem to be moving in circles: in order to form a conception of the notion of truth, we must look at its manifestations in our practice of assertion; and in order to understand the practice of assertion, we must credit the speakers with some notion of truth at which they can be seen as aiming and which fulfils a normative role. Is this circle a harmful one? And, if so, are there ways of breaking out of it? Or is there a possibility of passing by this circle by giving some of the fundamental notions concerned priority over the remaining ones?

The overwhelmingly simple suggestion which then comes to one's mind—and it did come to Frege's mind—is that the obsession with truth or the predicate 'is true' is but a superstition which natural language foists on us and that we should do better to direct our attention to assertoric force, for this is where the essence of logic is to be found. Thus Frege's choice of priorities at any rate is clear, and this is far from being a trivial choice. In Frege's own words:

... there is no doubt that the word 'beautiful' actually does indicate the essence of aesthetics, as does 'good' that of ethics, whereas 'true' only makes an abortive attempt to indicate the essence of logic, since what logic is really concerned with is not contained in the word 'true' at all but in the assertoric force with which a sentence is uttered.³⁸

³⁶ For a discussion of the undefinability thesis, cf. Dummett, *Frege: Philosophy of Language*, ch. 13.

³⁷ This formulation mirrors, I believe, Frege's views at the time of the BS; cf. BS, §13.

³⁸ PW, p. 252; NS, p. 272.

A Note on Dummett and Frege on Sense Identity

In many of his writings Frege urges that it would be desirable to lay down explicit criteria for telling whether two sentences express the same thought and whether two expressions can make the same contribution to the expression of a thought.¹ However, while he never tires of emphasizing that the discovery that proper names endowed with different senses have the same reference can add to our knowledge, he does not pay equal attention to the cognitive gain ensuing from the discovery of a sentence's analytic character or from finding out that two sentences are analytically equivalent.

Given that Frege sides with the *Umfangslogiker* à la Schröder against the *Inhaltslogiker* à la Husserl, one may well wonder why the issue of sameness of sense should have been of so acute an interest to him. The answer does not reside in any peculiarity of the formal systems he constructed but in the philosophical justification of his foundational programme.

Two features of this programme made it desirable that some criteria for sense equality and perhaps also some measure of cognitive significance should be available. The first was the claim that deductive reasoning and pure conceptual thought unaided by intuition can enlarge our knowledge; the second was the thesis that arithmetical propositions are analytic if they can be shown to be derivable from purely logical axioms by means of definitions of arithmetical concepts and rules of a purely logical nature. Arithmetical propositions—Frege surmised—are contained in logical axioms as plants are contained in a seed and not as bricks are in a house. What had to be explained was (a) the mechanism of deductive inference, i.e. the peculiar mode of containment of the conclusion in the premises of a deductive argument, in such a way as to account both for its validity and for its fruitfulness, and (b) the status of definitions. Perhaps the clearest statement of whence the value of deduction arises occurs in Frege's critique of Boolean logic:

If we compare what we have here [in Boole's algebra of logic] with the definitions contained in our examples, of the continuity of a function and of a limit, and again that of following in a series which I gave in §26 of my *Begriffsschrift*, we see

¹ A shorter version of this paper was read at the Fifteenth International Wittgenstein Symposium, Kirchberg am Wechsel (Austria).

that there is no question there of using the boundary lines of concepts we already have to form the boundaries of new ones. Rather, totally new boundary lines are drawn by such definitions and these are the scientifically fruitful ones. Here too we use old concepts to construct new ones, but in so doing we combine the old ones together in a variety of ways by means of the signs of generality, negation and the conditional.²

This programmatic statement receives a profound and elaborate interpretation in Dummett's book on Frege's philosophy of mathematics. Dummett's account stresses the tensions and changes in Frege's theory and proposes a resolution of them which does justice, in my opinion, to the bulk of his doctrines. In what follows I shall briefly touch on the bearing of synonymy on the issue of definition, pause to reflect on the significance of what Dummett has called the transparency thesis, and consider how Frege's own criterion of sense identity fares with respect to sentences other than definitions which he declares synonymous.

I

The reasons why the extensionalist logician, too, should be interested in the identity of senses are alluded to very concisely by Frege himself in a footnote to his essay 'Über Begriff und Gegenstand':

If all transformations of the expression were forbidden on the plea that this would alter the content as well, logic would simply be crippled; for the task of logic can hardly be performed without trying to recognize the thought in its manifold guises. Moreover all definitions would then have to be rejected as false.³

The definitions here alluded to seem to be what he later, in 1914, called 'analytic definitions' and contrasted with constructive ones, whose sole function is to serve as notational abbreviations; definitions of the latter kind cannot be rejected as 'false'. An analytic definition, on the other hand, purports to capture the sense of an expression to be defined and the question which arises concerns its credentials. This, in its turn, poses the problem of the general requirements which a definition has to fulfil if we are to accept it as providing an analysis of a concept already embedded in linguistic practice. It is natural to suppose that the definitions which occur in *Grundlagen* are of the latter type. However, the status of Frege's definitions was questioned by Husserl, among others, in his book *Philosophie der Arithmetik*.

² *Posthumous Writings*, p. 34; *Nachgelassene Schriften*, pp. 38–9.

³ 'Über Begriff und Gegenstand', p. 196; English translation, Geach and Black (1970), p. 46.

In his review of the book Frege represents Husserl's objections as follows:

[According to Husserl] in using the word to be explained, I either think clearly everything I think when I use the defining expression: we then have the 'obvious circle'; or the defining expression has a more richly articulated sense, in which case I do not think the same thing in using it as I do in using the word to be explained: the definition is then wrong. One would think that a definition was unobjectionable in the case where the word to be explained had as yet no sense at all, or when we are asked explicitly to regard its sense as non-existent so that it was first given a sense by the definition.⁴

However, instead of giving arguments to defend the conceptual credentials of his own definitions, he declares that mathematicians, unlike psychologistic logicians, concern themselves with the reference of words: it is the extension that gets defined, not the sense.

Dummett has drawn attention to a parallel between the problems which Frege (and Husserl) came up against when dealing with definitions, and the paradox of analysis. The problem which the paradox of analysis brings to the fore as regards the value of conceptual analysis has a counterpart in the problem of the justification of deduction, which so exercised John Stuart Mill and to which Dummett addressed himself in his essay on the justification of deduction. In order to be fruitful a deductive argument must tell us something that we did not grasp merely by apprehending the thoughts expressed by its premises, but if it is to count as valid its conclusion must not go beyond what is contained in the premises.

As Dummett and others have pointed out, after 1884 Frege's views on definition and very probably also on the whole issue of analyticity underwent a gradual change, culminating in the views expounded in 'Logik in der Mathematik', according to which definitions are merely notational abbreviations, which need not be responsible to the senses of the expressions as they are commonly understood in use.⁵ Here one might remark that the statement from the Husserl review just quoted suggests that Frege also regarded the definitions he had given at the

⁴ Frege (1894), p. 320; English translation in McGuinness (1984), pp. 199–200.

⁵ I should like to remark in passing that in stressing that the only point of definitions is that of notational abbreviation, Frege was also overlooking an important function of definitions which he himself had exploited and to which Quine was later to call attention, i.e. that definitions should better be seen as correlations between two languages:

The inclusive language, though redundant in grammar and vocabulary, is economical in message lengths, while the part, called primitive notation, is economical in grammar and vocabulary. Whole and part are correlated by rules of translation whereby each idiom not in primitive notation is equated to some complex built up of primitive notation. These rules of translation are the so-called *definitions* which appear in formal systems. They are best viewed not as adjuncts to one language but as correlations between two languages, the one part of the other. (Quine (1961), pp. 26–7)

Frege's own definition of membership (of an object in the extension of a concept under which it falls) given in *Grundgesetze der Arithmetik*, vol. I, §34, can be taken as a case in point.

time of *Begriffsschrift* and *Grundlagen* as 'constructive' in character. If this is a fair remark, Frege was not a reliable interpreter of his own work.

In 'Logik in der Mathematik' Frege states his conviction that we should be able to see at a glance, without proof, inference, or reflection, whether two words have the same sense and, in particular, whether the definiens captures the sense of the expression to be defined, as it is ordinarily understood. And yet, he concedes, doubts may occasionally arise whether the definiens really captures the sense of the definiendum. Should such a doubt arise, we need not agonize over it, for we can easily get round it: we introduce a completely new sign whose sense is composed out of the senses of the expressions used in the definiens and forget about the meaning of the expression as it is ordinarily understood.

No wonder that Rudolf Carnap, who attended Frege's lectures in 1914 and took notes of them in shorthand (still waiting, for all I know, to be deciphered), coupled Russell's craving for eliminative analysis with Frege's disdain for traditional conceptual analysis and became the champion of rational reconstruction. In a paper Carnap read at Königsberg in 1930 he contrasted postulationalism and logical constructivism:

The logicist does not establish the existence of structures which have the properties of the real numbers by laying down axioms or postulates; rather, through explicit definitions, he produces logical constructions that have, by virtue of these definitions, the usual properties of the real numbers. This 'constructivistic' method forms part of the very texture of logicism.⁶

It looks as if the entire justification of constructive definitions is that, added to the logical axioms, they can be used to yield the usual properties of numbers. There would then be no ground on which to choose between different reconstructions. In a similar vein one may argue that those readers who were bewildered by Frege's definition of number in *Grundlagen* were after all justified in protesting that it did not do justice to our intuitive conception of number, let alone to the psychological reality of the activity of counting. Frege had intended this definition to be merely formally adequate, not conceptually fruitful. And yet, even a superficial reader of *Grundlagen* realizes that this cannot really be the way Frege conceived of what he was doing. He obviously believed that the adequacy of his definitions was a consequence of his having found the correct logical analysis of sentences expressing an attribution of number. To be sure, he did not conceal from himself or the reader that his identification of numbers with classes of second-level concepts contained an element of arbitrariness. But this element of arbitrary stipulation in Frege's definitions should not blind us to the fact that most of the definitions

⁶ Carnap (1931); English translation in Pears (1972), p. 179.

which he gives in *Grundlagen* are of an analytic character and should be evaluated as such.

Dummett's argument in favour of this interpretation seems to me entirely cogent. He convincingly argues that the ground on which Frege's definition of number stands (or falls) is that it respects relationships of *conceptual priority* and of dependence between concepts. Among the examples of conceptual priority given by Frege and discussed by Dummett in his latest book, one may mention that of a concept (the reference of a predicate) over its extension, of parallelism over direction, of one-to-one mapping over cardinal equivalence. Now I do not propose to discuss the difficult and elusive notion of conceptual priority. It is rather the epistemological grounds on which Frege rejected analytic definitions that I should like to review.

II

In 'Logik in der Mathematik' Frege's official ground for preferring stipulative (constructive) definitions over analytic ones is the possible failure of what Dummett has called the 'transparency of sense' and contrasted with the opacity of reference.

Frege takes it for granted that a speaker who understands the sense of two expressions must be able to tell offhand whether they have the same or a different sense. Frege held that synonymy of words and sentences should be intuitively decidable, i.e. transparent, provided one had no difficulty in understanding the words in question. The latter proviso is, of course, essential. Naturally, in order to count as understanding a word one need not be aware of all the words synonymous with it; failure to realize that two sentences are synonymous does not show that one does not understand the thought expressed by the sentences concerned. A child who knows what a cube is but does not understand what 'regular hexahedron' means will show his ignorance when queried whether it has six square faces, while returning a positive answer to the question whether a cube has six square faces. Following Dummett's suggestion, we should distinguish between an extensional and an intensional construction of the expression 'knowing the meaning' of an expression: in the extensional sense we may say that the child knows the meaning of 'regular hexahedron' provided he has the concept expressed while ignoring that the word in question bears that sense.

The view that sense identity (and diversity) should be transparent is by no means peculiar to Frege, nor is it easily discarded. Wittgenstein's *Tractatus*, for instance, while calling into question the opacity of reference—i.e. that it may be possible to know the reference of two expressions without knowing whether they have the same reference—is explicitly committed to a version of the transparency

thesis as far as translation is concerned. At least, this is how I read 4.243 of the *Tractatus*:

Suppose I know the meaning of an English word and of a German word that means the same: then it is impossible for me to be unaware that they do mean the same. I must be capable of translating each into the other.

Dummett is surely right in challenging Frege's requirement that, given any two words that a speaker understands, he should be able to say offhand whether or not they bear the same sense. This requirement offends against the context principle, which demands that we should consider words in their appropriate settings, namely in sentences, not in isolation. Moreover, as regards definitions, it does not take into account their systematic role and the element of reconstruction which they inevitably entail. For to analyse a concept is not just to make explicit something that we implicitly know anyway, but involves an element of construction or reconstruction, of relating pieces of knowledge that are in our ken but which we may not be aware of having. In the case of analytic definitions Dummett proposes that their acceptance should be subordinated to the fulfilment of three requirements:

An analytic definition can therefore be required to do no more than to come as close as possible to capturing the existing sense. It does so if, first, any ordinary speaker can be brought to agree that it provides a necessary and sufficient condition for the application of the expression defined; if, secondly, it appeals only to connections which we make in practice, while someone's failure to make them would count as showing that he does not fully understand the expression; and if, thirdly, no rival definition is possible which has a better claim to capture the sense of that expression, as commonly understood. To this extent, analysis is necessarily reconstruction. (Dummett (1991b), p. 153)

In short, analysis purchases its fruitfulness at the expense of faithfulness. Much the same holds, as Dummett argues, for some of those 'transformations without which logic would be crippled' and to which I shall presently turn. In his 'Gedankengefüge' of 1923 Frege, oblivious of his own experience, still repeats that the hallmark of a logical axiom is its self-evidence; to compromise on this point would have seemed to him a concession to the formalists, who were guilty of blurring the distinction between axioms and definitions. However, as we have seen, even on his own grounds, there is no reason to require of definitions the evidence with which axioms are endowed and to surrender all attempts to look for definitions which are at least in part responsible to the meaning of words as they are used in ordinary language and in scientific practice. In fact, Frege's constant complaint was that mathematicians were unclear on the concept of number, not that they had not hit on an adequate set of constructive definitions.

Both in his book *The Interpretation of Frege's Philosophy* and in his latest book on Frege, Dummett draws a distinction between decomposition (dissection) and analysis to illuminate two notions that Frege failed to keep apart. The relevant distinction is that between extracting a predicate from a sentence, thereby bringing to the fore the pattern which the sentence shares with other sentences, and the process of grasping its sense by attending to the sense of its component words. The discernment of a pattern is not essential to our understanding of the sentence from which the pattern was extracted: in fact, we may understand the sentence without being aware that it can be seen as instantiating a given pattern which it shares with other sentences. This is why Frege can view the process of dissecting a sentence as a first step in the attainment of a *new* concept. Both the discernment of a pattern and the work of conceptual analysis embodied in definitions account for the fruitfulness of deductive reasoning.

III

Suppose, then, that following Dummett's advice, we relax the requirement of transparency and buy the services of conceptual analysis at the price of a modicum of infidelity to the pre-theoretical meanings of words. How does Frege's criterion of synonymy fare under this relaxed regime? In Frege's *Nachlass* we find a characterization of synonymy as a relation in which entire *sentences*, and not words in isolation, stand. This criterion appeals only to our occurrent grasp of sentences:

Now two sentences A and B can stand in such a relation that anyone who recognizes the content of A as true must thereby recognize the content of B as true straightaway [*ohne weiteres*], and, conversely, that anyone who accepts the content of B as true must immediately [*unmittelbar*] accept that of A. (*Equipollence*). . . . I assume that there is nothing in the content of either of the two equipollent sentences A and B that would have to be immediately accepted as true by anyone who had grasped it properly.

(*Posthumous Writings*, p. 196; *Nachgelassene Schriften*, p. 213)

The proviso that neither sentence should contain a 'self-evident' element, an element which commands immediate acceptance, is meant, I suppose, to avoid the collapse into the same thought of all sentences whose truth is self-evident or whose truth depends on the occurrence of a self-evident component. The requirement that the recognition of the equipollence should be possible without proof or inference improves on a similar criterion to be found in a letter to Husserl of the same year (9 December 1906). There Frege says that A and B express the same thought if by purely logical laws a contradiction can be derived from the assumption that A is true and B false and that A is false and B true. As both Jean van

Heijenoort (1977) and Dummett (1981) have pointed out, if we endorsed this criterion of sense identity we should be forced to the conclusion that any two analytically equivalent sentences have the same sense, in flagrant contradiction not only with Frege's claim that analytic sentences can extend our knowledge but with everything else he says about sense.⁷

At first blush Frege's approach to the issue of sense identity may appear almost psychologistic. However, when he alludes to 'immediate recognition' Frege certainly does not mean to refer to what psychologists nowadays call 'reaction times', but to a recognition which requires neither reflection nor inference. Frege was not interested in the skills and the mental equipment which enable us to judge whether two sentences express the same thought, but in uncovering those objective features of the meaning and structure of sentences on which such judgements rest. Sense is a cognitive notion that has to do with what is known by anyone who understands a sentence with that sense. This is perhaps the reason why in his scattered remarks on sense identity Frege never appeals to his conception of senses as a-spatial, a-temporal entities, which he was eventually to lodge in a third realm.

Are Frege's examples of synonymous sentences in accordance with the criterion mentioned above? His examples are somewhat heterogeneous. When he wants to convince the reader of a conceptual transition which he considers logically cogent he occasionally claims that what we have before us really is the *same content*, only clothed in different linguistic guises. At other times he realizes that if in order to acknowledge the legitimacy of a transformation of a sentence into another we need to invoke a logical axiom, we cannot possibly be dealing with the same

⁷ It is curious that this criterion of sense identity should have occurred in Frege's correspondence with Husserl. In his review of Husserl's *Philosophie der Arithmetik* Frege remarks that all that is required of a definition is that the definiens should have the same reference as the definiendum: two definitions which achieve this aim not only have the same extension but pick out the same concepts (in Frege's sense of 'concept'). There he says:

For the mathematician, it is no more right and no more wrong to define a conic as the line of intersection of a plane and the surface of a circular cone than to define it as a plane curve with an equation of second degree in parallel coordinates. His choice of one or the other of these expressions or of some other is guided solely by reasons of convenience and is made irrespective of the fact that these expressions neither have the same sense nor evoke the same ideas. (Frege (1894), p. 320; English translation in McGuinness (1984), p. 200)

In *Grundgesetze der Arithmetik*, vol. II, §58 Frege returns to this example in the course of his criticism of piecemeal definitions. He there says:

For if the first definition is already complete and has drawn sharp boundaries, then either the second definition draws the same boundaries—and then it must be rejected because its content ought to be proved as a theorem—or draws different boundaries and then it contradicts the first one. For example we may define a conic section as the intersection of a plane with a conical surface of rotation. When we have done this, we may not define it over again, e.g. as a curve whose equation in Cartesian co-ordinates is of second degree; for now this has to be proved. (English translation in Geach and Black (1970), p. 162)

On the assumption that Frege thought that this could be proved, and proved, moreover, by purely logical means, we would here have a striking example of two definitions which, according to the account given in 1894, exhibit no kinship of sense, while according to the criterion put forward in the letter to Husserl of 1906 they would count as expressing the same sense.

thought. The most notorious of his waverings concerns the transition from the generality of an equality to an equality of value ranges, that is, the question whether the two sides of Axiom V do or do not have the same sense. (Axiom V licenses the transition from the statement that two first-level functions have the same value for the same arguments to the statement that their value ranges are the same, and *vice versa* (*Grundgesetze*, II, §146).)

Another and perhaps less striking example of his waverings occurs in 'Gedankengefüge'. After having said that we see at a glance, without any help from a logical axiom, but only by attending to the sense, that 'A and B' and 'B and A' express the same thought (the difference in linguistic expression merely depends on the spatio-temporal medium into which thoughts are forced), he bluntly states that double negation and contraposition also leave the sense unaltered. And yet no help from a third realm can be expected in the latter case. Frege is far too prone to think that a logically evident step taken in thought is no step at all. And yet a series of small evident steps can make quite a difference to the thought concerned. Unless we revive the discredited law of the transformation of quantity into quality, we must admit that, if for ascertaining that two sentences express the same thought we need to appeal to a logical axiom, the sentences in question do not stand in a relation of synonymy. Otherwise it would not be clear where to draw the line.

On the basis of these considerations I should now like to propose that we class Frege's examples into five broad categories of equipollent pairs of sentences, as follows. Group A contains those sentences which differ only in colouring or tone; to group B belong those differing in grammatical construction; group C comprises paraphrases or elucidations (possibly involving formal concepts, such as 'function', 'object', 'concept'). Potentially problematic members of group C are the following pairs of sentences: 'The concept "square root of 4" is realized' and 'There is a square root of 4' on the one hand and 'Jupiter has 4 moons' and 'The number of Jupiter's moons is 4' on the other. Group D comprises sentences whose equipollence involves logical transformations, e.g. an appeal to a logical axiom. Group E contains sentences whose equipollence involves logical transformations involving new ontological commitments. As examples one may give the very pair of sentences I just mentioned, this time under the assignment of referents allotted to them in 'Über Begriff und Gegenstand' and in *Grundlagen*, respectively. Pairs of sentences belonging to the first three groups clearly pass Frege's test of synonymy; those belonging to the last two groups in my opinion should not. They can at best be regarded as analytically equivalent, in the sense of 'analysis' illustrated above.

Now, what about those sentences which come up twice in my list, once in the guise of elucidations and once ontologically burdened? In the intermediate group I classed such seemingly harmless transformations as 'Jupiter has 4 moons' and 'The number of the moons of Jupiter is 4'. The adjectival occurrence of the numeral '4' in the first sentence can be rendered by means of numerically definite

quantifiers along the lines suggested in *Grundlagen*, whereas the cardinality operator in the second sentence refers to a class of second-level concepts. Here we surely do not have synonymous sentences. However, if we read Frege's claim of synonymy in the context of §§57–60 and consider the way in which at this stage of the book he hedges his plea for the self-subsistence of number, a weaker interpretation in terms of philosophical elucidation is perhaps more to the point. At the close of §60 he writes:

The self-subsistence which I am claiming for number is not to be taken to mean that a number-word signifies something when removed from the context of a proposition, but only to preclude the use of such words as predicates or attributes, which appreciably [*etwas*] alters their meaning.

To my ear this statement sounds like a covert criticism of a statement made by Lotze in §7 of chapter 1 of his *Logik*. In classing number-words as adjectives, grammar—Lotze says—is true to the metaphysical status of numbers, for 'logically they doubtlessly belong to them, if one bears in mind that logically the adjectival form is appropriate to every not self-subsistent determination of a content, and not only to those determinations which attach to it in the sense of a property'. Frege, who probably shared Lotze's opinion that the logical sense of the parts of speech is a shadow of metaphysical concepts, wanted to make sure that we get at least the shadow right: the shadow cast by number-words is unmistakably substantial, as an unassuming grammatical transformation suffices to show.

By the same token, if the sentence 'The concept *square root of 4* is realized' is viewed as conveying more perspicuously than the sentence 'There is a square root of 4' the insight that we are predicating something of a concept, the two sentences should count as synonymous. On the other hand, under the interpretation actually offered by Frege, according to which the expression 'The concept *F*' functions as a name of an object of a very special kind, the sentences in question should not be viewed as conveying the same thought, while differing only in logical form and linguistic clothing. Frege's parallel between the pair of sentences mentioned above and the pair of sentences 'There is a square root of 4' and 'The number 4 has the property that there is something of which it is the square root' is misleading. The sentences belonging to the first pair differ among each other not only in logical form and ontological commitment, but, according to Frege's own analysis, in the very thought they purport to convey. For it is his chief claim that by using the phrase 'the concept *F*' we mean to refer to a concept, but *grammar* forces our words into the mould of singular terms. There is, however, no reason why we should acquiesce in the despotism of grammar, if in a particular instance it provably misrepresents thought.

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Michael Dummett's Interpretation of Frege's Context Principle

Some Reflections

In discussing Frege's context principle Michael Dummett often employed the contrast between a thin and a robust notion of reference, on the one hand, and between an idle and an operative conception of that notion, on the other. Is the contrast between *idle* and *operative* only a different formulation of the contrast between *thin* and *robust*? I seem to detect a difference. My positive suggestion is that only a notion of reference governed by the principle of compositionality counts as robust and can satisfy GGA's demands. As far as GLA is concerned my proposal is that we adopt the compositional reading of the context principle suggested by Dummett in chapter 16 of *Frege: Philosophy of Mathematics*. Under Dummett's interpretation of it, the principle suggests that a language be governed by a relation of relative priority and asymmetric dependence among the concepts it deploys. This shift of emphasis may help us in placing the notion of semantic value in sharper focus, and possibly disclose an *operative* construal of reference which is not hostage to the thin vs. robust divide.

'The Context Principle: Centre of Frege's Philosophy' is the title of the address delivered by Michael Dummett at the Frege Colloquium which took place in Jena in 1993. The title leaves little room for doubt as to the conclusion he had reached in 1993: the *Grundgedanke* encapsulated in the context principle informs the whole of Frege's philosophy, from *Die Grundlagen der Arithmetik* (= GLA) of 1884—where the principle is stated for the first and only time—to the *Grundgesetze der Arithmetik* (= GGA). The first volume of GGA was published in 1893, shortly after the publication of the seminal essays 'Funktion und Begriff' (1891), 'Über Sinn und Bedeutung' (1892 = SuB), and 'Über Begriff und Gegenstand' (1892). In these essays Frege offers a philosophical account of the new technical notions by means of which he had revised and extended his *Begriffsschrift* (= BS) of 1879.

The second volume of GGA was published in 1903, accompanied by an Afterword, where Frege tries, unsuccessfully, to circumscribe the disruptive effects brought about on his foundational programme by Russell's discovery that a contradiction was derivable in the language of GGA. The blame was put on Axiom V, which connects concepts to their value-ranges, by way of providing a criterion of identity for value-ranges in terms of the equivalence of the concepts of

which they are the extension. Already in 1885, in his review of *GLA*, Cantor had warned Frege against an unqualified use of the notion of *Umfang*, possibly a residue of old-fashioned logic books. In his reply, Frege (1885) rectifies Cantor's misunderstanding of the explicit definition of number presented in *GLA*, which involves the *Umfang* of a second-level concept, i.e. 'equinumerous to the concept *F*', and not the *Umfang* of a first-level concept *F*. He also reminds his unsympathetic reviewer that little hinges on that notion, and backs his statement with a reference to the notorious footnote appended to §68 of *GLA*, where we are told that a concept is logically prior to its *Umfang*, and that everybody knows what the *Umfang* of a concept is! On the other hand, value-ranges (the heirs to *Umfänge*) play a crucial role in *GGA*, and yet they remain as mysterious as ever.

In §29 of *GGA* I Frege stipulates under what conditions an expression of *GGA* counts as referential, and in §31 he attempts to show that all the expressions that can be constructed in *GGA* by means of the eight primitive functions are, in fact, referential. The case of names of value-ranges has to be handled with special care, since the abstraction operator introduces new objects in the domain, in addition to the two truth-values, whose significance (both philosophical and logical) is taken by Frege as already understood by anyone who is familiar with the practice of making judgements (*SuB*, p. 34, *KS*, p. 149). Had this proof been successful, names of value-ranges would have been admissible as featuring in the formulation of the logical axioms and definitions of *GGA*. Can we construe those sections of *GGA* I as informed by a generalized context principle? Interpreters of Frege's philosophy divide over this issue: for example, Resnik (1976) denies that this is the case, while at the same time providing an insightful account of the many good ideas displayed by Frege in the proof of referentiality offered in those sections of *GGA* I. In 1995, Dummett's considered answer is in the affirmative. However, the use of the generalized context principle, as it stands, is viciously circular, and Dummett's positive suggestion is that a reformed context principle should ban indefinitely extensible concepts and find the means for circumscribing in a non-circular way the domain of quantification:

The stipulations governing the primitive functors . . . could be determinate only if the domain, consisting wholly or largely of value-ranges, was determinate; but the domain was in the process of being determined by fixing the *Bedeutungen* of the value-range terms, and so the procedure went round in a circle.

(Dummett (1995), p. 18)

Unsurprisingly perhaps, Dummett's paper ends on a note of caution, indeed agnosticism:

The realist interpretation could be jettisoned without abandoning the context principle itself, but only if that principle, as here understood, can be shown to be

coherent; and this remains in grave doubt, And yet, it is hard to see how it can be abandoned, so strong is the motivation for it. The alternative is an apprehension of objects, including abstract objects, underlying, but anterior to, an understanding of reference to them, or, indeed, a grasp of thoughts about them: and this is a form of external realism too coarse to be entertained. I am therefore forced to conclude without either endorsing the central feature of Frege's philosophy or rejecting it; I can do no more than to say lamely that the issue is one of whose resolution is of prime importance to philosophy. (Dummett (1995), p. 19)

One is left with the impression that even if a practicable reformulation of the context principle were to become available, together with a non-circular way of circumscribing the domain of the quantifiers, Frege's claim to have secured the existence of numbers by purely logical means would not be vindicated. What stands in the way of such vindication is that only a *robust* notion of the reference could secure this result:

To justify a realist interpretation of a given theory, even if it be an internal version of realism, it is not enough that we should be entitled to ascribe a reference to the terms of that theory; the notion of reference, as so ascribed, must be sufficiently robust to bear the weight of a realist interpretation. This surely requires that the notion of reference should play a genuine role within the semantic theory; that is, that determining the reference of a term should be a step in determining the truth-value of a sentence containing it. But, if the referentiality of the term was justified by appeal to the context principle, its reference is semantically idle, since it was secured by specifying the truth-value of sentences of which the term is part, or the reference of more complex terms of which the term is part, by some means not involving the identification of the referent. (Dummett (1995), p. 18)

Here, I have touched on a host of difficult issues. What I would like to explore in what follows is the contrast between a thin and a robust notion of reference, on the one hand, and between an idle and an operative conception of that notion, on the other. Is the contrast between *idle* and *operative* only a different formulation of the contrast between *thin* and *robust*? I seem to detect a difference. And even if it is not there, I suggest that we make it up.

My positive suggestion is that only a notion of reference governed by the principle of compositionality counts as robust, and can satisfy GGA's demands. The principle of compositionality can be construed in many different ways (cf. e.g. Linnebo 2009). Linnebo's construal, for all I can judge, is appropriate to GGA, where the guidelines of a compositional semantics are laid down. As far as GLA is concerned, however, things are not so clear-cut. My proposal is that we adopt the compositional reading of the context principle suggested by Dummett in chapter 16 of *Frege: Philosophy of Mathematics* (= FPM), and discussed more

fully in chapter 10 of *The Logical Basis of Metaphysics* (= *LBM*). The principle of compositionality, under Dummett's interpretation of it, suggests that a language be governed, both in the order of acquisition and in the order of explanation, by a relation of relative priority and asymmetric dependence among the concepts it deploys. Semantic molecularism can be seen as governed by this principle, while, at the same time, allowing for local holisms, i.e. for sets of notions that can only be explained together. I suggest that from this vantage point we survey the collection of sentences expressing statements of number discussed by Frege in *GLA*. This shift of emphasis may help us in placing the notion of semantic value in sharper focus, and possibly disclose an *operative* construal of reference which is not hostage to the thin vs. robust divide. Whether a similar strategy can be applied in the setting of *GGA* I don't know, but I deem it unlikely. *GLA*, however, seems definitely hospitable to it.

Semantic Value and Reference

As I said, in many of his writings Dummett employs a distinction between a *thin* and a *robust* conception of *reference*, on the one hand, and an *idle* vs. an *operative* conception, on the other. These distinctions feature also in Dummett (1995), where, in addition, a parallel is drawn between a metaphysical conception of reference and a conception of reference *internal* to language, in the spirit of the internal realism put forward by Hilary Putnam in *Reason, Truth and History*. Dummett now argues that Frege's conception of reference is internal to language both in *GLA* and in *GGA*, but for different reasons, having to do with the lack of a distinction between object language and metalanguage in *GLA*, which makes it impossible to ask questions of reference in the formal mode, and with its presence in *GGA*, according to a very subtle construal of that distinction offered by Dummett (1995, p. 16).

I do not find the parallel with Putnam's Internal Realism particularly helpful, and as it stands the contrast between a thin and a robust conception of reference in the case of number words also appears elusive. Indeed, if, following the suggestion put forward by Dummett in *The Interpretation of Frege's Philosophy*, we construe Frege's notion of *Bedeutung* as semantic value, and ascribe a semantic value to incomplete expressions, there seems to be little point in denying a semantic value also to names of abstract objects. The ascription of semantic value to incomplete expressions is one of the main changes Dummett's interpretation of Frege underwent since the publication of *Frege: Philosophy of Language* in 1973. And it is, as I argued elsewhere (Picardi (2008)), a change for the better. Nor does Dummett wish to deny reference to number words, for he is out of sympathy with nominalistic strictures. But he is convinced that by appeal to the context principle only a

thin notion of reference is available, a notion too weak to satisfy the requirements laid down in *GGA*.

As I said, the discussion of this issue in Dummett (1995) is made more elusive by his appeal to the notion of reference featuring in Putnam's internal realism, as opposed to the notion employed by metaphysical realism. The contrast between the internal vs. metaphysical distinction as applied to reference must be kept apart from a similar distinction which Dummett draws as regards the role of the metalanguage in *GGA* with respect to the object language. He suggests that in *GGA* Frege was trying to explain in the metalanguage what the formulae of the object language were to mean, not from without, but from within the object language itself (Dummett (1995), p. 16).

Frege's notion of *Bedeutung* does not play just the role of a theoretical tool useful for laying down in the metalanguage the semantics of a formalized language such as *GGA*. The notion of *Bedeutung* is supposed to account for our actual grasp of the thoughts conveyed by the sentences of our language. It has a dual role. Our understanding of the thought expressed by a sentence requires that we have a conception of what it is for a given name to designate a certain object, what it is for a certain predicate expression to pick out a property of the objects that fall under a given concept: for the objects falling under a concept have as properties those indicated by the characteristic marks that make up the sense of the linguistic expression used to specify the concept in question. By introducing the notion of functional application Frege thought that he could avoid getting involved in the issue of whether and how, exactly, universals relate to particulars, how properties attach (inherence, instantiation, being part of) to the objects that fall under a given concept: what matters is that we, in judgement, can map objects onto truth-values. And this is why the notions of truth and assertion play a paramount role in Frege's epistemology. Anyone who performs an *act* of judgement must implicitly recognize the existence of two logical objects, the True and the False (*SuB*, p. 34). Frege certainly had no qualms in pleading the cause of what is *objektiv unwirklich*. Concepts are reason's proper study. Hopefully, many of our concepts capture objective features of the world. But I think that Frege would have emphatically resisted the suggestion that our grasp of concepts is *causally* dependent on the world's being a certain way, and if metaphysical realism entails that, he most certainly would have been out of sympathy with this brand of metaphysical realism. However, being out of sympathy with metaphysical realism does not turn Frege into an internal realist à la Putman: the opposition between metaphysical and internal conceptions of reference simply does not apply to Frege's notion of *Bedeutung*.

Are there candidates other than reference that can play the role of semantic value in the case of logical objects? A possible candidate could be the inferentialist account put forward by Brandom. As is well known, Brandom's hero is the author of the *Begriffsschrift*, for it is in this book that he detects the seeds of an expressivist

conception of logic in harmony with his inferentialist conception of semantic value. Perhaps it is not by chance that Frege's masterpiece of 1884 sits uncomfortably in Brandom's characterization. For, on the one hand, *GLA* is arguably the work in which the notion of content is given the richest inferential treatment, but, on the other hand, it is undeniable that making explicit the inferential potential enshrined in the concepts of arithmetic as a plant in a seed is conceived by Frege as a step in the process of establishing his claim that numbers are objects—a distinctly objectivist claim. Since in his paper 'Frege's Technical Concepts' Brandom (1986) disagrees with Hans Sluga concerning a purely non-relational reading of the referential idiom employed by Frege, something needs to be said concerning the way in which *GLA* fits in with the inferentialist framework. One may confine oneself to Frege One, the author of *BS*, and neglect Frege Two, the champion of compositional semantics. But this seems to me an absurd mutilation, for, as I have said, *GLA* offers the best examples of ways of extracting the inferential potential enshrined in certain concepts by way of a reconceptualization of the contents of sentences in which the relevant concept-words occur. Brandom believes that Frege's attempt to establish that numbers are (logical) objects is doomed to failure because of the high standards of individuation he sets on abstract objects. But this is a different issue altogether, an issue moreover that has been discussed intensely during the last thirty years or so, thanks to the revival of Neo-Fregeanism promoted by Crispin Wright (1983), and owing also to the results of George Boolos (1996), Wright himself, and many others. Unlike Dummett, I am not sceptical about the philosophical significance of principles of abstraction, though I am well aware that they can be very tricky, as the cases of Axiom V and of Boolos's (1998, p. 181) anti-zero demonstrate. This is the 'bad company objection' levelled by Dummett (1998) against abstraction principles.

But there are also proposals for reforming neo-logicism coming from a constructivist standpoint. For instance, there is Tennant's suggestion that we construe the crucial equivalences discussed by Frege in *GLA* as introduction and elimination rules à la Gentzen (cf. Tennant (2013))—a suggestion which I find very promising. The semantic value of a term would be given by its introduction and elimination rules, possibly in a framework of free logic. Here, however, I would like to remain as close as possible to a 'relational' construal of *Bedeutung*, for, unlike Dummett (1995), I believe that in 1884 Frege had already moved to a conception of identity as governed by Leibniz's law (cf. *GLA* §65), and very different from the one to be found in §8 of *BS*. As a matter of fact, the sign for sameness of conceptual content employed in *BS* is never used again after 1879. In *GLA* Frege seems to take it for granted from the very beginning that proper names and number words flanking an identity sign stand for objects. As far as sentences and predicate expressions are concerned, on the other hand, I fully concur with Dummett (1995) when he says that in *GLA* Frege deploys only a generic conception of the significance of an expression.

To be sure there are interpreters of Frege's philosophy, such as Joan Weiner, Thomas Ricketts, Warren Goldfarb, and many others, who deny that Frege's work contains the guidelines of a semantics for a formal (as well as) natural language of some kind. Ricketts (2012) in particular interprets the context principle as a version of the segmentation principle put forward in *BS*, and, as such, belonging to syntax (for an extensive discussion of this issue cf. section 4.5 of Heck (2012)). I will presently say something about this principle, but a full discussion of the tenets of what, after the model of 'the New Wittgenstein', might be called 'the New Frege' must be postponed to a different occasion.

Interlude: Some Tenets of Frege's Conception of Logic and Language

Before examining what it is for a notion of *Bedeutung* to play an operative role in semantics I would like to recall three tenets of Frege's conception of logic and language that Dummett (1981) has helped us to disentangle: (a) the context principle, formulated with great emphasis in *GLA*, but at work in an intricate way also in *GGA*; (b) the principle of compositionality which, after 1890, Frege applied at the level of both sense and reference, and which informs his conception of thought and meaning from 1890 onwards; and (c) the principle of extraction of a (complex) predicate from a sentence under a certain conception [*Auffassung*] of its content. We encounter principle (c) for the first time in §9 of *BS*; it puts in an appearance in §70 of *GLA*; and, with considerable shifts of emphasis, persists throughout Frege's philosophical development. Indeed, in retrospect, Frege came to consider it as THE distinctive feature of his conception of logic. It may be a good idea to spell out these principles which in the recent literature on the context principle often get confused or assimilated. So, let us briefly recall Frege's formulations of the extraction principle (c), the context principle (a), and the principle of compositionality (b), beginning with the last one. Frege's 'Third Logical Investigation' opens with the following observation:

It is astonishing what language can do. With a few syllables it can express an incalculable multitude of thoughts... This would be impossible if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure [*Aufbau*] of the sentence can serve as a picture of the structure of the thought. (Frege (1923), p. 36, (1984), p. 390)

To be sure, the application of the part-whole picture to thoughts is metaphorical. Strictly speaking, it is only sentences that have parts. A key word in the quoted passage is 'structure'. For, as Frege goes on to remark, 'the question now arises how a thought comes to be constructed, and how the parts are so combined

together that the whole amounts to more than the parts taken separately'. To answer this question Frege appeals to the contrast between saturated and unsaturated parts of a thought, where saturation and combination are not meant as temporal processes. 'It is natural', he goes on to say, 'to suppose for logic in general that combination into a whole always comes about by saturation of something else' (Frege (1923), p. 136, (1984), p. 390). This is, in a nutshell, Frege's answer to the ancient problem (or an aspect of the problem) of the unity of a proposition (or thought). It rests squarely on his insight into the *functional* character of concepts. And concepts are (among other things) the predicates of possible *judgements*, possibly *rules* for discerning and grouping individuals. And *judgement*, as Frege clearly saw, relates to *truth*, and truth relates to *assertion* and *justification*. Possibly, every assertion contains a predication (i.e. its content is amenable to the function-argument analysis), but predication and assertion must be kept apart: when a sentence occurs as the antecedent of a conditional or in the formulation of questions or in the laying down of definitions, we are dealing with instances of predication, but not of assertion. At the level of reference principle (c) asserts that the semantic value of a sentence or a complex expression depends only on the semantic values of its components and the way they are put together. The principle of compositionality forms the basis of (almost) all varieties of model-theoretic semantics.

Also the principle of extraction of a complex predicate from a sentence (what Ricketts (2012) calls the principle of 'logical segmentation') is involved in the issues of predication and of the unity of the proposition, which I for my part consider two separate issues. In one of its latest formulations this principle reads:

What is distinctive about my conception of logic is that I begin by giving pride of place to the content of the words 'true', and then immediately go on to introduce a thought as that to which the question 'Is it true?' is in principle applicable. So I do not begin with concepts and put them together to form a thought or judgement. I come by the parts of a thought by analysing the thought. This marks off my concept-script from a similar invention of Leibniz and his successors, despite what the name suggests; perhaps it was not a happy choice on my part. (Frege (1969) 1979, p. 253)

This is by no means an *ex post* 'rational reconstruction' of the spirit that had informed his logical work. In an unpublished essay, known as the 'Anti-Boole', young Frege, after having given an example of what he means by the several 'decompositions' to which the same equation, e.g. $2^4 = 16$, is amenable, goes on to say: 'Instead of putting a judgement together out of an individual as a subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgement' (Frege (1969) 1979, p. 17). This line of reasoning is present also in 'Über Begriff und Gegenstand',

marred, however, by the so-called paradox of the concept *horse* (cf. Picardi (2008)).

Frege's logic is from the very beginning conceived as a symbolic language suited to represent the structure of *sentences* which express *judgeable contents*. It carries with it from the very beginning an essential appeal to the notion of truth. In §9 of *BS* Frege's goal is to show that the argument–function distinction is to be preferred to the traditional analysis in terms of subject and predicate. In dropping one or more occurrences of the same proper name from a sentence such as 'Cato killed Cato', we highlight the argument-places of a function where substitutions can be effected. In so doing we extract different concepts from the original sentence (e.g. 'c killed x', 'x killed c', 'x killed x'). To the isolation of the characteristic pattern discernible in an array of sentences Frege attributed the utmost importance. The priority of the sentence over its constituents and components plays a crucial role also in Frege's epistemology. Once Frege had fully worked out the programmatic distinction between function and argument outlined in *BS*, he was able to give a full account of the semantic contribution of concept-words to the truth-conditions of sentences in which they occur in a predicative role. The extraction principle helps to highlight the different decompositions into function and argument to which a sentence gives rise. After 1890 Frege simply took it for granted that the different complex predicates resulting from different decompositions stand for concepts, relations, functions. These unsaturated entities form the domain over which second-order quantifiers range.

Although the principle (a) is stated by Frege as a methodological maxim which, together with the distinctions between concept and object, as well as between the logical and the psychological, should guide the enquiry into the concept of number, its significance is by no means confined to the philosophy of arithmetic. According to that principle, as stated in *GLA*, a content can be secured to numerical terms by securing a sense for sentences containing these terms. Of the three principles mentioned above, only the context principle has a clear ontological significance. By paying attention to the logical form of statements of number we should be led to the realization that numbers are independent logical objects.

The major changes with respect to the *BS* of 1879, apart from the sense–reference distinction and the new interpretation of identity, is that the notion of sentential *Inhalt* bifurcates into the thought expressed and its truth-value. A sentence is now conceived as a complex proper name of one of the two truth-values. The rules of substitution *salva veritate* licensed by Leibniz's law of identity of indiscernibles are applicable to sentences as well. In §65 of *GLA* Frege says that Leibniz's law offers a definition of identity, and that the rules of substitution *salva veritate* are constitutive of its very meaning—a bold statement indeed, and an easily falsifiable one in non-extensional contexts. In my opinion the appeal to Leibniz's law shows that, as far as singular terms are concerned, the distinction

between *Sinn* and *Bedeutung* is already operative in *GLA*, possibly in an inchoate form, together with the new interpretation of identity, very similar to the one which he was to employ after 1890.

To sum up: what (a), (b), and (c) have in common is the priority allotted to a sentence over its component words in the order of explanation. Only the context principle has also an ontological significance. The assimilation of sentences to names of truth-values seems to make the context principle redundant. But it still lingers in *GGA*, for what matters is the close connection between principles (a) and (b) and the concepts of truth and reference. For also sentences construed as complex names of truth-values express thoughts, and in *GGA* I § 32 we are told that the sense of a sentence is the thought that its truth-conditions are realized; each component part contributes via its sense to the thought that is being expressed.

Compositionality and the Context Principle

At first sight, the principle of compositionality suggests a different order of explanation, leading from the parts to the whole. However, I believe that it can be reconciled with (a) and (c), provided that by 'parts' we understand the *logically significant units* discerned in a sentence as a consequence of having applied either principles (a) or (c), or both. The parts thus discerned can, indeed must, occur as *independent* components of new sentences and contribute to the truth-value of new sentences via their semantic value.

Both at the time of *GLA* and, with much greater emphasis, at the time of *GGA* Frege strove for a construal of number words as semantic units that could be *detached* from the sentential contexts in which they occur and enter as components into new sentences. The context principle is of paramount importance in discerning the role that number words play in mixed statements of number and in equations, which he construes as identity statements.

Had the stipulations concerning value-ranges been successful in securing a *Bedeutung* to names of value-ranges, we would have earned an entitlement to use these names as standing for independent logical objects. Above all, we could have used them in the formulation of basic laws such as Axiom V. Moreover, in this way it would have been established indirectly that no proof would be needed for confirming their referential status in the case of extending the language by adding new functions. In short, nothing would have stood in the way of applying the principle of compositionality which, if my interpretation of Dummett's conception of a robust notion of reference is on the right track, would have shown how courses of values and number words *contribute via their reference* to the truth conditions of the sentences which can be constructed in the formal language. It

seems to me that Dummett in fact endorses Frege's diagnosis of what amending his conception of a logical object along Russellian lines would have entailed.

This line of argument is fully explicit in the Afterword to *GGA II*. To appreciate the connection between the context principle and the principle of compositionality let us pause to look at the following statement. In his *Nachwort* Frege writes (p. 255):

...presumably what is left is to consider class names as pseudo-proper names which would have in truth no reference. *They would have to be viewed only as parts of a sign which would have a reference [Bedeutung] only as a whole* [footnote 1: cf. §29, vol. I]. It may of course be considered advantageous for certain purposes to form distinct signs that agree in some part without thereby rendering them composite. The simplicity of a sign demands no more than that distinguishable parts in it do not autonomously have a reference. In this case, even if what we are accustomed to regard as a number-sign would not be in fact a sign. An explanation of the sign '2' would be impossible; instead, one would have to explain many signs which contain '2' as a non-autonomous [*unselbständig*] component, but are not to be thought as composed logically out of '2' and another part. It would be inadmissible to allow a letter to stand in for such a non-autonomous part; for as far as the content is concerned there would be no composition at all. The generality of arithmetical propositions would be lost. It would be also not be comprehensible how one could talk of the cardinal number of a class, of the cardinal number of cardinal numbers.

(*GGA II*, p. 255, trans. 2013, p. 255, emphasis added)

I interpret the above-quoted passage as follows. At the beginning of the Afterword we are reminded that an object is what can be the argument of a first-level function. Value-ranges too are suited for that role, on a par with individual objects (under a generous construal of this notion). Their ontological independence is attested by the circumstance that number words are suited to occupy a position where individual variables ('letters' in Frege's terminology) can occur and be bound by a quantifier. 'To be is to be the value of a (bound) variable' is one of the two famous dicta formulated by Quine in 'On What There Is'. The other dictum is 'No entity without identity': as regards the latter, it is Quine himself who, in 'Identity, Ostension and Hypostasis', recognizes his indebtedness to Frege; as regards the former dictum, you have Frege's own words as well as my interpretation of them. It is on individual variables bound by quantifiers that the ontological status of number words rests. A substitutional interpretation of quantification would be of no avail.

In *GLA* Frege had appealed to the grammatical distinction between singular and plural and to the function of the definite article, which, when placed in front of a number word, allows one to speak of 'the 1', 'the 2', etc., thus paving the way

for an understanding of identity as a relation between objects, and not between conceptual contents or conventional signs. A similar complaint was voiced at the close of §56 of *GLA*, where, after having explained how to render number ascriptions by means of (what today we call) numerically definite quantifiers, he points out that it is an illusion to have defined the number 0 or the number 1 once we have bestowed a sense to phrases such as 'the number 0 belongs to', 'the number 2 belongs to', etc., for here the number words occur only as components of larger sentences: 'We have no authority to pick out the 0 and the 1 here as self-subsistent objects that can be recognized as the same again' (trans. Austin, p. 68).

That some version of the context principle is operative in *GGA I* is confirmed also by the reference given in footnote 1 of Frege's Afterword to *GGA II*. Here, Frege refers to §29, which is reminiscent of the version the context principle put forward in *GLA* §60. Apparently Frege regards this section of *GGA I* as failing to secure an independent *Bedeutung* for class names for they would have to be regarded 'als Teile von Zeichen, die nur als Ganze eine Bedeutung hätten'. Here one is obviously reminded of *GLA* §60. In one of its formulations the context principle reads as follows:

But we ought always to keep before our eyes a complete proposition. Only in a proposition have the words really a meaning. ... *It is enough if the proposition taken as a whole has a sense; it is this that confers on its parts also their content.*
(*GLA* §60, trans. Austin, emphasis added)

Here we are told that 'Es genügt, wenn der Satz als Ganzes einen Sinn hat; dadurch erhalten auch die Theile ihren Inhalt.' This section suggests that if a sentence, considered as a whole, is endowed with sense, this should be sufficient for its component parts to receive a *content* as well. Should the word 'Inhalt' be taken here as alluding to what was later to become 'Bedeutung', or as synonymous with 'sense'? As Dummett (1991a) points out, in *GLA* Frege is working with the generic notion of the significance of an expression, and the use of the notion of 'Inhalt', as Frege himself pointed out, fails to draw a sharp line between what after 1891 was allotted either to *Sinn* or to *Bedeutung*. In 1903, on the other hand, there is no ambiguity left, and Frege's complaint is that under Russell's suggested reading number words would not receive a *reference* (*Bedeutung*) as detachable parts of sentences, and for Frege this amounts to no reference at all.

Should we come to the conclusion that in *GLA* Frege was operating with a thin notion of reference (whatever that is)? There seems to be little evidence for such a surmise. To be sure, at the beginning of section 60 of *GLA* we read:

The self-subsistence which I am claiming for number is not to be taken to mean that a number word signifies something when removed from the context of a

proposition, but only to preclude the use of such words as predicates or attributes, which appreciably alters their meaning.

It would be a mistake, I believe, to read this observation as suggesting that in *GLA* Frege was aiming at a holophrastic reading of sentences containing number words, say, in a Quinean spirit. What Frege is alluding to at this point is the contrast between an adjectival and a substantival use of number words in mixed statements of number. As a matter of fact, however, also in *GLA* Frege is eager to show that number words should be construed as detachable and recognizable parts of the sentences in whose context they occur. Failing that, they would not be available as flanking the identity sign and as being subject to Leibniz's law. What makes a decisive difference between *GLA* and *GGA*—as Dummett never tires of pointing out—is that by 1893 Frege had outlined a semantics for his formal language, whereas not a vestige of a semantic theory can be obtained from *GLA* (cf. also Heck (2007)).

If, following Dummett's suggestion, we construe the context principle in the form deployed in *GLA* as a principle concerning the reference, and not only the sense, of number words, the question arises as to its exact significance.

I have alluded to Quine's holophrastic construal of the utterances of the natives we come across in chapter 2 of *Word and Object*, because Dummett's contrast between a thin and a robust notion of reference may be interpreted in a way reminiscent of Quine's and Davidson's conception of reference. One can be a realist in the sense of Quine while at the same time allotting to the notion of reference only a subsidiary role—as the theses of the inscrutability of reference and of ontological relativity make clear. Semantic realism is fully compatible with a thin notion of reference, whereas metaphysical realism is not. However, according to Quine, an ontological commitment to abstract entities is unavoidable when it comes to number theory, and is justified by its entrenchment in the fabric of science. In 'Reality without Reference' Davidson argues that the notion of reference helps only to provide a smoother application of Tarski's Convention T in constructing an empirical theory of (radical) interpretation for sentences belonging to a natural language. He, too, however, has no qualms about accepting an ontology of events, should they prove useful in uncovering the logical form of action sentences. Why should a neo-logician be reluctant to treat numbers as objects if they can be purchased at so cheap a price? Because the ambition of neo-logicism is to offer an a priori justification of the logical laws and definitions, which, together with the conceptual truth delivered by principles of abstraction, warrants the ascription of reference to number words. The justification in question is not only a priori, but it should also qualify as analytic.

It seems unlikely, however, that Dummett (1995) has either Quine or Davidson in mind in the above-quoted passage, even though they both satisfy the description of philosophers who, while subscribing to realism, travel light as regards the

notion of reference. Possibly he is alluding to his own account in *FPM* where in chapter 15 he argues, both against radical nominalists, such as Hartry Field, and moderate Platonists, such as Crispin Wright and Bob Hale, that a tolerant reductionist should try to occupy a middle ground between these two extremes. Dummett suggests that we abandon nominalistic scruples, and take singular terms at face value, as purporting to refer to abstract objects, provided that they occur in an articulated corpus of true statements. However, in his opinion we should refrain from endowing such terms with robust reference. To a tolerant reductionist only a thin notion of reference is available, nor would he ask for more. Possibly a weaker (and in this sense 'thin') notion of reference is one that does not permit the elimination of the defined term—the numerical operator—by means of the definiens in all sentential contexts where it occurs. This weaker notion of reference might have saved Frege's foundational programme at the time of *GLA*, along the lines suggested by Wright (1983). Bearing in mind Frege's insistence that the concept is logically prior to its extension, and considering also his claim in *GLA* that the notion of extension is possibly dispensable, would he not have rejoiced, in retrospect, to be presented with such a brilliant result? Unlike Ruffino's (2003), I believe that he would.

In *GGA*, however, Frege's conception of semantics requires that reference be governed by the principle of compositionality. And this is, I surmise, what turns semantic value into robust reference. My conjecture is that only a semantics governed by the principle of compositionality can account for the robust notion of reference required to meet Frege's requirements (and Dummett's requirements on Frege's behalf).

What, then, is the role of the context principle, if, by Frege's own lights, its employment never secures the desired result? My surmise is that the context principle has a dialectical function: in *GLA* it serves to show that whenever a number word occurs in a statement of number its role is unmistakably that of a singular term purporting to stand for an object. The explicit definition of the numerical operator offered in §68 of *GLA* would hang in the air had we not been guided step by step, in §§46–62, to regard number words as behaving much like ordinary proper names, whose use is guided by criteria of identity and diversity.

The Context Principle in *Grundlagen*

According to Dummett, it is in *GLA* that for the first time in the history of philosophy an epistemological question, with an ontological question standing behind it, is transposed into a linguistic key. The linguistic turn was taken in *GLA* (Dummett (1993), ch. 2). In his account, Dummett draws attention to the Kantian flavour of the question Frege asks at the beginning of §62, 'How, then, are

numbers given to us, if we cannot have any ideas or intuitions of them?' Frege's answer is:

Since it is only in the context of a proposition that words have any meaning, our problem becomes this: To define the sense of a proposition in which a number word occurs. That, obviously, leaves us still a very wide choice. But we have already settled that number words are to be understood as standing for self-subsistent objects. And that is enough to give us a class of propositions that must have a sense, namely those which express our recognition of a number as the same again. If we are to use the symbol *a* to signify an object, we must have a criterion for deciding in all cases whether *b* is the same as *a*, even if it is not always in our power to apply this criterion.

The expression 'the same *F*', where '*F*' is a sortal concept, should be accompanied by a criterion of identity that enables us to recognize an object on various occasions and distinguish it from other objects belonging to the same sort. Frege's appeal to judgements of recognitions deployed in a logical setting reminds one of the recognitional abilities relied on in perception—a topic which was very near the heart of the psychologically minded authors of those logic books which Frege so vehemently criticized. But he admitted that a psychological investigation may have a point if the object of the investigation is to uncover causal laws or regularities that enable human beings to grasp a variety of thoughts. Logic, however, is concerned with a priori grounds of justification, not with empirical circumstances of learning and reasoning. In the epistemology of logic the importance of a sortal concept consists in this: that in order to understand the use of, say, a personal name, one has to know that there is such a thing as identifying its bearer. In the case of number, Frege's criterion appeals to the existence of a function which correlates one to one the objects that fall under *F* and the objects that fall under *G*. In §§ 63 of *GLA* Frege puts forward what in Dummett (1991a) calls 'the Original Equivalence', namely "The number of *F*s equals the number of *G*s iff there is a function which correlates one to one the objects that fall under *F* and the objects that fall under *G*.' We are confident that such a criterion is applicable, even if it is not always in *our* power to apply it. Most predicates have conditions of application, but perhaps only those that allude to substances or to countable collections have criteria of identity. A sortal predicate is one to which number words featuring in adjectival position can be attached. The concept of number itself is a sortal concept, since we can always decide issues of identity and diversity when number words flank the identity sign. Possibly, it is a sortal concept lacking clear-cut conditions of application when names of individuals that are neither names of the form 'the number of *F*s' nor previously defined numerals flank the identity sign—'Julius Caesar' may be a case in point (cf. MacBride (2003)).

But is talk of judgements of recognition at all tenable in the case of numbers and logical objects? We start from the paradigmatic case of identifying reference to physical bodies, which involves, as a rule, a demonstrative ingredient ('That chair', 'This shade of blue'). Possession of a criterion of identity enables us to reidentify them on future occasions, under different modes of presentation. According to Strawson (1959), our conceptual scheme allots a fundamental place to physical bodies and persons. Can numbers (positive integers) or abstract entities such as the equator or the direction of parallel lines aspire to the full status of objects? After all, by Frege's own lights they are not entities one can encounter in space or space-time, and the notion of causality does not apply to them.

The issue between nominalists, fictionalists, and Platonists revolves, among other things, around such matters. Frege was a realist concerning logical objects all right, and he was a realist in the sense of semantic realism. Semantic realism, as construed by Dummett, amounts to the assumption that a sentence which is neither ambiguous nor vague as well as devoid of indexical features is determinately either true or false. *Tertium non datur*. Would it not have been possible for Frege to continue to endorse semantic realism as far as statements belonging to number theory are concerned without at the same time requiring full-fledged reference to logical objects? Unfortunately, Russell's contradiction shattered his conviction that the stipulations of §31 of *GGA I* provide a reference (a truth-value) for all the sentences which could be constructed in the language. Semantic realism was crushed.

Presumably, a notion of reference counts as thin if it does not come up to standards when compared to the robust notion of reference we apply to proper names of concrete objects which can be pointed at and for which criteria of identity are available. But when it comes to abstract objects, one might think that thin reference is reference enough. Frege's appeal to judgments of recognition and criteria of identity deceptively assimilates what holds good for spatio-temporal particulars and what holds good for what is *objective unwirklich*—an umbrella word under which concepts, relations, numbers, and thoughts are meant to fall. Perhaps a better explanation of what is implicit in the contrast between the thin and the robust is the one Dummett alludes to in his 'Reply to Peter Sullivan'. Here, Dummett points out that in the case of empirical statements of identity such as 'Hesperus is Phosphorus' it may well require elaborate astronomical investigations to find out whether the identity holds, but that there is no need to specify in advance the domain over which the statement in question is defined; we are confident that a determinate answer to the identity statement will be forthcoming. In the case of fundamental mathematical theories, on the other hand, a circumscription of the domain is of the essence, for we do not have 'a sufficiently definite conception of what objects belong to the domain of such theories' (Dummett (2007), p. 791). And this is why, among other things, Dummett believes that an

intuitionistic understanding of the quantifiers, and of logic in general, is preferable to a classical one.

A weaker and, in this sense, thin notion of reference along the lines suggested by neo-logicists might have saved Frege's foundational programme at the time of *GLA*. In *GGA*, however, Frege's conception of semantics required a notion of reference as governed by the principle of compositionality. Presumably, this is what turns reference into the robust notion alluded to by Dummett.

A Different Interpretation of the Principle of Compositionality

The principle of compositionality can be given many different readings. What I am interested in is Dummett's own interpretation, which in my opinion chimes well with what goes on in *GLA*. In chapter 16 of *FPM* (pp. 202–3) he interprets the principle of compositionality thus:

It is meaningless to speak of grasping the sense of an expression conceived as standing on its own, independently of any sentence in which it occurs. Its sense just is its contribution to thoughts expressed by sentences of which it is part; to regard the expression as standing on its own, independently of any sentence, is to destroy the whole conception of its possessing a sense.

The escape from this dilemma requires us to regard sentences, and the thoughts they express, as ordered by a relation of dependence: to grasp the thoughts expressed by certain sentences, it is necessary first to be able to grasp the thoughts expressed by other, simpler, ones. To grasp the sense of a given expression requires us to be able to grasp the thoughts expressed by certain sentences containing it; if it did not, we should be able to grasp the sense in isolation, contrary to the context principle. Not, however, of *all* sentences containing it, but only of certain ones: those of a particular simple form, characteristic for the expression in question.

Dummett's understanding of the principle of compositionality requires that we discern a pattern of relative dependence among sentences, and that this pattern be faithful to relations of conceptual priority among them. The molecularist semantics outlined by Dummett in previous writings stands in contrast to Quine's holistic conception of meaning and reference. Dummett's conception of justification is indebted both to Gentzen's proof-theoretic interpretation of the meaning of logical constants and to Wittgenstein's conception of understanding and language mastery.

As is well known, since the publication in 1983 of Wright's book *Frege's Conception of Numbers as Objects* the specialized literature on Frege's Theorem

has reached a certain level of complexity and intricacy. What is usually meant by 'Frege's Theorem' is the result obtained by adding to full impredicative second-order logic what Dummett calls the Original Equivalence and Wright (1983) calls Hume's Principle (cf. also Parsons (1965)). The principle is stated in §62 of *GLA* and is interpreted by Wright as a conceptual truth, which added to a second-order logic allows the derivation of Peano's axioms. This is precisely what Frege was aiming at in *GLA* when stressing that definitions prove their worth by making the derivation of new theorems possible. By means of Hume's Principle we can derive Peano's axioms without going through Frege's explicit (and contradictory) definition of number as the extension of a second-order concept. In Dummett's opinion the Original Equivalence is not an implicit definition of the numerical operator, but a test that any correct definition of the cardinality operator should pass (Dummett, *FPM*, p. 201). Unlike Dummett, Wright takes Hume's Principle as a conceptual truth supposed to underpin the legitimacy of definitions by logical abstraction. From the point of view of a molecularist semantics the advantage of Wright's proposal is that it draws attention to a single logical device, namely logical abstraction, without incurring the arrogance of an outright stipulation of Peano's axioms as providing an implicit definition of the primitive notions of the theory (cf. Hale and Wright (2007)). Besides, one may embed principles of abstraction in a predicativist framework and investigate how strong or weak the resulting arithmetic will turn out to be. But, for reasons I have been unable to fathom, Dummett nowhere, to my knowledge, considers abstraction principles in a positive light.

What I propose to do in the last section of my paper is to look at what goes on in *GLA* from a quasi-Wittgensteinian perspective and to survey the collection of sentences which may lay claim to counting as *basic* and on which a molecularist semantics governed by the compositionality principle alluded to above can be built. If we are satisfied that in these basic sentences the reference of number words plays an operative role, we do not need to repeat the test of referentiality when it comes to more complex sentences. The meaning of number words was not fixed by means of an explicit definition, but by surveying its role in those sentences that can be constructed on the ground floor, as it were.

The Grammar of Statements of Number

Elsewhere (Picardi (2010)) I have drawn a parallel between Frege's and Wittgenstein's different ways of understanding the context principle at the time of the *Tractatus* (*TLP*) and of the *Philosophical Investigations*, respectively. More specifically, I concentrated on how the context principle works in connection with proper names in identity statements. Wittgenstein's thesis that a name is a representative of an object in a proposition suggests a considerable degree of

agreement with Frege. Among other things, it brings to the fore that a name contributes via its *Bedeutung* to the truth conditions of the sentences in which it occurs.

As we have seen, in *GLA* the context principle calls attention, among other things, to the explanatory priority of a sentence over its component words. One of the main claims of *GLA* is that number words are proper names of *objects*. If by means of purely logical laws and definitions we succeed in fixing the sense of identity statements in the context of which number words occur, the prospects of logicism as a foundational programme will be greatly enhanced. Frege's agenda was very familiar to young Wittgenstein. However, the *Tractatus*, while endorsing the context principle, emphatically denies that in arithmetical equations the symbol of identity should be construed along the lines suggested by Frege (1893). In fact, propositions of mathematics do not express thoughts (*TLP* 5.21) and in this respect differ dramatically from propositions in which names occur as representatives of objects in states of affairs.

Frege had voiced the hope that scrupulous observance of the context principle may prevent us from falling prey both to ideational and to 'physical' conceptions of meaning. According to the former, the meaning of a word is a mental image; according to the latter, the meaning is the physical object a word in isolation stands for. In addition, Frege believed that the context principle may also enable us to keep clear of the many pitfalls of psychologism. One of the most serious defects of psychologism is that it stands in the way of a correct construal of the notion of *objectivity* appropriate to the contents expressed by sentences accessible to (rational) language users (*GLA*, §26). In the course of his philosophical development Frege supplemented (or replaced—depending on one's interpretation of Frege's work) the context principle with a version of the principle of compositionality. Possibly, it is the latter principle to which Wittgenstein alludes approvingly in the *Tractatus*. With explicit reference to Frege (and Russell), Wittgenstein says that he considers the meaning of sentences a function of their component parts (*TLP* 3.318), and that to understand a proposition is to know what is the case if it is true (*TLP* 4.024). The possibility of understanding *new* sentences depends on our being familiar with their component parts. There is also agreement regarding the view that it is in virtue of being logically articulated that a sentence can say something true or false about the world (*TLP* 3.141): an isolated name or a string of names cannot fulfil such a function. However, Wittgenstein's emphasis on structure and composition is only a step in the process of establishing the picture theory. He holds that in order for a proposition to be able to communicate a *situation* to us, it must be *intrinsically* connected with the situation in question. Frege had failed to show how propositions connect with the world, for he had failed to realize that a proposition is a *fact*, and wrongly compared a proposition to a composite name (*TLP* 3.143). And this is why Frege's explanation of the concept of truth is unsatisfactory. Now, if we focus on

Wittgenstein's treatment of identity statements in general and of mathematical equations in particular, the differences clearly outweigh the similarities. Even though Wittgenstein himself states, in agreement with Frege, that the object is the *Bedeutung* of a name in the context of a proposition, he holds that in the case of mathematical equations all mention of reference drops out as irrelevant. In mathematics we are interested in internal relations. Wittgenstein here invokes the idea of a series of forms. In the case of the natural numbers, it is a series generated by an internal relation corresponding to the successor relation. The theory of formal series constitutes Wittgenstein's own account of the significance of mathematical statements (cf. *TLP* 6.24–6.241). It will later develop into the view that mathematical equations are expressions of rules, and hence have no propositional status. My suggestion in the earlier paper was that Wittgenstein's *Philosophical Investigations* are more hospitable to Frege's conception of the sense of proper names and to the varieties of our name-bestowing practices. What I propose to do now is to speculate on what Wittgenstein might have liked about the strategy Frege employed in *GLA* in order to uncover the grammar of statements of number.

In order to answer the question 'What is Number?' Frege singles out a set of sentences which are integral to our understanding of this concept. Let us call them *basic sentences*. In his effort to find out about the significance of such sentences Frege sets out to elucidate the logical form and semantic content of these basic sentences. An assumption of Frege's whole approach is that arithmetical equations express identities and that the equality sign is construed as indicating the relation of identity between objects. As I have said, I believe that in *GLA* Frege had moved to an objectual construal of the identity relation, and that the distinction between *Sinn* and *Bedeutung* is implicit in the train of thought underlying his masterpiece (even though it is not made explicit). To be sure, following in the footsteps of Leibniz, Frege seems occasionally to indulge in certain use–mention ambiguities. But it should be obvious to any reader (including those who, in the face of all textual evidence, stress Frege's debt to Kant) that Leibniz is the *spiritus rector* of both *BS* and *GLA*. In this I fully concur with Tyler Burge (2000), who stresses the importance of Leibniz to Frege's conception of logic. Besides, it is to Leibniz that the first definition of number goes back. Frege improves dramatically on it, thus opposing the main tenets of Kant's conception of number (and synthetic a priori judgements). In *BS* Frege had been content to conceive of identity as a relation between signs or, rather, their contents.

In the first part of *GLA* Frege examines various proposals to analyse and possibly define the concept of number, and he discards them all as defective. Eventually, in §46, possibly the most important section of the book and, at any rate, the one he refers back to in his preface to *GGA* as encapsulating the gist of *GLA*, he puts forward his own proposal, confident that his reader will now be in a position to appreciate its superiority. It is to concepts, and not material aggregates,

that number ascriptions apply. A second-level predicate containing a numeral as a component part is ascribed to a concept of first level. Sentences in the context of which number words occur in adjectival position are of the utmost importance. Now, §46 opens with an application of the context principle which is rarely commented on:

It should throw some light on the matter to consider number in the context of a judgement [*im Zusammenhange eines Urtheils*] which brings out its basic use [*Anwendungsweise*]. While looking at one and the same external phenomenon, I can say with equal truth both 'It is a copse' and 'It is five trees', or both 'Here are four companies' and 'Here are 500 men'. Now what changes from one judgement to the other is neither any individual object, nor the whole, the agglomeration of them, but rather my terminology [*Benennung*]. But that is itself only a sign that one concept has been substituted for another. This suggests as the answer to the first of the questions left open in our last paragraph, that the *content of a statement of number is an assertion about a concept*. (GLA, §46, emphasis added)

Two words should attract our attention: *Urteil* and *Anwendungsweise*. Here, the word *Urteil* has the desired ambiguity between act and content. Kant's failure to account for the content of a such judgements indicates a lacuna in Kant's conception of the way in which we employ concepts in acts of judging. In the concluding sections of the book (§§88 and 89) Frege himself points out that Kant had failed to account both for singular propositions and for existential statements. Singular predication and higher-order predication fall outside the scope of Kant's conception of logic (cf. MacFarlane (2002)). And this is why, Frege surmises, he gave so narrow a construal of analyticity. Finally, in §89 Frege feels entitled to rebut Kant's famous thesis, alluded to in §62, that without sensibility no object can be given to us. Although the *application* of numbers (Austin's translation of *Anwendung* as 'use' is not felicitous) in counting is not part of the essence of cardinal numbers, yet a definition of number which failed to account for the most basic *application* of numbers would be seriously defective.

But let us move on—we are only at the beginning of our survey. In §57 Frege considers sentences where number words occur in substantival position, a role which he considers truer to the essence of number. The substantival use of number words has the advantage over the adjectival use that the sentences in which they occur can be phrased as identities. In chapter 9 of *FPM* Dummett regards §57 as one the weakest and most contrived sections of the whole book. To be sure, such sentences are more naturally rendered by means of numerically definite quantifiers, and as Dummett points out, this strategy can be richly implemented. Frege on the other hand, is clearly interested in the substantival occurrence of number words in such sentences, for it adds further evidence to his syntactic conjecture that number words stand for independent objects. I suggested

elsewhere (Picardi (1993)) that we may regard those sentences where number words feature in substantival position as elucidations of what Frege means by 'self-subsistent' objects, and not as a way of smuggling in existential commitments to logical objects. Otherwise we would be faced with two synonymous sentences with different ontological commitments, whereas the natural relation of dependence would lead from the more basic use—the adjectival one—to the derivative use in substantival position.

In §63 Frege introduces the Cantor-Hume criterion for the equality of number (the Original Equivalence, in Dummett's terminology), and now the way is open for a discussion of the advantages and shortcomings of the use of logical abstraction in a definition of number. By the way, it is interesting to notice that it is Frege himself who mentions the pros and cons of the method of abstraction discussed in §§38–48 of *GLA* in a short paper devoted to making fun of the version of naïve formalism endorsed by Johannes Thomae, his colleague at Jena (Frege (1906), p. 327, Engl. trans. p. 345). In the following sections Frege examines one more variety of basic sentences to the effect that there are as many knives as plates (*GLA*, §70). In §63 Frege introduces the Cantor-Hume criterion for the equality of number, and now it will be possible to discuss the advantages and shortcomings of an implicit definition of number. That number words can flank the identity sign is no guarantee that number words stand for self-subsistent entities. Such an entitlement is earned only in §69, where the explicit definition of number is laid down by having recourse to the notion of an extension of a concept of second level. In the following sections Frege tests the fruitfulness of his definition by deriving from it, together with logical tools forged in the *BS*, some important theorems concerning equinumerosity and, above all, Peano's axioms. At this stage there are no more basic sentences in view, and the contextual explanation we were given in the previous sections is superseded by an explicit definition of number. As I pointed out elsewhere (Picardi (1993)), we do not expect an explicit definition to be faithful to the significance that we have provisionally ascribed to sentences belonging to the *basic set*. Rather, it must provide the means to account for their truth-values and, in so doing, for the semantic value of the singular terms that feature in them. A definition that failed to fulfil this requirement would disqualify as an analysis of the concept of number as we employ it in everyday life. Unfortunately, the explicit definition turns out to be inconsistent, partly because of the employment of the notion of an *Umfang* of a concept. However, in its place we can employ a predicativist version of *HP* and derive from it a number of fundamental theorems (cf. Heck (1996), Linnebo (2004)). To be sure, we have no guarantee that *HP* belongs to pure logic and is, in this sense, analytic. But this is a different issue altogether.

The only comment that we find in Frege's *unpublished* papers concerning the origin of Russell's paradox is that grammar is to be blamed for seducing us into thinking that the definite article in front of a concept-word signals the existence of

a logical object, a class, for which it stands, even though nothing whatsoever corresponds to it. Possibly, those who are enthralled by the paradox of the concept *horse* may rest content with this observation. However, as a diagnosis of the origin of the contradiction it must strike everybody as very poor. As far as I know, Frege never put forward such a diagnosis in the articles he saw to the press after 1906.

Now, wouldn't this collection of basic sentences (e.g. 'There are eight dogs in the yard', 'Eight is the number of dogs in the yard', 'There are just as many dogs in the yard as planets of the solar system', 'The number of dogs in the yard equals the number of planets of the solar system'), which Frege selects in order to find out what number is, by and large coincide with the collection of sentences that someone interested in a descriptive elucidation of the *grammar* of statements of number would opt for? Once we gain a bird's-eye view of the body of sentences that fall within its scope, light dawns from above. To use Wittgenstein's terminology, this survey may be part of a grammatical enquiry into the interlocking language games in which number words are used. In this setting the notion of reference plays an operative role.

Consider a parallel. Suppose that one were to offer an account of what words for sensations mean. The natural way to proceed is to single out two classes of basic sentences: the ones in which we self-ascribe sensations and the ones in which we ascribe them to other people. There may be disagreement over the conceptual priority of sentences in the first person over sentences in the third person, much as there may be disagreement over the conceptual priority of the adjectival use of number words over their substantial use. But words for sensations, like number words, occur also in contexts which belong to neither category, and the question arises whether a uniform semantic account is possible or whether we should simply rest content with listing various ways in which words for sensations are used in our language. Sometimes we are confronted with a new application of a word or concept, and we wonder whether the meaning of a given word has been extended to cover a new case or whether it has changed altogether. But since often no sharp line exists between a new concept and a new application of an old concept, it is up to us how to describe things.

Officially, Wittgenstein refused to acknowledge that there are sentential contexts which are more basic than others and which should be singled out as such in a grammatical survey of the way we talk about sensations. But in practice this is how he proceeded. If what I have said is on the right track, does not Frege's survey of the basic sentences in whose context number words occur provide an example of a grammatical investigation which should have attracted Wittgenstein's approval? Well, as a matter of fact, I am convinced that it did, although he always rejected Frege's conception of numbers as objects.¹

¹ I would like to thank Joachim Schulte and Andrea Sereni for helpful suggestions and advice.

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PART III

FREGE'S LEGACY

Carnap Interpreter of Frege

In the years 1910, 1913, and 1914 Rudolf Carnap attended as a student three cycles of lectures given by Frege at the University of Jena. The 1910 and 1913 courses were announced under the titles *Begriffsschrift* and *Begriffsschrift II*; the title of the 1914 course was *Logik in der Mathematik*.¹ There is no evidence of correspondence between Frege and Carnap. To the request, in 1963, by the curators of Frege's scientific epistolary of any letters in his possession, Carnap replied: 'Ich habe nie mit Frege korrespondiert. Ich habe aber noch Vorlesungsnotizen, in deutscher Stenographie, System Stolze-Schrey, mit Formeln in Frege's Symbolik' ('I have never corresponded with Frege. But I still have lecture notes, in German shorthand, Stolze-Schrey system, with formulas in Frege's symbolism').² These lecture notes today are part of Carnap's *Nachlass*, kept at the University of Pittsburg. So far, to my knowledge, no scholar has undertaken the task of producing a complete transcription of Carnap's notes, which would involve grappling with the difficulties presented both by the symbolism of the Fregean *Begriffsschrift* and by the Stolze-Schrey stenographic technique employed by Carnap.³ Fortunately, regarding the content of the lectures Carnap attended in the 1914 summer semester, other sources are now available to us. In 1969 the project of publishing Frege's scientific *Nachlass*, initiated by Heinrich Scholz in 1936, was finally realized, albeit

¹ Cf. R. Carnap, 'Intellectual Autobiography', in *The Philosophy of Rudolf Carnap*, edited by P. A. Schilpp, La Salle, 1963, pp. 5ff., Italian tr. *La filosofia di Rudolf Carnap*, edited by M. G. de Cristofaro Sadrini, Milan, Il Saggiatore, 1974, vol. 1, pp. 5–6.

² Cf. G. Frege, *Wissenschaftlicher Briefwechsel*, edited by G. Gabriel, H. Hermes, F. Kambartel, C. Thiel, A. Veraart, Hamburg, Meiner, 1976, p. 16. The (partial) Italian translation of Frege's epistolary, edited by C. Mangione, has been published as *Alle origini della nuova logica*, Turin, Boringhieri, 1983. According to Heinrich Scholz, among Frege's papers there was a postcard by Carnap dated 30 November 1921, in which the latter was asking Frege to send him a copy of his 1892 essay *Über Begriff und Gegenstand* ('Object and Concept'), published in *Vierteljahrsschrift für wissenschaftliche Philosophie*, 16 (1892), 192–205. Carnap's request is probably related the writing of the *Aufbau*.

³ One of the many questions on which Carnap's notes could shed light is which version of the *Begriffsschrift* Frege referred to in his lectures: probably not the 1879 one, but certainly not even the one of the *Grundgesetze*, which Frege tried in vain (at least until 1906) to save from the disaster of the contradiction discovered by Russell in 1902.

As is well-known, Frege abandoned the logicist programme and, after 1918, he worked on a 'new' foundation of arithmetic on a geometrical basis. Not by chance, Carnap's logicism is modelled on that of Russell and Whitehead, and not on Frege's. This, indirectly, clarifies the particular reading of the Fregean theory of concepts that Carnap adopts in the *Aufbau*.

in a radically reduced form.⁴ In fact, only a small part of Frege's copious *Nachlass* and scientific correspondence survived the vicissitudes of the Second World War. Among the surviving writings there is precisely one piece of about fifty pages, bearing the title *Logik in der Mathematik*, which is most likely the text used by Frege for the lessons followed by Carnap. As I will try to show, these lectures have left an indelible imprint on Carnap's philosophical style.

Carnap refers explicitly to Frege's work in two of his main works, namely in chapter 3 of his early work, *Der logische Aufbau der Welt*,⁵ and in chapter 3 of his mature work, *Meaning and Necessity*.⁶ These are the only two works by Carnap that I will deal with in the following pages, although references to Frege's work recur in many other works by Carnap.

I

As Alberto Coffa has observed,⁷ the claims contained in Carnap's biography about his intellectual debt to Frege's instruction, while quite appropriate regarding his 1947 work, *Meaning and Necessity*, may be misleading regarding the horizon of

⁴ G. Frege, *Nachgelassene Schriften*, edited by H. Hermes, F. Kambartel, F. Kaulbach, Hamburg, Meiner, 1969, Italian tr. edited by E. Picardi, *Scritti postumi*, Naples, Bibliopolis, 1987. In 1983, also by the publisher Meiner, a second edition of the *Nachgelassene Schriften* has appeared, enriched by an essay by Lothar Kreiser. For further information on Frege's *Nachlass*, see the introductions of the editors of the German edition and the remarks in my foreword to the Italian edition. The [Italian] translation of *Logik in der Mathematik* can be found at pp. 333–95 of *Scritti postumi*.

⁵ R. Carnap, *Der logische Aufbau der Welt*, Ullstein Materialien, Frankfurt, 1974; the first edition of the *Aufbau* appeared in 1928, the second (in German) in 1961, the third in 1966. The Italian translation of *Der logische Aufbau der Welt* by E. Severino was published in 1966, Milan, Fratelli Fabbri. In the text I give my translation of all the works, where they are not cited in the original. The works of Frege discussed and cited by Carnap in the bibliography of *Der logische Aufbau der Welt* are: *Die Grundlagen der Arithmetik*, Breslau, Koebner, 1884 (Centenar Ausgabe, edited by C. Thiel, Hamburg, Meiner, 1986); *Über Funktion und Begriff*, Jena, 1891; 'Über Sinn und Bedeutung', *Zeitschrift für Philosophie und philosophische Kritik* 100 (1892), 25–50; *Grundgesetze der Arithmetik*, Jena, Pohle, vol. 1 (1893), vol. 2 (1903); 'Kritische Beleuchtung einiger Punkte in E. Schröders Algebra der Logik', *Archiv für systematische Philosophie*, 1 (1895), 433–56. All Frege's minor articles and writings are collected in *Kleine Schriften*, edited by I. Angelelli, Hildesheim, Olms, 1990² (1967¹).

⁶ R. Carnap, *Meaning and Necessity*, Chicago, Chicago University Press, 1947¹, 1956², Italian tr. by A. Berra, *Significato e necessità*, edited by A. Pasquinelli, Florence, La Nuova Italia, 1976.

⁷ See J. A. Coffa, *The Semantic Tradition from Kant to Carnap to the Vienna Station*, Cambridge, Cambridge University Press, 1991, ch. 11, p. 207: 'When in his later years, recollecting his intellectual development, Carnap listed the major influences on his thought, the names he gave were those of Frege, Russell and Wittgenstein. But in 1920 he sent a very different list of names to Dingler. Kant, Riemann, Helmholtz, Mach, Avenarius, Poincaré, Natorp, Ostwald, Einstein, and Weyl were, he said, the people he was studying.' Interesting observations on Carnap's logical debt to Frege can be found in M. Friedman's article, 'Logical Truth and Analyticity in Carnap's "Logical Syntax of Language"', in W. Aspray, P. Kitcher (eds.), *History and Philosophy of Modern Mathematics*, Minnesota Studies in the Philosophy of Science, vol. 11, Minneapolis, University of Minnesota Press, 1988, pp. 82–94. For a critical evaluation of the project of the *Aufbau*, see J. Schulte, 'Rudolf Carnap: *Der logische Aufbau der Welt*', in *Interpretationen. Hauptwerke der Philosophie: 20. Jahrhundert*, 1992, Stuttgart, Reclam, pp. 123–43. Relevant observations can be found in the article by E. W. Beth in the volume edited by Schilpp mentioned in footnote 1. The most illuminating and thorough study of the *Aufbau* remains

ideas in which Carnap's first, and perhaps most original, work, *Der logische Aufbau der Welt*, matured.

Carnap chose as the motto for his book a famous maxim, stated by Russell on several occasions, which Carnap thought should inform 'scientific philosophy'. The maxim, known as 'constructions vs inferences', is the following: 'A supreme maxim in scientific philosophizing is this: Whenever possible, logical constructions are to be substituted for inferred entities.'⁸ As I have argued elsewhere,⁹ this maxim is, with its merits and limitations, the heart of Russell's philosophy: 'inferred entities' are entities that are either potentially suspect from the logical point of view, such as, for example, classes, which, if not treated carefully can entangle us into antinomies, or entities that are problematic from the epistemological point of view, such as, for example, other minds and physical objects. The ideal would be to 'reconstruct' all meaningful discourse, and therefore all scientific discourse, without resorting to inferred entities, but using only logical constructions starting from known entities. What are the known entities of which Russell is speaking? Essentially sense data: it is not by chance that the language at the basis of the *Aufbau* is also a phenomenal language, a language modelled on what C. I. Lewis in 1934 would call 'first person experience', which, as such, lends itself to the 'egocentric predicament'.¹⁰ It is worth remembering that the point of view of the kind of *methodological solipsism* advocated by Carnap in the *Aufbau* has essentially the value of a *methodological* injunction and does not indicate that a particular metaphysical privilege is granted to the subject or the ego.¹¹ It is rather, as Schlick acutely observed, a guide to the construction of new objects and concepts (those pertaining to inner experience, the external world, other minds, for example), which, if pursued consistently, involves the rejection rather than the acceptance of the solipsistic point of view.¹²

The point that I would like to emphasize in this context is how (and how much) the author of the *Aufbau* was indebted, on the one hand, to the philosophy of

that of Nelson Goodman, *The Structure of Appearance*, Cambridge, 1951¹, 1966²; *La struttura dell'apparenza*, tr. A. Emiliani, Bologna, Il Mulino, 1985, of the third edition of the work, appeared in 1977 with an introduction by G. Hellman, Dordrecht, Reidel. Carnap dwells on Goodman's work in the preface to the second edition of *Aufbau*. For a description of the cultural background in which the ideas of the *Aufbau* matured, see also F. Barone, *Il neopositivismo logico*, Turin, 1953 and A. Pasquinelli, *Carnap*, Bari, Laterza, 1972, pp. 22–38.

⁸ The passage quoted by Carnap is taken from Russell's article 'The Relation of Sense Data to Physics' (1914), reprinted in B. Russell, *Mysticism and Logic*, London, Unwin Books, 1917, 1963, p. 115.

⁹ See E. Picardi, Alfred North Whitehead/Bertrand Russell: *Principia Mathematica*, in *Interpretationen. Hauptwerke der Philosophie: 20. Jahrhundert*, 1992, Stuttgart, Reclam, pp. 7–42, and E. Picardi, *Linguaggio e analisi filosofica: elementi di filosofia del linguaggio*, Bologna, Patron, 1992, ch. 5.

¹⁰ C. J. Lewis, 'Experience and Meaning', *The Philosophical Review*, 43 (1934), reprinted, together with the article-reply by M. Schlick, 'Meaning and Verification', *The Philosophical Review*, 45 (1936), in the anthology edited by H. Feigl and W. Sellars, *Readings in Philosophical Analysis*, New York, Appleton-Century-Crofts, Inc., 1967. See pp. 130–6 of Lewis's article.

¹¹ *Der logische Aufbau der Welt*, §64, pp. 86ff.

¹² Schlick, *Meaning and Verification*, pp. 161–8. The object of contention is the 'egocentric predicament' inherent, according to Lewis, in the methodological solipsism theorized by Carnap.

Russell, and, on the other, to the inspiration that came to him from authors such as Mach, Avenarius, Schubert-Soldern, Driesch, Ziehen,¹³ who, to put it in very crude terms, were proponents of a sort of 'naturalized epistemology' *ante litteram*. The intrusion of psychology into logic was, in Frege's eyes, the main defect of these approaches: if logic had to do with psychology or could profit from psychology, it would have the status of an empirical science. The young Carnap accepted, at least in part, Frege's anti-psychological approach, without abjuring Ernst Mach's naturalistic approach. Carnap's idea was to use Frege's and Russell's new logic for the ambitious enterprise of rational reconstruction (*rationale Nachkonstruktion*) of the world from the elementary data of immediate experience. The bricks with which the definitions of the concepts we encounter in the *Aufbau* are built meet the requirements of the Russellian maxim, that is, they are 'safe' entities.

The invisible mortar that holds them together is Frege and Russell's logic, the technique used is that of '*konstitutionale Definition*'. In this perspective it is not logic that becomes empirical or that is contaminated (or enriched, depending on one's taste) by psychology, but rather experience that is endowed with more logical aspects than we can perceive at first sight. Moreover, the *logical* aspects of the world seem to prevail in importance over all others and seem to contain a sort of *clavis universalis* for the understanding of world and language. In attributing a central role to logic, Carnap deepens the point of view exposed by Wittgenstein in the *Tractatus Logico-Philosophicus*.

What distinguishes the originality of the young Carnap's project is, in my opinion, the new use he makes of two Fregean ideas: the first is the systematic request, leading to the building of systems (or languages, as Carnap prefers to call them); the second is the use of a particular kind of definition, the '*konstitutionale Definition*'. The task of a constitutive definition is not to render the scope of notions as they are used in ordinary linguistic or scientific practice, but to offer a purified version, the credentials of which are to be evaluated from a strategic point of view, so to speak. The importance of the construction of the system, or as Carnap says, echoing a metaphor of Frege's, the construction of a 'family tree of concepts' (*Stammbaum der Begriffe*) based on a small number of basic concepts (*Wurzelbegriffe*),¹⁴ is the main theme of the lectures held by Frege in Jena in 1914. Carnap himself mentions in his *Autobiography* the themes of Frege's lectures that impressed him:

He deplored the fact that mathematicians did not even seem to aim at the construction of a unified and well-founded mathematical *system*, and that because of this they showed a lack of interest in the foundations. He pointed

¹³ *Der logische Aufbau der Welt*, Part I, §3.

¹⁴ *Der logische Aufbau der Welt*, Introduction, §§1–2.

out a certain incorrectness in the common formulation of axioms, definitions, demonstrations even in works of the most eminent mathematicians... He criticized the lack of attention to certain fundamental distinctions, for example the distinction between symbol and symbolized, between a logical concept and a mental image or act, and that between a function and the value of a function.¹⁵

It is worth comparing this statement of Carnap's with a passage from *Logik in der Mathematik*:

Only in the system does science come to fruition. One must never give up on the system. Only through the system can one achieve full clarity and order. No science like mathematics can dominate its own matter and elaborate it in such a clear and perspicuous form, but perhaps no science like mathematics can get lost in such a thick fog when it renounces the construction of the system.¹⁶

The importance of systematic reconstruction will become a constant theme in Carnap's thought. He will apply this methodological intuition beyond the boundaries of logic, extending it to the reconstruction of scientific concepts and semantic notions. The other insight that Carnap drew from Frege's lectures was probably the conception of the role played by definitions in the enterprise of 'reducing' a concept to other more basic ones. The '*konstitutionale Definition*' or constitutive definition resembles very much what Frege in *Logik in der Mathematik* calls '*aufbauende Definition*', or constructive definition, and contrasts with the analytic or '*zerlegende Definition*', the purpose of which is to capture and specify the meaning of an expression as it is currently used. Carnap's constitutive definitions, as well as Frege's constructive ones, must satisfy a condition of material adequacy (extensional equivalence), but not one of intensional adequacy.¹⁷ The definitions found in the *Aufbau* are not to be evaluated one by one, but rather in view of their role in the overall construction of the system. The success of the 'reconstructed' system is revealed in the ability that it has *as a whole*, and not point by point, to account for the logical structure of the world, that is, to allow the 'translation' of any assertion about physical objects or other minds into an assertion about the elementary data of immediate experience ('*das Gegebene*', in Carnap's and

¹⁵ Carnap, 'Intellectual Autobiography', Italian tr., p. 6.

¹⁶ G. Frege, *Nachgelassene Schriften*, p. 382.

¹⁷ In the preface to the second edition of the *Aufbau* Carnap will declare the extensional method insufficient: for the reconstruction A of a given concept B to be valid, it is not enough that A and B have the same extension, but A and B must necessarily have the same extension, where 'necessarily' refers to logical rules or laws of nature. We cannot mention here the difficulties inherent in the Carnapian conception of the definition as 'explication'. Not dissimilar problems arise already regarding the status of the definition in Frege's logicist conception; see in this regard M. Dummett, 'Frege and the Paradox of Analysis' (1987), in *Frege and Other Philosophers*, Oxford, Clarendon Press, 1991, pp. 17–52, and, also by the same author, *Frege: Philosophy of Mathematics*, London, Duckworth, 1991, ch. 11.

Schlick's terminology). What must be preserved is a structure, not a content. Like Carnap's definitions, Frege's too contain an essential element of conventionality and arbitrary stipulation. The following excerpt taken from *Logik in der Mathematik* effectively illustrates the analogy between the 'konstitutionale Definition' employed by Carnap in the *Aufbau* and the kind of 'aufbauende Definition' indicated by Frege as the only scientifically admissible definition:

Suppose that A is a simple sign (expression) that has long been in use, whose meaning we have tried to analyse logically; to represent this analysis we have formed a compound expression. Since we are not sure whether the analysis is right we dare not present the compound expression as substitutable for the simple sign A. If we want to give a real definition we cannot choose the sign A, which already has a sense, but we must choose a completely new sign B, which with the definition only gives the sense of that compound expression. Now the problem arises whether A and B have the same sense. But we can get around this problem completely, rebuilding the system from the foundations, without using the sign A, but only the sign B. We have arbitrarily introduced the sign B for that compound expression and thus given it a sense. This is a definition in the proper sense, a constructive definition [aufbauende Definition].¹⁸

The young Carnap's debt to Frege's teaching does not end, however, in the assimilation of a methodological lesson. In fact, Carnap skilfully adapted some of Frege's ideas to his own ambitious project of logical reconstruction of the world, after having tacitly 'reconstructed' them. An illuminating example of this tacit work of rational reconstruction is represented by the reading that Carnap proposes in the *Aufbau* of the functional conception of concepts advanced by Frege. To this aspect of Frege's doctrines Carnap returns also in *Meaning and Necessity*, in a theoretical framework very different from that of the *Aufbau*. What I would like to highlight in the following pages are the similarities and the differences between the interpretations that Carnap gave in 1928 and 1947 of the semantic value of predicates—'Begriffswörter' in Frege's terminology and 'predicators' in Carnap's.

II

At first glance, the *Aufbau* opens with a declaration of anti-Fregean flavour: 'object' must be understood as everything that can be an object of discourse, i.e. events, properties, relationships, concepts, etc. In §5, entitled 'Begriff und Gegenstand', the relationship between a concept and 'its' object is illustrated: we

¹⁸ Frege, *Nachgelassene Schriften*, Italian tr., p. 343.

are told that each concept has only one object, which obviously should not be confused with the objects that fall under the concept. However, Carnap insists that, unlike what is stated in other theories of concepts, in his opinion there would be no intrinsic difference between general concepts and individual concepts:

Whether a certain object symbol indicates an object or a concept, whether a certain proposition applies to concepts or to objects, does not constitute a logical difference, but at most a psychological one, i.e. a difference in the ideas employed in the representation (*repräsentierenden Vorstellungen*). It is not fundamentally a matter of two different conceptions, but of two different formulations of the interpretation.¹⁹

It is not a matter—says Carnap—of reifying concepts, but of ‘functionalizing’ objects. These somewhat sibylline statements become clearer in the light of the way in which, in §27 of chapter 3, Carnap interprets Frege’s conception of unsaturated expressions. Carnap agrees with Frege that only utterances, strictly speaking, have an autonomous meaning, but that, nevertheless, one can recognize an autonomous meaning also to the noun precisely with respect to the ‘unsaturated’ part of the sentence. So, for example, ‘Luchs is a dog’ and ‘Karo is a dog’ coincide in what Frege called the unsaturated part, that is ‘... is a dog’, and differ in the autonomous or complete part, i.e. in the proper name. The unsaturated part also occurs in other contexts, for example, in subject position, when we say ‘a dog is a mammal’. Although here the unsaturated term appears in subject position, it does not designate an object at all, but what Carnap proposes to call a ‘quasi-object’. These ‘quasi-objects’ are nothing but classes, i.e. extensions of predicates and ranges of values of functions. The unsaturated term in the first pair of utterances, which Carnap proposes to render as ‘ $x \text{ E dog}$ ’, designates an enunciative function, while the expression ‘a dog’ in ‘a dog is a mammal’ designates a quasi-object, i.e. a class, and the expression as a whole should be rendered as ‘Dog C Mammal’. The legitimacy of formulations containing a reference to quasi-objects rests, according to Carnap, on the possibility of transforming these sentences into sentences containing individual variables, which can be replaced, in turn, by proper names of objects. As Carnap himself points out, his interpretation of quasi-objects is indebted to Russell’s ‘no-class theory’, but it differs from it by an even more accentuated nominalism. Classes, therefore, conceived as extensions of propositional functions, are nothing but quasi-objects. The function of a term of class (with respect to the corresponding unsaturated expression) is to indicate ‘to a certain extent what is common to objects, that is, to elements’.²⁰ In this Carnap follows Frege in repudiating the *aggregative* conception of classes and

¹⁹ *Der logische Aufbau der Welt*, §5, p. 5.

²⁰ *Der logische Aufbau der Welt*, §33.

in indicating the decisive step in what Frege would have called the concept, that which 'holds together' the elements of the collection. According to Carnap, classes, understood as quasi-objects, are distinct from the elements, in the sense of being '*sphärenfremd*', unlike the aggregates that he declares '*sphärenverwand*'.²¹

A useful place to start when focusing on both the points of contact and those of divergence for Frege's and Carnap's conceptions is the statement at the end of §33 of the *Aufbau*, according to which 'the fact that expressions indicating extensions, and therefore class symbols, are unsaturated symbols has already been shown by Frege'. Here one has the impression that Carnap is assimilating Russell's incomplete symbols to Frege's unsaturated expressions. Except that, while Frege's unsaturated expressions show by virtue of their syntactic form that they do not designate objects, but concepts (functions), Russell's incomplete symbols refer to *contextual definitions* through which they are systematically eliminable from the formulas that contain them. Even a superficial reading of Frege's writings suffices to show that Carnap's interpretation, far from being a faithful mirror of Frege's doctrines, is instead an adaptation of them, perhaps the best adaptation possible in the light of the contradiction discovered by Russell.²² The contradiction had shown precisely that the way in which Frege characterized extensions (classes, value ranges, logical objects) was untenable. Yet, Frege in all his writings, beginning with the *Grundlagen der Arithmetik*, emphasizes that, from the logical point of view, there can be no greater difference than that between objects and concepts. This difference is sometimes obscured by grammar. Nonetheless, Frege identifies in the definite article of natural languages a reliable marker to distinguish an expression indicating an object from one indicating a concept. A proper name designates an object, an autonomous constituent of reality, while an unsaturated expression designates a concept.²³ In *Über Begriff und Gegenstand* Frege embroiled himself in the notorious 'paradox' according to which 'The concept "horse" is not a concept' would be, at first glance, a true sentence since the presence of the definite article in front of the expression 'horse' transforms the unsaturated expression into a saturated one, which is, presumably, the name of the extension of the concept. When we say 'The concept "horse" is easy to grasp' what we mean to refer to is an unsaturated entity *par excellence*, a concept, but language forces us to form an expression that indicates an object instead. Carnap is a faithful interpreter of Frege's

²¹ This contrast between the logical conception and the aggregative one of classes is exposed incisively by Frege in the essay on Schröder's algebra (quoted above), which Carnap mentions in fact in the bibliography.

²² See in particular Frege's essay *Über Begriff und Gegenstand*, mentioned in the footnote. Frege's article was intended as a response to a series of eight articles by the Austrian philosopher Benno Kerry, which appeared between 1885 and 1891 in the *Vierteljahrsschrift für wissenschaftliche Philosophie*, entitled *Über Anschauung und ihre psychische Verarbeitung* (Intuition and its Psychic Processing). On the Frege-Kerry controversy see E. Picardi, 'Kerry und Frege über Begriff und Gegenstand', *History and Philosophy of Logic*, 15 (1994), 9–32. *Über Begriff und Gegenstand* can be found reprinted in *Scritti postumi*, together with an early draft of it.

²³ See *Logik in der Mathematik*, Italian tr. pp. 346–7 and 364–5.

thought when he emphasizes that an unsaturated expression designates first of all a *concept* (a propositional function), but he ceases to be a faithful interpreter of Frege's thought when he considers that classes and ranges of values are 'quasi-objects', that is concepts in disguise. Classes were for Frege the logical objects *par excellence* with which he proposed to reconstruct arithmetic on a purely logical basis, and once abandoned, the logicist programme has no hope of success. The correspondence with Russell and the appendix attached to the second volume of *Grundgesetze* show *ad abundantiam* that the idea of conceiving classes as 'improper objects' (*uneigentliche Gegenstände*), organized in a hierarchy of types, was deeply alien to Frege.²⁴ The analogy between Carnap's 'Quasi-Gegenstände' and the 'uneigentliche Gegenstände' proposed by Russell and rejected by Frege, catches the eye. In conclusion, we can say that Carnap's interpretation is faithful to those parts of Frege's doctrines that survive the disaster of the contradiction, but not to the way Frege understood his own doctrines. Carnap's Frege is, so to speak, a 'Russellified' Frege.

In the *Aufbau* Carnap not only dwelled on Frege's theory of unsaturated symbols; he also reformulated Frege's distinction between the 'truth value' (*Wahrheitswert*) and the 'cognitive value' (*Erkenntniswert*) of a sentence in terms of the distinction between 'logical value' (*logischer Wert*) and 'cognitive value'. *Truth value* and *logical value* are identified by Carnap in accordance with the Russellian principle of extensionality which, not by chance, informs the whole *Aufbau*.²⁵ In §44 of the *Aufbau* the distinction between *Sinn* and *Bedeutung* of a simple or complex sign is illustrated just as it is explained in the famous essay of 1892, *Über Sinn und Bedeutung*. This part of Carnap's exposition also contains points of considerable interest, and moreover lends itself to a direct comparison with the theories of *Meaning and Necessity*.

III

In §44 of the *Aufbau* Carnap refers to the different relations of sameness of meaning that we can find among linguistic expressions—a theme that will have a central role in the work of 1947. In *Meaning and Necessity* the relations of 'congruence' between sentences range from material equivalence, to logical equivalence, to L-equivalence that takes into account the rules (or postulates) of meaning, and, finally, to synonymy in the strict sense, which is explained through the notion of *intensional isomorphism*. In §44 of the *Aufbau*, after observing that the signs 'VII' and '5 + 2', while designating the same object, and therefore

²⁴ See E. Picardi, 'Alcune osservazioni sulla corrispondenza Frege-Russell', in *Atti del Convegno Nazionale di logica*, 1–5 ottobre Montecatini 1979, Naples, Bibliopolis, 1982, pp. 717–35.

²⁵ See *Der logische Aufbau der Welt*, §50.

coinciding in their *Bedeutung*,²⁶ differ both concerning the sign and the sense expressed by the respective signs. Carnap observes:

By the *sense* of a sign we mean what is common to the intentional objects of those representations, thoughts, or anything else, that the signs are meant to evoke.²⁷

Carnap's formulation is undoubtedly original and differs from the explanation offered by Frege in two important ways, namely in the mention of intentional objects and in the juxtaposition of representations to thoughts. As is well known, according to Frege two signs can differ in three respects: in the sense, in the meaning (*Bedeutung*, which in 1947 Carnap will render as '*nominatum*'), and in the representation that the individual speaker associates with the sign.²⁸ The sense is for Frege objective and communicable, while the representation is subjective and incommunicable, and what is more important, it is irrelevant when considering both the truth value and the cognitive value. Carnap's idea seems to be this: signs evoke representations or thoughts; thoughts have an intentional object; meaning is the common element of the intentional objects of thoughts or representations evoked by signs. Of course, one could object that Carnap does not attribute to the term '*Vorstellung*' the psychological connotations that Frege attributed to it, and this is a perfectly legitimate objection. But the decisive point is that in the context of a phenomenal reduction of the world to the data of immediate experience we would not know how to draw the distinction between thoughts and representations that seemed essential to Frege to build logic and not to slip into idealism and solipsism. Perhaps it is no coincidence that in the work of 1928 Carnap does not mention Frege's first logical research, *Der Gedanke*, which

²⁶ In the *Aufbau*, §44 Carnap employs angle brackets to indicate the sense of a sign and square brackets to indicate the meaning. The meaning of $\langle 7 \rangle$ is different from that of $\langle 5 + 2 \rangle$, while the meaning of $[7]$ is the same as that of $[5 + 2]$. The meaning of $\langle 7 \rangle$ is the same as that of $\langle VII \rangle$. See *Meaning and Necessity*, ch. I, §14, where the notion of intensional isomorphism serves to define a stronger notion of L-equivalence: the expressions $\langle 5 + 2 \rangle$ and $\langle 7 \rangle$, while being L-equivalent, are nevertheless not 'isomorphic' because their ultimate subcomponents are different, which helps explain why they are not intersubstitutable *salva veritate* in belief contexts. For modal contexts, the substitution of L-equivalent expressions is instead adequate. See, however, the possibility of stricter substitutability conditions left open in §40 of ch. 5 of *Meaning and Necessity*.

²⁷ 'Unter dem *Sinn* eines Zeichens verstehen wir das Übereinstimmende an den intentionalen Gegenständen derjenigen Vorstellungen, Gedanken oder dgl., die hervorzurufen der Zweck des Zeichens ist' (*Der logische Aufbau der Welt*, §44, p. 61).

²⁸ See *Über Sinn und Bedeutung*, p. 31: 'We can identify three levels of difference in words, expressions or entire utterances. Either the difference concerns representations at most, or the sense or, finally, also the meaning (*Bedeutung*). . . . In the following we will not mention any more representations or images: they have been named here so that the representation that a word evokes in a listener is not mistaken for its sense or meaning.' A thorough discussion of Frege's essay and of Carnap's criticism of the theory of senses and of indirect meanings and senses can be found in my work '*Über Sinn und Bedeutung*: An Elementary Exposition, Part I', in *Lingua e Stile*, 24, 3 (1989), 331–64, and Part II in *Lingua e Stile*, 25, 2 (1990), 159–99.

contains a strong criticism of those 'naturalistic' and 'psychologistic' points of view so dear to the author of the *Aufbau*.

Among the examples given by Carnap of expressions with different meanings there are examples of names and definite descriptions and among the examples of expressions with the same meaning there are 'der Abendstern' and 'the evening star', i.e. expressions that are the translation of each other. Unfortunately, Carnap does not give us any further information about *intentional objects*—what they are and where they come from (a Brentanian or Husserlian reminiscence?). This silence perhaps helps to explain the curious metamorphosis that the objects in question undergo in the English translation that Rolf George prepared in 1967 of Carnap's work. Here, in fact, on p. 75 we read:

By the sense of a sign we mean that which the intensional objects, i.e. representations, thoughts, etc. which the sign is to evoke, have in common.²⁹

The translation, besides being inaccurate, has miraculously transformed intentional objects into intensional objects. It cannot be excluded that the method of intension and extension, theorized by Carnap in *Meaning and Necessity*, has consciously or unconsciously influenced the translator.³⁰

As we have said, in 1947 the theoretical framework in which Carnap works is very different from that of *Aufbau* and the spirit with which he approaches Frege's theories is also different. In §29 of chapter 3 of his 1947 work Carnap compares his own method of extension and intension with Frege's method of sense and meaning. The latter, unlike the former, is based—Carnap argues—on the name relation and this is a fundamental defect, as it is responsible, among other things, for the baroque complications that plague the Fregean treatment of *oratio obliqua* and belief contexts.³¹ As is known, with *Meaning and Necessity* Carnap aims to provide a semantics, as well as a philosophical interpretation, for the systems of modal logic formulated by C. I. Lewis and other authors from the early decades of the twentieth century. The project of 1947 constitutes therefore a more suitable theoretical framework than the one of 1928 to discuss Frege's theories.

First of all, we observe that the way Carnap illustrates the notion of sense in 1947 is, at least at first sight, more adherent to the spirit of Frege's work. For example, in § we read: 'The concepts of sense and intention refer to meaning in the strict sense, to what we grasp when we understand an expression without knowing

²⁹ R. Carnap, *The Logical Structure of the World and Pseudoproblems in Philosophy*, English tr. R. A. George, Berkley and Los Angeles, University of California Press, 1967.

³⁰ An excellent summary of the many good reasons for not confusing 'intentional' and 'intensional' is the entry 'Intentionality' written in 1967 by R. Chisholm for the *Encyclopedia of Philosophy* edited by P. Edwards.

³¹ I have dealt with the Fregean theory of *oratio obliqua* and Carnap's criticism in ch. 8 of *Linguaggio e analisi filosofica*.

the facts; the concepts of *nominatum* and extension refer to the application of the expression, depending on the facts.' Yet it is far from clear that the Carnapian concept of intension has much to do with what we grasp when we understand an expression. Intensions, as Carnap in fact employs them in modal logic, are functions from possible worlds to truth values, which determine, so to speak, of their own accord, and not by virtue of the speaker's knowledge, the possible worlds in which they are realized. Moreover, since Frege's conception, as is well known, is not modalized, it is a bit misleading to say that the '*nominatum*' depends on the application of an expression according to the facts. Since for Frege a sentence is true (absolutely) and not true in this or that world (or state description), grasping its sense is not an operation that must be completed by information about how things are in the world. Obviously, we must be able to grasp the sense of an utterance before we know whether it is true (or false), but the determination of facts or extensions adds nothing to what we grasp when we understand its sense. In any case, these are marginal subtleties with respect to the aspect of the Carnapian interpretation of Frege's doctrines that I would like to discuss here. The main differences between Frege's and Carnap's methods can be summarized in three points: (a) in the Carnapian system an expression has always the same intension and the same extension, which therefore does not change, as it does in Frege, according to the enunciative context; (b) in non-extensional contexts the semantic value of an expression is constituted by its intension; (c) in oblique contexts (and, indeed, in all contexts) the semantic relation between the linguistic expression and the intension attached to it cannot be assimilated to a name relation.

If the parallel drawn by Carnap between the pairs intension/extension and sense/meaning is quite adequate for what concerns sentences and proper names,³² the same cannot be said about the sense and the *Bedeutung* (*nominatum*) of concept terms (*Begriffswörter*) (predicates). And this is really surprising, if you think that in the *Aufbau*, as we have seen, Carnap had grasped exactly what Frege meant by the *nominatum* of an unsaturated expression, i.e. a concept (a propositional function). In *Meaning and Necessity*, however, Carnap adheres (cautiously) to Church's interpretation and proposes to identify the *nominatum* of a predicate (*Begriffswort*) with the class it determines and its meaning with a property and therefore with the intension. Things are otherwise: a predicate has a concept, an unsaturated entity, as its *Bedeutung*. In a passage of the *Nachlass* Frege deals explicitly with this problem and his answer could not be clearer:

³² Subject, of course, to the reservations I mentioned above regarding the conditions of identity of sense (in Frege) and of identity of intension (in Carnap; see note 27) and the relativization of the truth of a sentence to a possible world (or state descriptions).

In an essay entitled *Über Sinn und Bedeutung* [Sense and Meaning] I introduced the distinction between sense and meaning at first only for proper names (or, if you prefer, singular terms). The distinction can be extended to predicates [*Begriffswörter*]. Here misunderstandings can easily arise, if the distinction between sense and meaning is confused with the division into objects and concepts, and sense and concept are conflated on one side and meaning and object on the other. To each predicate corresponds, as a rule, according to my use of these terms, a sense and a meaning. . . . the meaning of a proper name is the object which it designates or denominates. A predicate has as its meaning a concept, supposing that this term is used as is appropriate in logic.³³

Since for Frege a concept is nothing but a function whose values are only truth values, it becomes also clearer in what sense it constitutes the reference of a predicative expression, and not its sense.³⁴ As is well known, Axiom V of *Grundgesetze* legitimizes the transition from a statement relating to the generality of an equality to a statement relating to an equality of value ranges (classes), and thus, *de facto*, allows one to transform a statement concerning concepts into a statement concerning classes (the statements, of course, are different, but, according to Frege, they always have the same truth value). What Frege insists on—even after the discovery of the contradiction—is that from his point of view the concept, understood as the reference of a predicate, has the logical (and foundational) priority over the class.

Yet, one may ask, why does Carnap espouse Church's interpretation? The reason given by Carnap is that Frege never says clearly what is meant by the sense of a predicate (*Begriffswort*). According to Carnap, this question can be answered as follows: the sense is what L-equivalent expressions have in common, i.e. an intension. It is undeniable that in Frege's texts we do not find a satisfactory formulation of synonymy, but it is also true that in the *Aufbau* Carnap had acutely observed that for Frege there is a difference between the sense of '5 + 2' and '7', and therefore he was perfectly aware that the conditions imposed by Frege on the identity of sense resemble more those of intensional isomorphism than those of L-equivalence. It is true that Frege does not say in clear terms what should be meant by the 'sense' of a predicate; but it is possible without too much difficulty to extend to predicates the characterization given for the sense of a proper name and to suggest that the sense of a predicate is the way in which the function (the concept)

³³ *Nachgelassene Schriften*, Italian tr., p. 222.

³⁴ One might think that Carnap has in mind a difference between property and concept, but *Meaning and Necessity*, §4 disproves this hypothesis. Moreover, Carnap, like Frege, does not mean by 'concept' and 'property' something mental or linguistic, but something objective, which is exemplified in nature. Also interesting is the characterization of (factually) true propositions as states of affairs, found in §6.4, which has considerable affinities with the way Russell, before 1905, conceived of propositions.

which it designates is given to us. A concept (function), like an object, can be given to us in many different ways. It can be a genuine discovery to learn that two predicates indicate the same concept (function).

In any case, what is most striking (and disorienting) about the reading of Frege's ideas presented in *Meaning and Necessity* is precisely the interpretation of the *nominatum* of predicates, which is different from (and opposite to) that given in the *Aufbau*. In fact, while in the *Aufbau* classes (value ranges) were lowered to the rank of quasi-objects in favour of unsaturated entities (concepts, propositional functions), in *Meaning and Necessity* they acquire the role of genuine *nominata*, while unsaturated entities are equated with intensions. But surprises do not stop here. In fact, if we read §27 carefully we discover that, in the end, the semantic conception of the predicators that Carnap presents as his own (and distinct, therefore, from Frege's) is not too different from the one we encountered in the *Aufbau*. He argues, in fact, (a) that it is unimportant whether we interpret the predicate 'human' as referring to a property or to a class, and that therefore the alternative that was so dear to Frege was a pseudo-alternative; and (b) that the language that he considers preferable is the one that contains properties, but not classes (§§27-2–27-6). It is not by chance that the definition of cardinal number offered in §27 of *Meaning and Necessity* is identical, as Carnap himself points out, to the one given by Frege in *Grundlagen der Arithmetik*. The missing step in Carnap's reconstruction is, not by chance, the introduction of classes (i.e. of the extensions of second-level concepts). Whether the logical resources with which Carnap reconstructs arithmetic in *Meaning and Necessity* are richer (and therefore more adequate) than those mentioned in the *Aufbau* is a question I do not intend to discuss here. What I would like to emphasize, in conclusion, is the continuity of Carnap's logical inspiration although almost twenty years separate the *Aufbau* from *Meaning and Necessity*, together with a certain predisposition to infidelity as far as the interpretation of Frege's theory of concept is concerned. In both works, in fact, Carnap, far from being a faithful interpreter, as he tends to be, proves to be an original and not always benevolent one. Faithfulness, as is obvious, is not necessarily a virtue. In some cases, it may be worthwhile to ask about the reasons for infidelity, since they generally arise from dissatisfaction and not from distraction, and are therefore instructive. However, to dissect these reasons would require another article, and, moreover, one that I would not be able to write.

Frege and Davidson on Predication

1. Predication: What Is the Issue and What Is at Issue

The ancient problem of predication, i.e. the most basic connection in which words can enter to form a semantic unit, may be seen as bearing on at least four clusters of issues: (i) What requirements on word meaning and grammatical form have to be satisfied in order for a collection of words to count as a *sentence* of a given language? (ii) What constraints on word meaning and logical form have to be satisfied for a sentence to express a proposition upon which a thinker may pass *judgement*, i.e. accept it as true or reject it as false? Plausibly, our acceptance of a given sentence as true and our rejection of it as false require that we *understand* at least the meaning of its component words and the way they are put together. This brings us to the third and fourth issues that a theory of predication has to address: (iii) How do the syntax and logical form that we ascribe to sentences relate to their truth-conditions? (iv) How does the syntactic structure that we read into (simple) sentences relate both to their inferential potential, i.e. to their occurrence as premises and conclusions of deductive arguments and to the vast variety of compound and quantified sentences that may be seen as ‘constructed’ out of them? For, obviously, an account of syntactic structure that did not explain how sentences connect with one another would be of no help for casting light on the notions of semantic consequence and valid argument. It is essential that we bear in mind the *architectonic demands* that any account of predication should satisfy, in order that we may be able to evaluate its success or failure. The main claim of this paper is that, in spite of all its difficulties, including those pointed out by Davidson, Frege’s account of predication is the only candidate at our disposal that satisfies such demands.

Broadly speaking, we may distinguish *deflationary accounts of predication* of the Quine–Davidson variety, with their attendant disquotational schema:

- (a) ‘F’ is true of x iff x is (an) F

and *inflationary accounts of predication*, with their attendant ‘robust’ non-disquotational schema:

- (b) ‘F’ is true of x iff x instantiates the *property* of being F.

(MacBride 2006) offers a less committal formulation of non-deflationary accounts of predication, for he is concerned chiefly with the *semantic* issue of predicate reference, and not with the metaphysics of particulars, universals, properties, tropes, facts, states of affairs. Perhaps, if we can provide a convincing account of (b), not all is lost for correspondence theories of truth, heirs to the logical atomism of Russell and Wittgenstein (cf. e.g. Newman 2002).

As an alternative to (b), (MacBride 2006) suggests the following schema:

(b*) 'F' is true of x iff x instantiates the referent of F.

According to MacBride, the issue of predicate reference is only loosely connected to the ontological commitments undertaken by the bound variables of quantification, and pertains chiefly to questions of identity and identification concerning the referents of predicates (if any). Quine held such questions intractable, and this was one of the reasons why he rejected quantification over attributes, relations, and propositions, and second-order logic in general. MacBride is inclined to think that quantification does not commit us to the entities our bound variables range over unless such a commitment had already been undertaken in language or thought. However, the lively contemporary debates in meta-ontology on the one hand, and the renewed interest in second-order logic and abstraction principles on the other, show that there is no consensus as to how ontological issues should be framed, let alone settled. Quine's criterion is still, in my opinion, a valuable one.

I have mentioned (b*) mainly because it is against *this* way of construing Frege's account of the role of concept-words in predication that Davidson's criticisms in his lectures on predication are directed. However, I am inclined to think that (b*) does not capture the functional model relevant to Frege's account of predication. Besides, talk of 'instantiation' is reminiscent of traditional theories of universals, and is wedded to the subject-predicate model, which Frege considered inadequate to explain both the logical role played by concept-words in deductive argument and the way *concepts* are given to us in thought and language. Frege held that in order to account for predication in general, and for singular predication in particular, we ought to refrain from assimilating the logical role of predicates to the role played by proper names. It is a well-known (and questionable) feature of Quine's (and Davidson's) account of predication that the referential role is discharged not by singular terms but by individual variables occurring in a position accessible to first-order quantifiers.

Schema (b*), on the other hand, may be relevant to Frege's epistemology of language, and can be seen as offering an answer to his characterization of what it is for an object to fall under a concept, and derivatively, what it is for an object to belong to the *extension* (*Umfang*) associated with the concept in question. As is well known, after Russell's discovery of the contradiction, Frege no longer availed himself of the notion of *extension* conceived as a *logical* object associated with a

given concept, while holding fast to his view that concepts are a special kind of function. These two issues (the semantic issue of predication and the epistemological issue of how concepts are given to us in thought and language), though closely linked, should not be conflated. One may subscribe to the functional model¹ without endorsing the basic tenets of Frege's epistemology of language.

2. Compositionality and Truth-Theories

Davidson's seminal paper of 1967, 'Truth and Meaning', opens with the question: How do the meanings of sentences depend upon the meanings of individual words? This is possibly *the* fundamental question of semantics. If wonder is what sets philosophy in motion, this is the appropriate place for it to set in. Frege's *Third Logical Investigation* opens with the observation:

It is astonishing what language can do. With a few syllables it can express an incalculable multitude of thoughts, so that even if a thought has been grasped by an inhabitant of the Earth for the very first time, a form of words can be found in which it will be understood by someone else to whom it is entirely new. This would be impossible, if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure (*Aufbau*) of the sentence can serve as a picture of the structure of the thought. To be sure, we talk figuratively when we transfer the relation of whole and part to thoughts.

(Frege 1923–6: 390)

A key word in the above quoted sentence is *structure*. For, as Frege goes on to remark, 'the question now arises how a thought comes to be constructed, and how the parts are so combined together that the whole amounts to more than the parts taken separately'. This is a version of what (Russell 1903) called the problem of 'the unity of the proposition', which is at the centre of Davidson's enquiry in *Truth and Predication*. Frege's answer to the question of compositionality rests on his distinction between the different semantic role played by saturated vs. unsaturated parts of a thought, where saturation and combination are not meant as temporal processes.² It is well to bear in mind that the part-whole model is not applicable at

¹ By 'functional model' I mean the model to be found in Frege's mature writings, e.g. in (Frege 1891) and (1893), as distinct from the programmatic distinction between function and argument that he put forward in his *Begriffsschrift* of 1879.

² The metaphor of 'saturation' made its appearance very early in Frege's writings, earlier, that is, than his mature view that a concept can be conceived as a special kind of function. As early as 1882, in his letter to Anton Marty, Frege writes: 'A concept is unsaturated, in that it requires something that falls under it; hence it cannot subsist by itself. That an individual falls under it is a judgeable content, within which the concept appears as the predicate, and is always predicative. In this case, where the subject is an individual, the relation of subject to object, is not a third added to the two of them, but belongs to the

the level of *Bedeutung* or semantic value: it does not make sense to say that we discern parts within a truth-value (a logical object). The functional model is not a part-whole model, as Frege himself was at pains to point out, after having helped himself to such an interpretation.³ This is by no means a minor quibble, for Davidson's interpretation of Frege's functional character of concepts rests on the assumption that a function (the *Bedeutung* of a predicate) is a separate ingredient of reality, alongside its arguments and values. To be sure, concepts and functions must be in the domain of second-order quantifiers, but it is not through *this* feature that they discharge their role in predication. At any rate, functions and concepts are better conceived as an *objective pattern* that we discern in the world (or in a structure), not as a separate ingredient of it. Frege thought that to us human beings most concepts are accessible only through the medium of language. The ability to correctly apply a concept-word to an object can be seen as the ability to fit things into a pattern. Concepts, as much as objects, must be given to us under a mode of presentation. He held, moreover, that only concept-words with sharp boundaries are suited to pick out concepts. The semantic role played by a concept-word in predication could be conceived as an *objective method* for fitting arbitrary objects into a pattern: we ourselves may lack the means for finding out whether an arbitrary object fits into the pattern, hence such a method falls short of being a constructive one. Frege's (or Cantor's, for that matter) conception of functions takes it for granted that there is a determinate value that a well-defined function yields for any arbitrary object as argument.⁴ Davidson's picture, on the other hand, fits very well Russell's account of the unity of the proposition to be found in his masterpiece of 1903, *The Principles of Mathematics* (Russell 1903). I will return to this in section 4.

Some philosophers (e.g. Geach 1975) have found the metaphor unintelligible that a thought should have *parts*, and have suggested that we apply the functional model also to the thought. This idea should be resisted, for it misconstrues the relation between sense and reference and takes us back to a conception of content as a complex of representations (a 'Vorstellungsverbinding') which Frege (Frege 1879: §2) had inherited from the Kantian tradition and was deeply dissatisfied with. In fact, he discarded it as soon as he had the functional model fully worked out, i.e. after 1890. As Dummett has pointed out:

The condition for the truth of the thought has to be viewed as the satisfaction by a particular object, given in a certain way, of a certain condition on it. ... In more Fregean language, the sense of the proper names determines an object as its

content of the predicate, which is what makes it unsaturated' (Frege 1976 (tr. 1980): 101, my emphasis). Empty concepts, unlike vague concepts, are admissible in logic, for they too are capable of an ascription of number.

³ Cf. in his essay 'Über Sinn und Bedeutung'.

⁴ For a discussion of this issue cf. (Dummett 1991).

referent; and the sense of a predicate determines a mapping from objects to truth values, that is to say, a concept. The sentence is true or false according as the object does or does not fall under the concept, that is according as it is mapped by it on the value *true* or on the value *false*. The mapping of objects on to truth-values is not the sense of the predicate, but its referent: the sense is, rather, some particular way, which we can grasp, of determining such a mapping. But the sense of the predicate is to be thought of, not as being given directly in terms of a mapping from the sense of proper names on to anything, but rather in terms of mapping of *objects* onto truth-value. (Dummett 1981: 252–3)

I have mentioned the Geach–Dummett controversy over the part–whole model because when reading Davidson I sometimes have the impression that he too, like Geach, construes Frege’s principles of compositionality and functionality as if we could determine what thought is expressed by a sentence *in advance* of determining how its constituent parts—i.e. proper names, quantifiers, identity and predicate expressions—contribute to its truth-conditions. Moreover, Geach’s model suggests that a predicate contributes via its *sense* to the truth-conditions of the sentences in which it occurs, contrary to Frege’s explicit endorsement of the extensional point of view against the ‘intensional’ logicians.⁵ Frege’s statement that the thought expressed by a sentence could be rendered as the thought that its truth-conditions are satisfied (*Grundgesetze der Arithmetik*, vol. 1, §34) could not be more explicit, and has an obvious bearing on the present discussion. This strategy of explanation is not available to those philosophers (such as e.g. (Künne 2005) and (Horwich 1998)) who maintain that the concept of truth and the concept of a proposition can, and hence must, be explained *independently* of each other. Neither Frege, nor Davidson, nor Dummett, for that matter, are among them.

An additional reason for being dissatisfied with Frege’s doctrines is presented in the second part of *Truth and Predication*, and concerns the problem of predication. Predication had already been mentioned in *Truth and Meaning*, for there Davidson wrote:

One proposal is to begin by assigning some entity as meaning to each word (or significant syntactic part) of the sentence; thus we might assign Theaetetus to ‘Theaetetus’, and the property of flying to ‘flies’ in the sentence ‘Theaetetus flies’. The problem then arises how the meaning of a sentence is generated from these meanings. Viewing concatenation as a significant piece of syntax, we may assign to it *the relation of participating in or instantiating*: however it is obvious that we have here the start of an infinite regress. Frege sought to avoid the regress... but this doctrine seems to label a difficulty rather than solve it.

(Davidson 2001: 17, my emphasis)

⁵ I have dealt with such matters elsewhere, e.g. (Picardi 1983) and (1987).

In the course of that paper the building-block conception of meaning—which he attributes to Frege, among others—is criticized and discarded: to be told that the meaning of a sentence depends on the meanings of its component words and the way they are put together does not carry us anywhere, for it appeals to an unspecified notion of meaning. Frege's metaphor of saturation vs. insaturation merely *labels* the difficulty, but does not solve it.⁶ Admittedly, the model of saturation applies also to non-sentential units, such as functional expressions whose value is not a truth-value. Hence the distinction between saturated and unsaturated parts of a compound, without an appeal to truth and truth-conditions, does not, as it stands, 'solve' the problem of predication.

However, it does a good deal more than merely 'labelling' the problem. For, on Frege's own grounds, we cannot account for the predicative role of a concept-word without putting forward a picture of the way in which it contributes systematically to the wealth of thoughts expressible in a language. Since Frege's account of the contribution of the sense of a predicate expression to a judgeable content is inseparable from his conception of how it contributes, in virtue of its semantic value (*Bedeutung*), to the truth-conditions of the sentence in which it occurs, the link is forged between predication and truth, which Davidson himself considers the key to the problem.

3. 'True of'

Davidson admits that Frege had made enormous progress over his predecessors:

So far, Tarski's method has not been distinguished from Frege's except that it associates no entities which express generality with predicates or any entities at all with sentences. The focus on the role of variables or the spaces they occupy is analogous to Frege's, and was inspired by him. Tarski's essential innovation is to make ingenious use of the idea that predicates are *true of* the entities which are named by the constants that occupy their spaces or are quantified over by the variables which appear in the same spaces and are bound by the quantifier.

(Davidson 2005: 159)

Davidson then goes on to mention infinite sequences: it is such sequences that *satisfy* sentences, whether open or closed. Sequences are abstract objects, just as Frege's mappings. However, the satisfaction relation belongs to the meta-

⁶ And yet, as (Sainsbury 1996) points out in a paper devoted to Russell's multiple relation theory of judgement, and sympathetic *ante litteram* to Davidson's approach to the problem of predication, many philosophers nowadays have no qualms in helping themselves to the ingredients of Russellian singular propositions. They hardly pause to reflect on what turns such items into the particular proposition which happens to be the object of belief or judgement for a thinker.

language, and the ontological commitments undertaken in the meta-language should not be mixed up with the ontological commitments undertaken in the object-language. Quite so, but, as we shall see, Frege's notion of *Bedeutung* too, as applied to predicates of a formalized language, can bear a quasi meta-linguistic construal. Davidson does not deny that predicates have 'extensions', for it would be very strange indeed that within an extensional framework co-extensional predicates should not be intersubstitutable *salva veritate*. Co-extensional predicates are true of the same objects. Frege, for one, denied that unsaturated expressions could flank the identity sign; functions can be said to be equivalent, but not identical. Davidson, too, assumes that predicates have extensions, but he does not tell us what sort of abstract objects extensions are; only he insists that we should not think of extensions as sets.⁷

To appreciate the semantic role of predicates is to appreciate how they contribute to the truth-conditions of the sentences in which they occur. Tarski's account provides such a general *method* for specifying the role of each predicate in the language, plus 'a non recursive axiom which says under what conditions it is true of any number of entities taken in the order in which its blanks occur' (Davidson 2005: 161). The only way of avoiding the regress involved in inflationary explanations of predication, while, at the same time, offering an account of the semantic role of predicates, is to employ the 'true of' idiom. Since a language couched in first-order logic does not need set-theory to model its semantics, at no stage in the object-language a commitment to an ontology of sets is undertaken. Doubtlessly, however, Davidson was well aware that there is much in natural language that goes beyond the expressive bounds of first-order logic.⁸

It may not be amiss to recall the reasons why the *Urheber* of the 'true of' coinage thought that we should excise all reference to attributes in dealing with predication. In *Methods of Logic*, section 39 Quine writes:

Despite this correspondence the singular abstract term differs from the general concrete term in an important way: it purports to name one and only one object, abstract object though it be, while the general term does not purport to name at all. The general term may indeed 'be true of' each and *many* things, viz. each red thing, or each man, but this kind of reference is not called naming; 'naming' at least as I shall use the word, is limited to the case where the named object purports to be unique.

⁷ Could they be (as large as) classes, as Parsons surmises on p. 146, footnote 5 of (Davidson 2005)?

⁸ (Lepore and Ludwig 2007) seem to have waved aside Davidson's preference for first-order logic. Moreover, Davidson follows Quine in considering substitutional quantification unintelligible. Whether he follows Quine also in taking second-order quantifiers as first order-quantifiers over sets is unclear to me, owing to the lack of an explanation of what the extension of a predicate is supposed to be.

This is one of the early occurrences of the expression 'true of' in Quine's writings addressed to a general public; the expression had already occurred in his early logical writings. Charles Parsons justly complains that Davidson gives too much philosophical credit to Tarski and too little credit to Quine and to himself (Davidson 2005: 152, footnote 14). It is to Quine that we owe the appreciation of the philosophical significance of Tarski's technical notion of satisfaction. At any rate, most of us were instructed by Quine as regards the advantages of this way of formulating matters over the traditional logic of terms. Peter Geach's work, *Reference and Generality*, has contributed like few others to expose the pitfalls into which the doctrine of supposition can lead us. Quine closes section 39 with an observation characteristic of his whole approach to issues of semantics and ontology:

The distinction between general terms and abstract singular terms is a remnant of medieval logic which some modern logicians do not share my concern to preserve. Actually the significance of the distinction is clearer since the rise of quantification theory than it had traditionally been; singular terms are accessible to positions appropriate to quantifiable variables, while general terms are not. In the foregoing paragraphs it has accordingly been urged that general terms have the virtue, as against abstract singular terms, of letting us avoid or at least postpone the recognition of abstract objects as values of our variables of quantification. Some logicians, however, attach little value to such avoidance or postponement. This attitude may be explained in some cases by a Platonic predilection for abstract objects; not so in other cases, however, notably Carnap's. His attitude is rather that quantification over abstract objects is a linguistic convention devoid of ontological commitment; see his 'Empiricism, semantics, and ontology'. (Quine 1972³: 220–1)

Those who want to dissociate quantification from ontological commitment, and, so to speak, 'travel light' on variable-binding operators, should side with Carnap. Davidson is on Quine's side as regards the philosophical advantages of the 'true of' idiom. However, his emphasis differs from Quine's, because, unlike the latter, he is interested in the construction of theories of meaning and he is wary of the slogan that truth is disquotation that some have read into (Quine 1970). Also his attitude concerning ontological and metaphysical issues is different from Quine's (and Carnap's). 'There is', he says, 'the metaphysical question how particulars are related to properties, and the semantic question how subjects and predicates are related' (Davidson 2005: 83). The metaphysical question, Davidson surmises, may be relatively easy to answer, whereas the semantic question is the difficult one. Hard-boiled students of Quine and Carnap may very well wonder whether, once the semantic question has been settled, there is any (easy or difficult) metaphysical question left to be answered. And, above all, who is supposed to answer *that*

question, and by what means? Davidson's main thesis, however, is not just that the metaphysical question may be easier to answer than the semantic one, but that whatever answer we favour—no matter whether we are nominalists, Platonists, trope theorists, arch-realists, scientific materialists, and what not—it has no bearing on the issue of predication.

But are the metaphysical and epistemological issues which Davidson does not address really irrelevant to the problem of predication? Plainly, those who expect from a theory of predication a clue as to the difference between essential predication and contingent predication will be dissatisfied with Davidson's proposal. Their dissatisfaction, however, does not concern me. Equally dissatisfied will be those who expect from a theory of predication a clue concerning how to handle vague predicates, predication in fictional and metaphorical contexts, predicates with a tendency to generate paradoxes, etc. My concern is with the epistemology of the satisfaction relation.

Davidson's deflationary account of predication is counterbalanced by an 'inflationary' account of truth. It is via the theory of radical interpretation that the connection is forged between truth and belief. But can a theory of belief afford to be silent about the epistemology of concepts? For, after all, in predication we display our mastery of conceptual connections, possibly (though not for Davidson) of conceptual schemes.⁹ Is it really credible that questions concerning concept formation and concept identification should be irrelevant to the *semantic issue* of predication? What is the epistemology of the relation of satisfaction? How does it connect with belief and acceptance? Do all predicates conform to the same pattern when it comes to satisfaction? Towards the close of his lectures on predication Davidson remarks:

It will be noted that I have explained satisfaction in terms of truth. If I were out to define truth, this would be circular. But defining truth is not my aim, for it cannot be done. I was not defining but *using* the concept of truth, which, however beset by paradox, is the clearest and most basic concept we have. What my strategy amounts to, then, is to show how a grasp of the concept of truth can explain predication. (Davidson 2005: 160–1)

But, as Davidson himself had pointed out (Davidson 2005: 75), the concept of truth must connect *essentially* with belief, and belief and acceptance are epistemic notions *par excellence*. It is *this* connection that is left unexplained by Tarski's account. The whole construction would be, as it were, hanging in the air unless we

⁹ For a theory of concept possession that aims to capture certain features of Frege's semantic (and metaphysical) realism cf. (Peacocke 1992). Under the heading of 'concept formation' fall issues relating e.g. to conceptual and objectual abstraction, and to the various kinds of definition by means of which new concepts are added to a given language or theory.

were able to show that the concept of truth is really involved in the *practice* of speaking a language. This is the task of the theory of radical interpretation, whose goal is to give empirical substance to the truth-theory for a language, by connecting semantic notions with speakers' attitudes of holding true directed at sentences. A semantic theory which purports to give an account of what it takes to *understand* the meaning of the most basic form of linguistic unit cannot afford to be silent on questions concerning concept formation and concept identification. For, clearly, Davidson sees more in Tarski's notion of satisfaction than a mere technical device, and this is why more should be said in support of the claim that it is through this relation that language hooks upon the world. If all we can say about the satisfaction relation is exhausted by a statement of the satisfaction clauses, the promise that the theory of radical interpretation will cast light on the connection between truth and belief (and hence meaning and use) is bound to remain unfulfilled.

4. Regress Arguments

In *Truth and Predication* Davidson touches on Plato's truly striking (and soon forgotten) mention of *singular predication* in the last sections of the *Sophist*, rehearses the probable and improbable deeds of Theaetetus, and goes through the regress argument implicit in considering instantiation and participation as the semantic counterparts of concatenation. The regress argument is the master-argument that Davidson employs to undermine a number of attempts made in the history of philosophy to 'solve' the problem of predication by resorting to an assignment of referents to predicate expressions. The problem of the unity of the proposition was especially pressing for the founding fathers of modern logic, and for the author of *Principles of Mathematics* in particular. Indeed, Davidson's regress argument seems tailored on Russell's own account of it, as put forward in chapter 4 of (Russell 1903), and it is well worth reviewing in broad outline.

In §54 of (Russell 1903) Russell remarks:

The verb, when used as a verb, embodies the unity of the proposition, and is thus distinguishable from the verb considered as a term, though I do not know how to give a clear account of the precise nature of the distinction.

(Russell 1903: §54, 50)

In Russell's opinion the difference between a concept occurring as a predicate in the proposition 'Socrates is human' and as a term in the proposition 'Humanity belongs to Socrates' has nothing to do with 'the intrinsic nature of the terms' (Russell 1903: §49, 46). The concept is the same in both instances: the difference is merely grammatical, and has to do with 'external relations'. And yet, this

conception of 'humanity' and 'human' as indicating terms of equal standing causes troubles for the unity of the proposition. We have just reviewed Quine's reasons for *not* making this move. For, given Socrates and humanity, one may be tempted, in order to restore the unity of the proposition, to invoke the *copula* of instantiation, or an abstract relation of participation. Davidson takes it for granted that all the linguistic 'connectors' or copulae that one may be tempted to invoke for connecting terms introduce new *components* into the sentence, which call for new *constituents* in the realm of reference, thus starting off a search for new copulae or higher-order connectors. An alternative open to Russell would have been to enter the difference between object and concept among the 'indefinables of logic', as Frege had suggested.¹⁰ According to Frege (as read by Russell in 1903) concepts, owing to their predicative nature, can only appear as *relating* relations (or as Frege, and later (Russell 1924), will put it, *always* with an indication of their argument-places and range of significance). However, far from maintaining that concepts themselves cannot figure as subjects of predication, Frege insisted that the relation in which a first-level concept stands to a second-level concept under which it falls is analogous to the relation in which an object stands to a first-level concept under which it falls. Frege's hierarchy of functions, much as Russell's later theory of types, prevents the occurrence of sentences such as

'() is mortal is mortal'

In Appendix A of *The Principles of Mathematics*, devoted to an examination of Frege's logical doctrines, Russell comes to the conclusion that 'on the whole, the doctrine of concepts which cannot be made subjects seems untenable' (Russell 1903: §483, 510). Probably Russell's failure at the time to appreciate the beauty of Frege's theory of first- and second-level concepts was due to Frege's own bogus 'paradox' of the concept horse.

As a matter of fact, however, in a letter to Russell, dated 19 June 1902, Frege had already pointed out in passing that the concept of a function is a second-level concept, whereas in spoken language it often masquerades as a first-level concept. Similar consideration might be brought to bear on the so-called paradox of 'the concept *horse* is not a concept'. Frege tried various ways out of his self-inflicted puzzle. For instance, we may try the rendering: 'That which "is a horse" refers to is a concept'¹¹; its second-level formulation as ' $(\exists F) (x) (F(x) \text{ iff } \text{Horse}(x))$ ', sounds tame enough. In the first of his three essays 'The Foundations of Geometry',

¹⁰ For a discussion of this and related issues cf. (Picardi 2003).

¹¹ For a discussion of Frege's way out the bogus paradox of the concept horse cf. (Dummett 1973) and (1981), (Picardi 1994), (Mendelsohn 2005). (MacBride 2006) offers a useful discussion of the relevant literature.

published in 1903, Frege comments in a footnote on Russell's way of using the word 'term' in *The Principles of Mathematics*. He writes:

Russell does not want to concede that a concept is essentially different from an object; concepts too are always supposed to be *terms*. I have treated of this difficulty in my essay *Über Begriff und Gegenstand*. It is clear that we cannot present a concept as independent, like an object; rather it can occur only in connection. One may say that it can be distinguished within, but it cannot be separated from the context in which it occurs. All apparent contradictions that one may encounter here derive from the fact that we are tempted to treat a concept like an object, contrary to its unsaturated nature. This is sometimes forced on us by the nature of our language. But it is merely linguistic. (Frege 1903: 282)

Those who have taken the so-called paradox of the concept horse as a *reductio ad absurdum* of Frege's conception of the predicative character of concepts are, I think, badly off the mark.¹²

Russell, accordingly, discards Frege's solution, but the dual nature of the verb poses problems for his own account of the unity of the proposition:

By transforming the verb, as it occurs in a proposition, into a verbal noun, the whole proposition can be turned into a single logical subject, no longer asserted and no longer containing in itself truth or falsehood. ... 'Caesar died' and 'The death of Caesar' will illustrate this point. If we ask: What is asserted in the proposition 'Caesar died'? the answer must be 'the death of Caesar is asserted'. In that case, it would seem, it is the death of Caesar which is true or false; and yet neither truth nor falsity belongs to a mere logical subject. The answer here seems to be that the death of Caesar has an external relation to truth or falsehood (as the case may be), whereas 'Caesar died' in some way or other contains its own truth or falsehood as an element. (Russell 1903: §52, 48)

Transforming the 'relating' verb into a verbal noun is one of the two ways in which the unity of the proposition is lost. For the function of the verb is not just that of 'relating' the terms but also that of conveying an *assertion*. It appears to Russell

¹² During the last twenty years or so Jaakko Hintikka, Thomas Ricketts, Joan Weiner, Warren Goldfarb, Cora Diamond, and others have claimed that those who praise Frege for his insights on the semantics of natural languages are mistaken. Allegedly, the absence of a distinction between object-language and meta-language together with the lack of the conception of variable domains render semantics *de facto* ineffable for Frege, and *de iure* ineffable for early Wittgenstein. I, for one, fail to see why Frege's contention that only informal elucidations of the 'indefinables' of logic are available should be incompatible with the construction of a formalized language endowed with a semantics. For a different, and more balanced, appraisal of Frege's contribution to the philosophy of language cf. (Heck and May 2006). Linsky's valuable paper of 1992 on the unity of the proposition in Frege and Russell takes the paradox of the concept horse to be fatal for Frege's conception.

that it is the act of *assertion* (in the logical sense) that brings it about that a proposition is *internally* related to truth or falsity. 'The death of Caesar' and 'Caesar died' seem to denote the same complex, and yet only the latter has an *internal* relation to truth and falsehood:

There appears to be an ultimate notion of assertion, given by the verb, which is lost as soon as we substitute a verbal noun and is lost when the proposition in question is made the subject of some other proposition...

Thus the contradiction which was to have been avoided, of an entity which cannot be made a logical subject, appears to have become inevitable. This difficulty, which seems to be inherent in the very nature of truth and falsehood, is one with which I do not know how to deal satisfactorily. The most obvious course would be to say that the difference between an asserted and an unasserted proposition is not logical, but psychological. (Russell 1903: §52, 49)

This is not the *regress* on which Davidson focuses in *Truth and Predication*, but is possibly more instructive as a way of showing that by turning a verb into a verbal noun we sever the connection with the concept of truth, which was later to become also for Russell a defining feature of a proposition (an 'internal' relation, as he and Wittgenstein put it). Russell (tentatively) suggests that what must be added to the complex is not a copula in its role of a connecting glue, but the copula as a bearer of assertoric force. Frege, on the other hand, had split the two roles that traditional logic assigned to the copula thus: in its 'connecting' role 'is' forms part of the predicate, whereas in its role of force indicator it is represented by the vertical stroke of the assertion sign, whose scope is the whole sentence. Hence predication and assertion part company. The 'is' of identity and of concept-subordination are not 'copulae', but expressions indicating the identity function and the conditional connective, which in Frege's formalized language is a function of the first level of two arguments.

The unity of the proposition is lost also by treating the relation as a term, as metaphysical pluralism requires:

Consider... the proposition 'A differs from B.' The constituents of this proposition, if we analyse it, appear to be only A, difference, B. Yet these constituents, thus placed side by side, do not reconstitute the proposition. The difference which occurs in the proposition actually relates A and B, whereas the difference after analysis is a notion which has no connection with A and B... A proposition, in fact, is essentially a unity, and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition.

(Russell 1903: §54, 49–50)

As soon as we analyse the proposition into its constituents, the *relating* relation, expressed by the verb and responsible for binding the constituents into a unit, turns out to be a term on a par with the remaining constituents. In 1903 Russell, who was well conversant with Bradley's regress arguments, failed to see a satisfactory way out of the problem. We cannot discuss all the variants of Russell's accounts of the unity of the proposition during the years 1910–18. We may surmise that Wittgenstein's conception of the relation between propositions and states of affairs, as put forward in the *Tractatus Logico-Philosophicus*, did, to a considerable extent, satisfy Russell. His former pupil, however, did not approve of the theory of types—Russell's full account of predication—but we cannot here go into such matters.¹³ However a hint to the theory of types may not be amiss, for, as I said, this is Russell's full account of predication, an account, that is, that satisfies the architectonic demands I mentioned in section 1 above.

In the essay 'Logical Atomism' of 1924 Russell admits that in 1903 he had badly underestimated the importance of language in general, and had overlooked the many traps that grammar has in store for philosophers and logicians. He writes:

A very unfortunate effect of the peculiarities of language is in connection with adjectives and relations. All words are of the same logical type; a word is a class of series, of noises or shapes according as it is read or heard. But the meanings of words are of various different types, an attribute (expressed by an adjective) is of a different type from the objects to which it can be (truly or falsely) attributed... a relation is of a different logical type from the terms... There is not one relation of meaning between words and what they stand for, but as many relations of meaning, each of a different logical type, as there are logical types among the object for which there are words...

'Brutus killed Caesar' is significant, but 'killed killed Brutus' is nonsense, so that we cannot replace 'Brutus' by 'killed', although both words have meaning. This is plain common sense, but unfortunately almost all philosophy consists in an attempt to forget it. The following words for example by their very nature sin against it: attribute, relation, complex, fact, truth, falsehood, not, liar, omniscience. To give a meaning to these words we have to make a detour by way of words or symbols and the different ways in which they mean...

Attribute-words and relation-words are of the same logical type, therefore we can say significantly 'attribute-words and relation-words have different uses'. But we cannot say significantly 'attributes are not relations'. By our definition of type, since relations are relations the form of words 'attributes are relations' must not be false but meaningless, and the form of words 'attributes are not relations' similarly must

¹³ See e.g. (Griffin 1985), (Candlish 1996), (Sainsbury 1996).

be not true, but meaningless. Nevertheless the statement 'Attribute-words are not relation-words' is significant and true. (Russell 1924 (1972): 153)

Russell's defence, in 1924, of the theory of types, and his new way of addressing Bradley's regress arguments, bear a striking resemblance to Frege's complaints about the logical 'imperfections' of ordinary language.¹⁴ Russell's essay foreshadows Carnap's distinction, as put forward in the *Logical Syntax of Language*, between formal and material modes of expression. Carnap's doctrine of 'Allwörter' echoes, in turn, Wittgenstein's distinction between formal concepts and genuine concepts, as put forward in the *Tractatus Logico-Philosophicus*, while advancing a criticism of Wittgenstein's distinction between saying and showing.

In the *Tractatus Logico-Philosophicus* the saying/showing distinction together with the distinction between formal and genuine concepts enabled Wittgenstein to place the function/argument model in a new light. It is an *Urbild*, a sort of logical archetype, *displayed* in all propositional functions, not a *constituent* of any of them. A sentence does not contain among its constituent the *logical form* it displays: and this is why, among other things, there can be no representative in the proposition of the logic of facts. The logical constants do not *stand* for anything, and certainly do not stand for functions, as Frege had maintained. This is a criticism of Frege different from Davidson's, which cannot be discussed in the present paper. Yet, Wittgenstein's idea that a sentence *displays* its logical form is in harmony with Frege's conception of functional application as the logical *Urform* of simple sentences.

5. Frege's Epistemology of Language

What enabled Frege to get round a problem that was so vexing for young Russell? The answer is suggested by Frege himself. Unlike the author of *The Principles of Mathematics*, he did not begin with the constituents (terms) given in advance, but extracted them out of complete sentences by means of decomposition and analysis. Frege always took pride in this achievement, indeed, he thought that this was a (the?) peculiarity of his approach to logic:

What is distinctive about my conception of logic is that it begins by giving pride of place to the content of word 'true' and then I immediately go on to introduce a thought as that to which the question 'is it true?' is in principle applicable. So I do not begin with concepts and put them together to form a thought or judgement. I come to the parts of a thought by analysing the thought. This marks off my

¹⁴ 'But with regards to unities the question is more difficult. The topic is one with which language, by its very nature, is peculiarly unfitted to deal. I must beg the reader, therefore, to be indulgent if what I say is not exactly what I mean, and try to see what I mean in spite of the unavoidable linguistic obstacles to clear expression' (Russell 1924 (1972): 157).

concept-script from a similar invention of Leibniz and his successors, despite what the name suggests; perhaps it was not a happy choice on my part.

(Frege 1919: 253)

This is no means an *ex post* 'rational reconstruction' of the spirit that had informed his logical work. In an unpublished essay, written around 1882, known as the 'Anti-Boole', young Frege, after giving an example of what he means by the several 'decompositions' to which the same equation, e.g. $2^4 = 16$, is amenable, writes:

Instead of putting a judgement together out of an individual as a subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of a possible judgement. Of course if the expression of the content of a possible judgement is to be analysable in this way, it must already be itself articulated. We may infer from this that at least the ideas of these properties must have their own simple designations. But it does not follow that these properties and relations occur apart from objects: on the contrary they arise simultaneously with the first judgement in which they are ascribed to things. (Frege 1976 (tr. 1980): 17)

The contrast between young Frege and young Russell could not be sharper. Frege's logic is from the very beginning conceived as a symbolic language suited to represent the structure of *sentences* which express *judgeable contents*. It carries with it from the very beginning an essential appeal to the notion of truth which should have received Davidson's enthusiastic approval. In section §9 of *Begriffsschrift* Frege's goal is to show that the argument/function distinction is to be preferred to the traditional analysis in terms of subject and predicate. In dropping one or more occurrences of the same proper names from a sentence (such as 'Cato killed Cato'), we highlight the argument-places of a function where substitutions can be effected. In so doing we extract different concepts from the original sentence (e.g. '*c* killed *x*', '*x* killed *c*', '*x* killed *x*'). To the isolation of the characteristic pattern discernible in an array of sentences Frege attributed the utmost importance. The priority of the sentence over its constituents and components plays a crucial role also in Frege's epistemology of language. Once Frege had fully worked out the programmatic distinction between function and argument outlined in the *Begriffsschrift*, he was able to give a full account of the semantic contribution of concept-words to the truth-conditions of the sentence in which they occur in a predicative role.

He held that concepts are given to beings with our physical and mental constitution only via the medium of language. To be sure, they are not *created* by language, but without a symbolic medium (without signs) we would not have access to them. Young Frege started with the Kantian distinction between

Anschauungen and *Begriffe*, only to find it unsatisfactory on several grounds. Aristotle, as much as Boole, operated with a conception of concepts as given in advance of the sentences by means of which we formulate our judgement, or in restricting artificially the range of such sentences to those exemplifying the subject–predicate model. While we may possibly have access to (spatio-temporal) objects (the ‘unbestimmtes Etwas’ or the ‘determinable X’, or the ‘something = x’ invoked as a substratum in traditional accounts of predication) independently of singular terms, our access to concepts, functions, and relations is made possible only through the employment of a symbolic medium. Only language users can form *sentences* endowed with a grammatical and logical structure that enables them to express a judgeable content, i.e. a content capable of being accepted as true or rejected as false. This claim was perhaps at the time a striking and controversial one. Nowadays, after the so-called ‘cognitive turn’, it is almost universally rejected, except by ‘linguistic’ philosophers, among which the writer of this paper counts herself. Frege found it hard to see how one can do mathematics without a symbolic medium. However, he did not confine his claim to the province of mathematics.

There is no finite (or infinite) stocks of concepts given in advance of our engaging in the practice of forming a *judgement* and putting it into words in some language. Frege replaced the Kantian question: How do concepts combine with intuitions in the synthesis of judgement? With the question: What is the role played by concept-words in the context of a declarative sentence capable of being accepted as true or rejected as false? The answer young Frege suggested to this question in a short essay, ‘On the Scientific Justification of a Conceptual Notation’, published in 1882, is this:

Without symbols (*Zeichen*) we could hardly attain conceptual thinking. Thus in applying the same symbol to different but similar things we actually no longer designate the individual thing, but rather what they have in common: the concept. And we attain the concept only in the process of designating it; for the concept itself is not something that can be apprehended in intuition (*unanschaulich*), but it stands in need of a sensible representative (*er bedarf eines anschaulichen Vertreters*) in order that it may appear to us. Thus a sensible element (*das Sinnliche*) discloses to us the world of what is not sensible.

(Frege 1882: 84, translation modified and completed).¹⁵

In his First Logical Investigation, *Der Gedanke*, of 1918 Frege returns to this cluster of problems, and fits them into his new conception of the third realm.

¹⁵ In the Bartlett/Bynum’s translation the last sentence in the quoted passage is omitted, and, as a result, the reader cannot appreciate the connection between (Frege 1882) and (Frege 1918). Also the allusion to the Kantian *Anschauung* is lost.

Young Frege, however, was no such arch-realist, though he plainly was a conceptualist of sorts. In section 26 of *Grundlagen der Arithmetik* he characterizes 'objectivity' as independence from sensation, intuition, and ideas, but not as independence from reason. The objectivity of arithmetic rests on the objectivity of the *concepts* involved in statements of number, and concepts fall within the province of logic. Logic does not traffic in 'pure forms': it is from the very beginning contentual logic, i.e. a logic apt to express the judgeable contents conveyed by our sentences, irrespectively of their subject matter. In this respect, Frege's conception of logic is very different from Kant's. At the heart of Frege's logicism there is the conviction that concepts, functions, and relations (of first and second level) have the *epistemological* priority over classes, be they conceived as sets, aggregates, or collections. Concepts are apprehended by us (are 'given to reason') in the process of issuing a *judgement* on the content expressed by a sentence that satisfies certain syntactic and *semantic* requirements. A class, as Frege never tires to emphasize, has its *Bestand* in the concept whose extension it is: the latter should not be conceived as the *collection* of objects that fall under the concept in question, but as an individual (logical) object. However, with the help of concepts alone, we are unable to justify the claim that numbers are individual logical objects—a claim that Frege regarded as crucial for his foundational programme.

Frege's logicist epistemology mandates an objectivist conception of concepts, functions, and relations. His *epistemology* requires that concepts be construed as that which concept-words refer to in the context of a sentence. The notion of reference as applied to predicates is therefore forced on us. However, predication requires that the semantic role of predicates should not be construed according to name-bearer prototype. Perhaps one could try to develop an *objectivist* conception of functions and relations, short of a full-fledged Platonist construal of them. My suggestion that the semantic role of predicates be seen as an *objective method* for fitting objects into a pattern goes in that direction.

6. The *Bedeutung* of Predicates in Formalized Languages

In *Truth and Predication* Davidson writes:

... any attempt to give full explications of the semantics of predicates by associating them with objects of any kind is doomed. ... Nor will it help to distinguish, as Frege did, between objects, which is what singular terms refer to, and concepts, which is what one-place predicates refer to. To say that predicates are functional expressions, and that therefore are incomplete or unsaturated, and that what they refer to is similarly full of holes or spaces waiting to be filled in, does not help: entities are entities, whatever we call them. (Davidson 2005: 136)

Now, if all that Frege had to offer were a bunch of metaphors, few would disagree with Davidson. But before we dismiss Frege's insight into the functional character of concepts, we should try to understand the rationale behind the metaphor by recalling the peculiar features of Frege's formalized language, as put forward in the first volume of *Grundgesetze der Arithmetik*. Dummett's emendation in *The Interpretation of Frege's Philosophy* (1981) of his earlier (Dummett 1973) criticism of Frege's functional characterization of concepts is now seen by Davidson as a move backwards. One of Dummett's reasons for being ill at ease with Frege's account of concepts as functions is Frege's assimilation of sentences to complex terms, standing for one of the two logical objects, the True and the False. But both the idea that True and False are logical objects, and the conception of sentences as complex names of such objects can be given up or reinterpreted, without abandoning Frege's important insight into the semantic role of predicates. The counterintuitive construal of sentences in Frege's formalized language could be (and has been) remedied by appeal to categorial grammar or type-theoretic approaches, which make finer distinctions within the category of terms. Besides, Frege himself emphatically denied that in *asserting* a sentence we are *naming* anything: this is what the assertion sign is there for. It is a fact, however, that Frege's syntax is hospitable to many semantic interpretations—including combinatorial, type-theoretic, constructivist ones. Under all semantic interpretations we need to know what the *semantic value* of a predicate is supposed to be. Frege took the semantic value of a concept-word (an expression with one or more argument-places) in predicative position as consisting in a mapping from objects to truth-values, whereas Tarski, like most mathematicians, took the semantic value of the predicates in the object-language to be a set. Davidson, too, maintains that predicates have an extension. His main claim is that no matter how such 'extensions' are construed, they play no role in *explaining* predication. There is a further charge that Davidson levels against Frege, i.e. the duplication of semantic entities corresponding to predicates. For he writes:

If we take predicates as referring to entities we introduce a shadowy level of explanatory machinery between the expressions and the work they do. . . . But if predicates have a referent, this is in *addition* to their sense and extension. This is the wheel that becomes redundant: to describe the semantic value of a predicate is not to introduce another level of explanation. (Davidson 2005: 139)

Here Davidson mentions the notion of extension, but 'extensions' were conceived by Frege as the referents of names of value ranges, and they never played a role in his account of what it is for an object to fall under a concept, or what it is for a first-level concept to fall within a second-level concept. Because of extensionality, all concepts-words that take the same value for the same arguments count as standing to each other in a relation analogous to that of identity.

In his book *The Interpretation of Frege's Philosophy* Dummett argues that the correct construal of the notion of *Bedeutung* is that of semantic value—a *pro-grammatic* notion, which can be variously interpreted, depending on the theory of meaning one subscribes to. I would go even further. Frege's notion of *Bedeutung* has a dual role. It functions as a *theoretical* concept required for laying down the semantic contribution that different types of expressions make to the truth-conditions of sentences belonging to a formalized language. In *Grundgesetze der Arithmetik* Frege employed the notion of *Bedeutung* for spelling out, within the object-language, as it were, an equivalent of the recursive clauses in the definition of satisfaction that Tarski formulated in the meta-language. Frege's specification is viciously circular, but this is a different matter.¹⁶ Thus in stating in the *formal mode*, relative to a given formalized language, the truth-conditions of 'Theaetetus flies' we can say that the sentence in question is true iff the function designated by 'x flies' maps Theaetetus on to the True, and is false otherwise.

Let us consider more closely the notion of satisfaction. We may say that a predicate is true of an object or a sequence of objects, but also that there is a *function* that assigns objects to variables as satisfying a predicate; it satisfies it when the result of interpreting the free variables in it as referring to the object assigned to them by the function is true. This formulation in terms of functions is the one actually used by Tarski in his essay of 1936. Thus, to put it a bit impressionistically, we may say that Frege's mistake, according to Davidson's reconstruction, is due to his ignorance of the distinction between meta-language and object-language. It is in the meta-language that we need to appeal to the existence of a function of the sort just described: no function needs to be mentioned in stating, within the object-language, the truth-conditions of simple and compound sentences. Once we appreciate that the *theoretical* role of the notion of *Bedeutung*, as applied to a functional expression within a formalized language, does not consist in assigning a referent to the predicate, but rather in spelling out, in the object-language, how e.g. the predicate 'sits' contributes to the truth-conditions of sentences which can be constructed in the formalized language, the contrast between Frege and Tarski appears less stark. The matter is, I think, really entangled, for though it is true that the distinction between object-language and meta-language appears only very late in Frege's writings, he made an attempt to state the semantics of his formalized language within the very same language. It is by no means clear that all such attempts are doomed to failure, and that Frege's strategy is hopeless. However, if we see matters from this angle, would Frege's appeal to functions as the semantic value of concept-words still be unwarranted *also* from Davidson's own perspective?

¹⁶ Cf. (Kutschera 1989: ch. 7) and (Dummett 1991).

Frege's notion of *Bedeutung*, however, does not play just the role of a theoretical tool, useful for laying down a systematic assignment of truth-conditions for the sentences of a formalized language. The notion of *Bedeutung* is meant to account for what it is to *understand* a sentence belonging to a natural language, built out of words whose sense and mode of composition is familiar to the speakers of the language. The understanding of the thought expressed by a sentence of a given language requires that we have a conception of what it is for a given name to designate an object and of what it is for a certain predicate to determine a method for fitting objects in a pattern of discerned similarities. The pattern is not of our own invention: it is the joint work of mind and world.

Davidson's criticisms notwithstanding, Frege's account of predication still strikes me as the best one at our disposal. One may wonder whether the metaphor of saturation is a felicitous one for elucidating predication. Here opinions may differ. Nonetheless, it may not be amiss in this context to recall Frege's conception of the constraints by which the originators of new metaphors should abide:

The purpose of elucidations is a pragmatic one; and once it is achieved, we must be satisfied with them. And here we must be able to count on a little goodwill and cooperative understanding, even guessing; for frequently we cannot do without a figurative mode of expression. But for all that, we can demand from the originator of an elucidation that he himself know for certain what he means; that he remain in agreement with himself; and that he be ready to complete and emend his elucidation whenever, given even the best of intentions, the possibility of misunderstanding arises. (Frege 1906: 301, translation slightly modified)¹⁷

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¹⁷ For criticisms and suggestions I wish to thank Fabrizio Cariani, Antonio Ferro, Daniele Mezzadri, Carlo Penco, and Elisabetta Sacchi.

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Davidson and Frege on the Unity of the Proposition

Some Remarks

Davidson's seminal paper of 1967, 'Truth and Meaning', opens with the question: How does the meaning of a sentence depend upon the meanings of the words that occur in it? This is possibly *the* fundamental question of the philosophy of language. If wonder is what sets philosophy in motion, this is the appropriate place for it to set in. In that paper Davidson already alluded to the problem of predication, for there he writes:

One proposal is to begin by assigning some entity as meaning to each word (or significant syntactic part) of the sentence; thus we might assign Theaetetus to 'Theaetetus', and the property of flying to 'flies' in the sentence 'Theaetetus flies'. The problem then arises how the meaning of the sentence is generated from these meanings. Viewing concatenation as a significant piece of syntax, we may assign to it the relation of participating in or instantiating, however it is obvious that we have here the start of an infinite regress. Frege sought to avoid the regress by saying that the entities corresponding to predicates (for example) are 'unsaturated' or 'incomplete' in contrast to the entities that correspond to names, but this doctrine seems to label a difficulty rather than solve it.

(Davidson (1967), 2001, p. 17)

That paper contains an early criticism of Frege's 'building block' account of sentence meaning, and a plea for Tarski's semantic conception of truth as a key to a theory of meaning for natural languages. In Davidson's Lectures on Predication, delivered in Perugia in 2001 (and which appeared in print in 2005, as the second part of his posthumously published book, *Truth and Predication*), the sentences 'Theaetetus flies' and 'Theaetetus sits' occupy once more the centre of the philosophical stage. One of the problems addressed in those lectures is what fosters the unity of a proposition, a question which was left hanging in the paper of 1967. It will not come as a surprise to students of Davidson to learn that an additional bonus of Tarski's theory of truth is that in the process of establishing a close connection between the semantic role of predicates and the notion of truth, it also illuminates the problem of what turns a collection of words into a unity

capable of expressing a sentence that can be used in the performance of speech acts.

As is well known, Frege never tired of emphasizing that propositions (thoughts, in his terminology) are the only semantic units that can feature unasserted as antecedents of conditionals and as sub-sentential constituents of compound sentences. If our goal is to understand deductive inference, it is to such contents that we have to pay attention, and not to acts of synthesis or judgement. Once this step is taken, the concept of truth almost inevitably forces itself upon us in an account of predication and inference. How this concept is to be interpreted is a different and more difficult question. A caveat is in order here. According to Robert Brandom (2000), Frege's early writings, and his *Begriffsschrift* of 1879 in particular, lend themselves to an inferentialist reading (as opposed to a representationalist reading). If we endorse Brandom's interpretation, Davidson's main criticism of Frege, i.e. that in construing predicates as *standing for* incomplete entities (such as functions or concepts) he falls victim to the very regress he had helped uncovering, loses much of its force. In 1879 Frege had not yet formulated the sense-reference distinction, and hence he could not make the mistake that Davidson attributes to him. Robert Brandom has urged that we consider Frege as a proto-inferentialist. The locus classicus on which Brandom rests his interpretation of Frege as a proto-inferentialist is §3 of *Begriffsschrift*, where he writes:

A distinction between *subject* and *predicate* finds *no place* in my representation of judgement. To justify this, I note that the contents of two judgements can differ in two ways, either the conclusions that can be drawn from one when combined with certain others also always follow from the second when combined with the same judgements, or else this is not the case. The two propositions 'At Plataea the Greeks defeated the Persians' and 'At Plataea the Persians were defeated by the Greeks' differ in the first way. Even if a slight difference in sense can be discerned, the agreement predominates. Now I call that part of the content that is the *same* in both the *conceptual content*. Since *only this* has significance for the *Begriffsschrift*, no distinction is needed between propositions that have the same conceptual content. (Frege (1879) 1997, p. 53)

Here Brandom detects the seed of the idea that concepts are individuated by their inferential role. In 1879 Frege does not appeal to semantic notions for elucidating the semantic role of predicates. Correctly so, in Brandom's opinion, for he believes that the notion of reference plays a subsidiary role in explaining the characteristic features of the conceptual. It is not as if Brandom held that this notion should be excised as irrelevant, on the contrary, it has a role to play, but only a subsidiary one. His logical expressivism demands that we are to keep clear of the conception of logic as concerned with the laws of being true held by Frege after 1879. Logic has to do with the most general laws of judgement, as Frege tells us in his

Begriffsschrift. Frege's semantic doctrines are seen by Brandom as backsliding into a representationalist, pre-Kantian conception of content. Indeed, Brandom goes so far as to suggest that since Frege deemed truth undefinable and opposed correspondence theories of truth, his conception of a thought is not 'representational'. To be sure, 'representational' is as tricky term. As is well known, Frege held that the word 'true' should not be construed as expressing a relation between a thought and a fact. However, the claim that truth is primitive and undefinable has no tendency to show that there is no *representational* ingredient in Frege's concept of a thought. According to Frege the essence of the *concept* of truth is enshrined in assertoric force and not in the word 'true'—which he held (wrongly) unable to make a meaning contribution of its own to the sentence in which it occurs. Judging is characterized by Frege as a mental act in which thinkers engage when aiming at truth. But assertion is not just the outer manifestation of such inner acts; it involves the social dimension of language, which Frege saw as the only (albeit imperfect) vehicle both for the expression and the communication of thoughts. Truth and assertion have, indeed, to be elucidated together, but this does not in the least indicate that Frege held that inferential connections are metaphysically prior to semantic relations. At any rate, since I don't want to depart too much from Frege's conception of semantics, the inferentialist strategy is not open to me.

As is well known, Frege appealed to the metaphor of saturation in order to elucidate the reasons why construing a concept (the *Bedeutung* of a predicate expression) as a function from objects to truth-values offers a solution *both* to the problem of predication (i.e. what it is for an object to be *subsumed* under a concept) and to the problem of propositional unity (what turns a list of words into a sentence capable of expressing a thought). However, in Davidson's opinion, Frege's conception of predication is marred by a number of serious defects. Davidson holds that the assignment of a *Bedeutung* to a predicate expression not only conflicts with Frege's own emphasis on the predicative character of concepts, but threatens to reinstate some form or other of the regresses which Plato made us familiar with in his dialectical dialogues. Unlike Davidson, I believe that Frege has done a good deal more than merely 'labelling' a difficulty, or, rather, a cluster of difficulties. For, on Frege's own grounds, we cannot account for the predicative role played by a concept-word in a singular sentence without putting forward a picture of the way in which it contributes systematically to the wealth of the simple and compound sentences (including quantified ones) expressible in a given language. Since Frege's account of the contribution of the sense of a predicate expression to a judgeable content is inseparable from his conception of how it contributes, in virtue of its semantic value (*Bedeutung*), to the truth-conditions of the sentence in which it occurs, the link is forged between predication, truth, and propositional unity which Davidson himself considers the key to the constellation of problems addressed in *Truth and Predication*. Indeed, Davidson admits that Frege had made enormous progress over his predecessors:

So far, Tarski's method has not been distinguished from Frege's except that it associates no entities which express generality with predicates or any entities at all with sentences. The focus is on the role of variables or the spaces they occupy are analogous to Frege's, and was inspired by him. Tarski's essential innovation it to make ingenious use of the idea that predicates are *true of* the entities which are named by the constants that occupy their spaces or are quantified over by the variables which appear in the same spaces and are bound by the quantifier.

(Davidson (2005), p. 159)

One of the advantages of Tarski's account of predication (in terms of satisfaction) over Frege's is that it does not require that the predicate that occurs in an open sentence be assigned a referent of any kind: the predicate is said to be 'true of' objects in a sequence, according to a certain assignment of referents to the free variable(s) occurring in the open sentence. In Davidson's opinion this move has the chief advantage of preventing a vicious regress in the explanation of what it is for a predicate to apply to an object. The regress argument is the master argument used by Davidson for undermining Frege's account of the semantic role played by concept-words in predication. As a matter of fact, it is to Quine, and not to Tarski, that we owe the appreciation of the philosophical merits of the 'true of' idiom. However, it is not ontological parsimony in Quine's spirit that motivates Davidson's rejection of predicate reference. Davidson sides with Quine on many issues, but differs from him in his attitude as regards ontological and metaphysical problems. 'There is', he says, 'the metaphysical question how particulars are related to properties and the semantic question how subjects and predicates are related' (Davidson (2005), p. 83). The metaphysical question, Davidson surmises, may be relatively easy to answer, whereas the semantic question is the difficult one. Whatever answer we give to the metaphysical question—no matter whether one is a nominalist, a Platonist, a trope theorist, a scientific materialist, or what not—it has no bearing on the issue of predication, though it may well bear on other issues, such as causation and scientific explanation. Frege's hierarchy of concepts requires that concepts be in the domain over which the bound variables of quantification range if the theory is to provide an account of the meaning and truth-conditions of arithmetical statements. This was the point behind Frege's distinction between characteristic marks and properties of a first-level concept (cf. Frege 1884). Frege's logic was born as second-order logic, and this is why it was so natural for him to connect two issues that Davidson insists that we should keep separate, the semantic role of concepts-words in predication (their yielding a truth-value when their argument place(s) are filled with proper names) and their semantic role in singling out first-level functions from objects to truth-values for second-level functions to take as their arguments. In a full-fledged theory of second-order logic Frege's strategy is the natural one. To be sure, the interpretation of second-order logic is riddled with problems, but this is a different

issue. Quine (1953) argued (a) that quantification gives us a clue concerning the ontological commitments undertaken by a theory, and (b) that quantification over properties, concepts, functions, universals is unintelligible, since the alleged entities have no clear-cut conditions of identity. In the absence of such conditions, we have no grounds for treating properties and universals as part of our ontology. Classes, unlike properties, are provided with conditions of identity, and Quine has no qualms concerning classes, provided that they prove indispensable in the best scientific theory at our disposal. Davidson, as I said, differs from Quine in this respect, and I take it that it is not because he follows Quine in his mistrust of second-order logic that he denies that predicates have a reference of sorts.

While I agree with Davidson that it is not via *naming* a referent that a concept-word discharges its role in first-order predication, I remain unconvinced by his claim that functions (properties, relations, concepts, or whatever one chooses as the referent of a predicate) have no role to play in an account of singular predication. As Tyler Burge (2007) has pointed out, this is a very strong and controversial claim. A skeletal account of predication as the one recommended by Davidson tells us nothing about the epistemology of predication and gives us no clue concerning our mastery of the variety of concepts involved in acts of judgement. The robust conception of truth which Davidson emphatically endorses, though pivotal in his enterprise of radical interpretation, seems to place no constraint on metaphysics and epistemology. As a result, the notion of proposition (and of propositional unity) to which Davidson alludes remains purely programmatic. I have discussed elsewhere (Picardi 2008) the reasons why Frege's account of predication still seems to me the best at our disposal, Davidson's criticisms notwithstanding. In the present paper I would like to dwell a bit on the connection between the issue of predication and the issue of propositional unity. Davidson describes the connection thus:

If we can show that our account of the role of predicates is part of an explanation of the fact that sentences containing a given predicate are true or false, then we have incorporated our account of predicates into an explanation of the most obvious sense in which sentences are unified, and so we can understand how, by using a sentence, we can make assertions and perform other speech acts.

(Davidson (2005), p. 155)

In the passage quoted above Davidson connects, indeed, seems to identify, the problem of predication with the problem of the unity of the proposition. The problems are indeed connected, but I suggest that we keep them apart. A follower of Frege may subscribe to the functional model of predication at the extensional level, without accepting his picture of what turns the component senses expressed by the logically significant parts of a sentence into a unity: she may find talk of unsaturated and saturated parts of a thought unhelpful, if not straightforwardly

unintelligible. Similarly, one may agree with Davidson's criticism of Frege's account without conceding that Tarski's account of truth (and satisfaction) also solves the problem of the propositional unity. A lot depends on what we mean by *proposition*. It is relatively easy to criticize Frege's conception of an unsaturated sense as a mode of presentation of an unsaturated entity; however, since we have no clue as to what, if anything, Davidson wants to put in place of Frege's saturated and unsaturated senses, we have no clue as to what he means by a propositional unity either, though we know full well how he thinks that the recursive account is supposed to work at the extensional level.

In Davidson's writings there is a fluctuation between talk of sentences and talk of propositions, which in the context of the present discussion may be confusing. As is well known, Davidson is wary of propositions conceived as abstract objects. The goal of his 'interpretative' truth-theoretical account of meaning is to connect speech with belief and action within the framework of a general conception of rationality. To invoke the notion of a proposition (or of a thought, as Frege did) in order to spell out what a sentence of a given language *says* is a move devoid of *explanatory* value in Davidson's truth-theoretical account. And this seems to me undoubtedly correct. However, on accounts of meaning different from Davidson's the notion of a proposition can be given a different interpretation, and our understanding of matters is better served by a modicum of terminological accuracy. The philosophical literature is full of such accounts, but Davidson takes no notice of them. All these caveats must be borne in mind especially when we review the issues of predication and propositional unity from a historical perspective.

For, one may argue that the issue of what turns a string of words into a sentence of English or Italian belongs to grammar or syntax, whereas the issue under what conditions a sentence of English or Italian expresses a proposition belongs to the philosophy of language. As Davidson points out in his breathtaking historical excursus from Plato's *Sophist* to the present day's literature, such problems were intensely debated by the pioneers of modern logic, and by Frege, Russell, and Wittgenstein in particular.¹ Only Frege and Russell feature in Davidson's historical narrative, and this is a great pity, for Wittgenstein's *Tractatus Logico-Philosophicus* contains the seeds of a solution to some of the problems left open by Frege and reviewed by Davidson. In his *Tractatus Logico-Philosophicus* Wittgenstein argued that Frege had misconstrued both the notion of truth and the role it plays in an account of logic and language. Davidson thinks that Tarski too had failed to appreciate the role of the concept of truth in language and thought, but for reasons very different from those given by Wittgenstein. Tarski's failure consists in ignoring the connection between truth and belief, thus

¹ A vast literature has piled up in recent times on this topic. Cf. for instance Gibson (2004), Gaskin (2008), Candlish (2007), Potter (2009), just to name a few. Bell (1979) and Dummett (1973 and 1981) offer illuminating discussions of Frege's conception of propositional unity.

overlooking the possibility of a fruitful application of the recursive apparatus he had deployed in his account of the truth-predicate in formalized languages to the construction of empirical truth-theories for natural languages. With Wittgenstein Davidson also shares a misgiving concerning Frege's notion of assertion and assertoric force. And yet, in my opinion, appeal to the notion of *assertoric force* is precisely what is required in order to shed light on the *normative* dimension that Davidson and Frege were eager to read into our employment of the concept of truth in thought and talk.

But there are various reasons, independently of the exegesis of Frege's work, why the two issues should be kept apart. For one thing, many linguistic utterances can be used in communication for conveying a content that are not amenable to the format of Tarski's truth-theory. One needs considerable theoretical ammunition to back the claim that (a) singular sentences (sentences obtained by filling in the blanks of a predicate with the same or with different proper names) are the basic semantic units in all natural languages, and (b) that all remaining sentential forms are either reducible to, or explainable in terms of, such basic units. Luckily, the grammatical theory of ellipsis has fallen today into a state of well-deserved disrepute. Suffice it to think of the opening paragraphs of Wittgenstein's *Philosophical Investigations* or of Austin's performative utterances to appreciate the point. We may perform all sorts of linguistic acts by using utterances that are not amenable to a representation into function and argument. And yet, a representational content may perhaps be detected in an utterance even in the absence of a canonical representation *à la* Frege/Tarski. What the pioneers of modern logic were looking for was a representation of the content of a sentence that could feature not just in isolation (as it is the case with utterances of 'Slab', 'Pillar', 'Help', 'Fire!', 'Ouch', 'Ah!'), but as the antecedent of a conditional, and, more generally, as a component of sentences built up by means of sentential connectives and quantifiers. In those days the problem of the unity of the proposition merged naturally with that of finding a symbolic representation of the structural features of sentences that are relevant to an account of deductive inference and logical consequence.

Now, if one defines a proposition as that to which the calculus of truth-functions is applicable, then the link between truth and predication is forged from the very beginning, and the function/argument model offers a satisfactory account of predication which smoothly generalizes to sentences with multiple quantification. This is Frege's deep and lasting insight, as Davidson himself recognizes. However, there is no reason outside logic (and logic books) why a proposition should be *identified* with that which is expressed by a declarative sentence capable of being accepted as true or rejected as false. No doubt, this characterization has a very ancient tradition, and if we consider it as a stipulation concerning the way one intends to use the word 'proposition' there is little room for disagreement. However, Frege himself was ready to acknowledge as 'thoughts'

the contents expressed by sentences with truth-value gaps. On the face of it, many singular sentences with empty names are *thinkable*: it looks as if an object is subsumed under a concept. And yet either we give up bivalence, or the functional model is not applicable as it stands to predicates whose blanks are filled with proper names that, for whatever reason, turn out to be empty. Many philosophers have found this aspect of Frege's conception untenable; yet, there is little doubt that he put it forward in his published writings, and that he did not use the notion of truth and falsehood as a defining mark of what he called a 'thought', although in his 'First Logical Investigation' of 1918, *Der Gedanke*, he averred that truth stands in the *closest* connection with thoughts. The alternative is to deny that under such circumstances we are actually dealing with singular thoughts. This way out has some plausibility in the case of singular perceptual judgements, but cannot be generalized to all sentences with empty names. Russell's way out of Plato's worry how something true can be said as regards that which does not exist, without endowing the latter with some kind of being, is committed to a variant of the latter move. There may be predication in the presence of truth-value gaps, and there may be truth in the absence of the sort of propositional unity that a sentence bears on its grammatical surface. Davidson does not pronounce on sentences with empty names, and in consequence we are not in a position to tell how the connection between predication, truth, and assertion is to be construed in such cases.

Moreover, Frege offered many different characterizations of what he called a 'thought'. He described a thought as the truth-evaluable core that synonymous sentences of the same language, stripped of all ingredients of tone or colouring, have in common and that which ought to be preserved under translation into different languages. He also characterized the thought expressed by a sentence as that which sentences put forward in making assertions and asking sentential question have in common. A thought is also what is named by a that-clause, introduced by verbs of saying and of propositional attitudes. A feature of Fregean thoughts, conceived as the objects of belief and knowledge, is that they are very fine-grained. A truth-theoretical account *à la* Davidson is bound to deliver truth-conditions which are much less fine-grained. A thought is also that which results when one completes a sentence with indexicals with the information extracted from the context of utterance. What Frege called 'thought', contemporary philosophers call 'proposition': one of the issues currently under discussion is whether there is *one* notion of proposition which can fulfil all the different tasks that Frege assigned to thoughts.

A further reason why we may want to distinguish the issue of predication from that of propositional unity is that much more has to be said if we want to give a philosophical account of how it comes that in uttering a *token* of a sentence *type* on a given occasion we perform a linguistic act. To offer an account of the formal unity of sentence types is well within the reach of formal semantics, but as

Davidson himself has emphatically stressed after 1986, to account for the *use* of sentence tokens in communication we cannot just rest content with such a meagre structural insight. In the course of an article devoted to literary language Davidson wrote:

Words, Frege emphasised, have meaning only in the context of a sentence. . . . It is at the sentential level that language connects with the interests and intentions language serves, and this is also the level at which the evidence for interpretation emerges. But just as words have meaning only in the context of a sentence, a sentence has meaning only in the context of use, as part, in some sense, of a particular language. There would be no saying what language a sentence belonged to if it were not for the actual utterances or writings, not, perhaps of that very sentence, but of other sentences appropriately related to it. So in the end the sole source of linguistic meaning is the intentional production of *tokens* of sentences. If such acts did not have meanings, nothing would. *There is no harm in assigning meaning to sentences*, but this must always be a meaning derived from concrete occasions on which sentences are put to use.

(Davidson (1993) 2005a, 170; my emphasis)

Like Davidson, I believe that no harm is done in assigning meaning to sentences but, unlike him, I believe (a) that this move commits us to the recognition of semantic entities, i.e. word-meanings and sentence meanings, and thus involves a radical departure from robust meaning holism; and (b) that the order of explanatory priority suggested by Davidson conceals an ambiguity. The meaning of words cannot be derived *just* from the concrete occasions in which sentences are put to use, for in order to understand what a *newly* uttered sentence means we have to resort to our knowledge of the meanings of its component words. It is only at the very early stages of the enterprise of radical interpretation (and radical translation) that the order of priority suggested by Davidson (and Quine) is appropriate; but as soon as we apply our provisional truth-theory to new utterances, we avail ourselves of the clauses of satisfaction and designation concerning individual words. Such clauses are revisable, but are clauses concerning word-meaning nonetheless. In Frege's terminology, our grasp of the *thought* expressed by the utterance of a given sentence is made possible, at least in part, by the grasp of the meanings of its component words and of the pattern discernable into it. By 'pattern' I here mean, following Frege, one of the various decompositions into function and argument to which a sentence lends itself. However, as almost everybody is ready to admit nowadays, grasp of sentence meaning is often insufficient to determine the *content of the assertion* made by means of it—i.e. the particular thought conveyed on a certain occasion made in uttering the sentence. This is invariably the case when token-reflexive words are explicitly involved, but, as I said, the phenomenon is much more widespread. Indeed, one may wonder to what extent one is entitled to

apply the type/token distinction to describe language *use*, if a gap is generally to be expected between sentence meaning and utterance meaning.

All the different conceptions of propositions available in contemporary philosophical literature have the status of theoretical tools that we use in our attempt to understand and systematize linguistic practice, and their success or failure is to be measured on a global scale. I do not here mean to enter the maze of the intricate issues I have just alluded to.² I only want to point out that the connection between truth and predication on the one hand, and between predication and the unity of the proposition on the other, is not as straightforward as Davidson intimates. As I said, Frege's (and Tarski's) functional model of predication can be incorporated into many accounts of propositions that differ greatly from one another. Whether Frege's notion of unsaturatedness and incompleteness helps us in approaching the issue of the unity of the proposition is what I propose to discuss in the next section. Frege explicitly touches on the problem of the unity of the proposition in the opening remarks of his 'Third Logical Investigation'. There he writes:

It is astonishing what language can do. With a few syllables it can express an incalculable multitude of thoughts, so that even if a thought has been grasped by an inhabitant of the Earth for the very first time, a form of words can be found in which it will be understood by someone else to whom it is entirely new. This would be impossible, if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure [*Aufbau*] of the sentence can serve as a picture of the structure of the thought. To be sure, we talk figuratively when we transfer the relation of whole and part to thoughts.

(Frege 1967 (1984), p. 390)

We discern a pattern in a sentence, and with its help we identify relevant logical units. But to what extent do the discerned parts of the sentence contribute to the identification of the thought expressed? For, after all, Frege laid emphasis on the possibility that the same thought be expressed by different sentences of the same language. He also laid stress on the possibility that the same sentence be analysable in such a way as to give rise to different decompositions into argument and function. What appear as linguistic subject and predicate of the sentence give us only a hint as regards logical arguments and functions.

A key word in the above quoted sentence is *structure*. For, as Frege goes on to remark, 'the question now arises how a thought comes to be constructed, and how the parts are so combined together that the whole amounts to more than the parts taken separately'. This is a version of what Russell (1903) called the problem of 'the unity of the proposition'. One of the reasons why we must refrain from

² But cf. Picardi (2006) and Picardi (2009).

conceiving of saturation as a temporal process is that the unity of a thought is not brought about by our putting together parts in acts of affirmation, nor is its unity dissolved by breaking it up into parts in acts of denial. Frege urges that we conceive of the structural *constitution* of thoughts as independent both of acts of judgement and of the particular means of expression employed. And this is why the problem of the unity of a thought (or proposition) does not reduce to that of the unity of a sentence that can be seen as expressing it.

But how is the notion of unsaturation to be interpreted at the level of sense? What is it that Frege wanted to elucidate by means of this notion? What seems involved here is a part-whole model, which, as Frege pointed out, differs from the functional model. The functional model is not a part-whole model, as Frege himself was at pains to point out, after having helped himself to such an interpretation.³ If the semantic value of a sentence such as 'Theaetetus sits' is a logical object (one of the two truth-values), it does not make sense to say that we discern parts with it. However, if the semantic value of the sentence in question is construed as a Tractarian state of affairs, or as a structured Russellian proposition, the idea of discerning parts is not *prima facie* implausible. The problems that beset such accounts, however, are well known, and relate to the difficulty of singling out the situation to which the sentence in question corresponds, if true, from the rest of all possible candidates. According to Davidson, it is futile to appeal to facts or situations in order to explain truth (and hence predication), and this, of itself, provides a further reason for rejecting those accounts of predication that ascribe a *referent* both to the predicate expression ('sits') and to the proper name 'Theaetetus', conceived as ingredients of what has to be the case for the sentence to be true. If we conceive the sentence itself as a fact, along Tractarian guidelines, it is the grasp of the different *symbolic* roles of capital letters and small letters that accounts for the unity of the proposition.

Both in his essay of 1967 and in *Truth and Predication* Davidson seems to hold that the regress problems that affect meanings, conceived as referents, apply just as well to meanings conceived as senses. And this is also why, I suppose, he holds that the problem of predication is closely connected to problem of the unity of the proposition. But this connection is by no means obvious. In fact, I would like to argue, on the one hand, that the model of functional application faces no regress at the level of predication, and, on the other, that no such model is operative at the level of sense. Hence, if a regress is lurking somewhere, it must be of a very different kind. It is true, though, that the conflation of the two sorts of regress goes back to Frege himself, and it is instructive to quote in full Frege's diagnosis of the regress, for a second dubious feature of the alleged regress comes to the fore, i.e. a confusion between what a sentence literally *says* and what its philosophically

³ Cf. in his essay 'Über Sinn und Bedeutung' (1892).

enriched *paraphrases* construe it as saying. As Burge (2007) has pointed out, at times Davidson seems to make a similar mistake.

The notorious paradox of the concept horse ('The concept *horse* is not a concept') rears its ugly head also at the level of sense. Towards the end of his essay 'Über Begriff und Gegenstand' Frege contends that the difficulties that we face in our attempt to refer to the *Bedeutung* of a concept-word by using the phrase 'the concept *F*' apply also to the *sense* of the expression in question. In the previous sections of his essay Frege had argued, against Benno Kerry, that the distinction between object and concept is a substantive, and not a relative, one. However, he had conceded to Kerry that whenever we want to refer to a concept (or function), language forces us to use means of expressions that seem unfit to the task. This linguistic awkwardness, however, by no means impugns the ontological difference between concept and object. Frege's own *Begriffsschrift* is immune from such shortcomings. At the end of his article Frege writes:

For not all the parts of a thought can be complete, at least one must be 'unsaturated' or predicative; otherwise they would not hold together. For example, the sense of the phrase 'the number 2' does not hold together with that of the expression 'the concept *prime number*' without a link. We apply such a link in the sentence 'the number 2 falls under the concept *prime number*'; it is contained in the words 'falls under' which need to be completed in two ways—by a subject and by an accusative—and only because their sense is thus 'unsaturated' are they capable of serving as a link. Only when they have been supplemented in this two-fold respect do we get a complete sense, a thought. I say that such words or phrases stand for a relation. We now get the same difficulty for the relation that we are trying to avoid for the concept. For the words 'the relation of an object to a concept it falls under' hold aloof from one another just as much as the first two by themselves: however we put them together, we get no sentence. It is thus easy for us to see that the difficulty arising from the 'unsaturatedness' of one part of the thought can indeed be shifted, but not avoided. 'Complete' and 'incomplete' are of course only figures of speech; but all I wish or am able to do here is to give hints (Frege (1892a) 1984, pp. 193–4)

Harold Noonan (2001, pp. 180–6) has argued that what makes the paradox of the concept *horse* unavoidable is that our grasp of the functional character of concepts (the referents of concept-words) is made possible *only* by our grasp of the sense of a linguistic expression that presents the function as incomplete. But as soon as we try to spell out what this incompleteness of the sense of the linguistic expression amounts to, we inevitably apply words that are fit to indicate the sense of complete expressions. The distinction between saying/showing invoked by Wittgenstein in his *Tractatus Logico-Philosophicus* seems to offer a honourable way out of this quandary. I agree that this is a very useful distinction and I have used it myself on

various occasions (Picardi 1994 and 2008). However, I remain unconvinced that we are here confronted with a genuine paradox.

For (a) there is not even the shade of an argument in support of the claim that the incompleteness of the sense of a predicate should replicate that of the function (the *Bedeutung* of the predicate) or be of the *same* kind. Nor is there any compelling reason to assume that the sense of a predicate is a special sort of function, a function, i.e., that takes the sense of a proper name into the sense of a sentence (a thought).⁴ Moreover, (b), the sense of the sentence '2 is a prime number' is not the same as the sense of the sentence that Frege puts forward as its more explicit counterpart. Here a weakness of Frege's conception of synonymy comes to the fore. As Burge (2007) has pointed out in his critical review of Davidson's book, the sense of a sentence differs from that of its formal counterpart. A sentence such as '2 is a prime number' does not *say* that there are two objects (senses) indicated or expressed by '2' and by 'the concept *prime number*', nor is it synonymous with such a juxtaposition or concatenation of senses. And much the same holds at the level of reference. The sentence does not *say* that the *object* 2 falls *under the concept* '– is prime'. And even if it did, the sense of the latter sentence should not be rendered by concatenating 'the sense of the proper name 2', 'the sense of *x* falling under *y*', and 'the sense of the concept-word *prime*': in so doing we have listed the senses of the expressions that make up the sentence. But in a sentence the words are *used* not to talk about their senses, but about their subject matter. The relation of functional application is not a further ingredient connecting the arguments and values of a given function. If this were the case we would have a list, and not a thought, as Frege is quick to point out. In order to connect such sense items into a thought we would have to invoke a third concept, namely the doubly unsaturated relational concept '*x* falls under *y*'. But as soon as we try to say something about this relational concept we are once more forced to employ the definite article in front of the expression 'relation of an object to the concept it falls under', thereby naming a saturated entity, which, as such, is unfit to bring about, at the level of sense, the propositional unity, and at the semantic level the application of a function to an argument to yield a truth-value.

Frege's train of reasoning seems to run as follows: since the sense of a linguistic expression is the way in which its referent (*Bedeutung*) is given to us in the context of a sentence, a mapping from objects to truth-values (a function) cannot possibly be given to us by an expression whose sense is 'complete' or 'saturated'. But does this really follow? Are concepts and functions given to us *only* as the referents of incomplete expressions? Is it really the case that epistemology and metaphysics do not bear on the issue? As I said, Frege's argument contains a number of questionable assumptions. First of all, the predicate of the sentence '2 is a prime number' is

⁴ Cf. the discussion in Dummett (1981), Noonan (2001), Geach (1975), Rumfitt (1994), MacBride (2007), just to name a few.

'() is a prime number', and not 'the concept *prime number*'. A predicate, as Frege emphasized, is an expression with blanks, is what remains when one or more occurrences of the same proper name or of different proper names have been removed from a sentence. In Noonan's (and Dummett's) terminology a predicate is not a quotable bit of language but a pattern that we discern in an array of sentences. Moreover, even under the assumption that senses are objects (as Frege's theory of *oratio obliqua* requires), there is no reason why the sense of a concept-word should be given to us as that which is referred to by means of the expression 'the sense of the concept *F*'. On the contrary, we grasp the sense of a concept-word in the process of understanding a variety of sentences in the context of which it occurs *as* a predicate. Plainly, there are other roles in which a concept-word may occur. Frege's insight is that in order to appreciate the functional character of concepts we have to focus our attention on singular sentences in the context of which a concept-word occurs in predicative position. Something similar can be said of proper names: in order to appreciate their role in language we have to focus on identity statements, for it is only proper names that, in Frege's opinion, can flank the identity sign. But proper names can occur also in other roles, as part of predicates, for instance, such as 'identical to Vienna'. I am not here claiming that Frege's argument is waterproof; I am only trying to understand the source of what seems to me a flawed argument.⁵

The metaphor of incompleteness is most vivid at the level of linguistic expression, while its explanatory potential is strongest at the level of predication and semantic value. In fact, it seems to me that the model of functional application carries conviction only at the level of reference and semantic value, at least when we deal with simple predicates. Not only does it help to underline the dependence of the truth-value of the whole on the semantic values of its parts, but it also suggests that we think of this dependence in functional terms, that we conceive of predicates and sentential connectives as devices that take certain inputs and return certain outputs.

To be sure, there is no guarantee that what looks like a predicate or a sentential connective actually singles out *a* function and only *one* function, otherwise the whole machinery would be perfectly idle. And this is what, from a different perspective, speaks against the assimilation of concepts to functions. Predicates which display indeterminacy, under-determinacy, vagueness, and ambiguity are the common coin of linguistic exchange. Frege's characterization fares reasonably well for the purpose of logic, but requires considerable changes and adaptation when applied to natural languages. The least that we should expect is the failure of bivalence.

⁵ I have dealt with this issue in Picardi (2009). Cf. also Gaskin (2008).

But in what respect is the sense of a predicate more incomplete than the sense of a proper name? It is the function of proper names in referential position that differs radically from that of concept-words when they feature as predicates in the context of a proposition. The dictum that only in the context of a sentence does a word have any meaning (sense) applies to all words, be they proper names or concept-words, as Frege had stressed in his masterpiece of 1884, *Die Grundlagen der Arithmetik*. Thus, in a way, there is some truth in Davidson's protest that the metaphor of incompleteness is of little use in accounting for the unity of the sentence. All the logically relevant parts of a sentence can be described as 'incomplete' in so far as none of them amounts to a full sentence, and as 'complete' in so far as we can recognize them as identifiable components of the large varieties of sentences constructive in a language.

Frege's formulation contains important insights. The role of the metaphor is twofold: it serves as a reminder of (a) the different role that words belonging to different syntactic categories play in the context of a sentence, and (b) of the systematic character of a word's contribution to the truth conditions of the sentences in which it may occur. Predicates are unique in that they are applicable without change of meaning to an infinite variety of objects while at the same time indicating a pattern of similarity common to all such objects and in virtue of which something true is said in each case.

Frege's metaphor of unsaturation serves to elucidate notions that are fundamental to the fabric of our thinking. Wittgenstein's distinction between saying and showing, together with his distinction between formal and material concepts, go in the direction suggested by Frege. The function/argument distinction is an *Urbild*, a sort of archetype, displayed in all propositional functions of one argument, not a *constituent* of any of them. A sentence does not contain among its constituents the logical form it displays, and this is why, among other things, there are no representatives in the proposition of the logic of facts. The world is the totality of facts, not of things. We cannot possibly be presented with objects in isolation, but only with objects as constituents of states of affairs. The ontology of the *Tractatus Logico-Philosophicus* has structure and modality built into it from the very outset. Section 2.0122 reads as follows:

Things are independent in so far as they can occur in all possible situations, but this form of independence is a form of connection [*Zusammenhang*] with the states of affairs, a form of dependence. (It is impossible for words to appear in two different roles, by themselves and in a proposition.)

Here Wittgenstein suggests, as it were, a metaphysical foundation for Frege's Context Principle. Unlike Frege, however, Wittgenstein urges that we apply the dependent/independent contrast to both objects and concepts (Frege's unsaturated entities). What accounts for the way in which the component names are

bound together into a propositional unity are the 'modes of signification' (*Bezeichnungsweise*) appropriate to them (3.322).

There are two tenets on which Frege's approach to logic and language rests that make the problem of the unity of the proposition much more tractable for him than it was for young Russell at the time when he published *The Principles of Mathematics*. The first tenet is the Context Principle, the second is the explanatory priority allotted to sentences over their constituents in his account of predication and concepts. To be sure, these two tenets should not be confused with an early version of the rampant meaning holism to which both Quine and Davidson subscribe. The functional model of predication does not require that we subscribe to either of these tenets. But if we want to appreciate Frege's insight into what turns a list of words into a unity capable of expressing a thought it is to them that we have to pay attention, rather than to the metaphor of saturation.

Unlike the author of *The Principles of Mathematics*, he did not begin with the constituents (terms) given in advance, but extracted them out of complete sentences by means of decomposition and analysis. Frege always took pride in this achievement, indeed, he thought that this was a (the?) peculiarity of his approach to logic:

What is distinctive about my conception of logic is that it begins by giving pride of place to the content of the word 'true' and then I immediately go on to introduce a thought as that to which the question 'is it true?' is in principle applicable. So I do not begin with concepts and put them together to form a thought or judgement. I come to the parts of a thought by analysing the thought. This marks off my concept-script from a similar invention of Leibniz and his successors, despite what the name suggests; perhaps it was not a happy choice on my part. (Frege 1918–19, p. 253)

This is by no means a 'rational reconstruction' *ex post* of the spirit that had informed his logical work. In an unpublished essay, written around 1882, known as the 'Anti-Boole', young Frege, after giving an example of what he means by the several 'decompositions' to which the same equation, e.g. $2^4 = 16$, is amenable, writes,

Instead of putting a judgement together out of an individual as a subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of a possible judgement. Of course if the expression of the content of a possible judgement is to be analysable in this way, it must already be itself articulated. We may infer from this that at least the ideas of these properties must have their own simple designations. But it does not follow that these properties and relations occur apart from objects, on the contrary they arise simultaneously with the first judgement in which they are ascribed to things. (Frege (1976) 1980, p. 17)

Frege's logic is from the very beginning conceived as a symbolic language suited to represent the structure of *sentences* which express *judgeable contents*. It carries with it from the very beginning an essential appeal to the notion of truth which should have received Davidson's enthusiastic approval. In section 9 of *Begriffsschrift* Frege's goal is to show that the argument/function distinction is to be preferred to the traditional analysis in terms of subject and predicate. In dropping one or more occurrences of the same proper name from a sentence (such as 'Cato killed Cato'), we highlight the argument-places of a function where substitutions can be effected. In so doing we extract different concepts from the original sentence (e.g. 'c killed x', 'x killed c', 'x killed x'). To the isolation of the characteristic pattern discernible in an array of sentences Frege attributed the utmost importance. The priority of the sentence over its constituents and components plays a crucial role also in Frege's *epistemology*. Once Frege had fully worked out the programmatic distinction between function and argument outlined in the *Begriffsschrift*, he was able to give a full account of the semantic contribution of concept-words to the truth-conditions of sentences in which they occur in a predicative role.

He held that concepts are given to beings with our physical and mental constitution only via the medium of language. To be sure, they are not *created* by language, but without a symbolic medium (without signs) we would not have access to them. Young Frege started with the Kantian distinction between *Anschauungen* and *Begriffe*, only to find it unsatisfactory on several grounds. Aristotle, as much as Boole, operated with a conception of concepts as given in advance of the sentences by means of which we formulate our judgement, or in restricting artificially the range of such sentences to those exemplifying the subject-predicate model. While we may possibly have access to (spatio-temporal) objects (the 'unbestimmtes Etwas' or the 'determinable X', or the 'something = x' invoked as a substratum in traditional accounts of predication) independently of singular terms, our access to concepts, functions, and relations is made possible only through the employment of a symbolic medium. Only language users can form *sentences* endowed with a grammatical and logical structure that enables them to express a judgeable content, i.e. a content capable of being accepted as true or rejected as false. This claim was perhaps at the time a striking and controversial one. Frege found it hard to see how one can do mathematics without a symbolic medium. However, he did not confine his claim to the province of mathematics. There is no finite (or infinite) stock of concepts given in advance of our engaging in the practice of forming a *judgement* and putting it into words in some language. Frege replaced the Kantian question, How do concepts combine with intuitions in the synthesis of judgement? with the question, What is the role played by concept-words in the *context* of a declarative sentence capable of being accepted as true or rejected as false? The answer young Frege suggested to this question in a

short essay, 'On the Scientific Justification of a Conceptual Notation', published in 1882, is this:

Without symbols [*Zeichen*] we could hardly attain conceptual thinking. Thus in applying the same symbol to different but similar things we actually no longer designate the individual thing, but rather what they have in common, the concept. And we attain the concept only in the process of designating it; for the concept itself is not something that can be apprehended in intuition [*unanschaulich*], but it stands in need of a sensible representative [*er bedarf eines anschaulichen Vertreters*] in order that it may appear to us. Thus a sensible element [*das Sinnliche*] discloses to us the world of what is not sensible. (Frege 1882, p. 84; translation modified and completed).⁶

There is no trace in young Frege of the conception put forward in his 'First Logical Investigation' of 1918, where he described thoughts as entities outside space and time, belonging to a third realm. However, he always laid stress on their being objective, accessible to many through the medium of language, and endowed with a structure. The metaphor of 'saturation' too made its appearance very early in Frege's writings, earlier, that is, than his mature view that a concept should be conceived as a special kind of function. In 1882, the same year when the paper 'On the Scientific Justification of a Conceptual Notation' was published, in letter to Anton Marty, Frege wrote:

A concept is unsaturated, in that it requires something that falls under it; hence it cannot subsist by itself. That an individual falls under it is a judgeable content, within which the concept appears as the predicate, and is always predicative. In this case, where the subject is an individual, the relation of subject to object, *is not a third added to the two of them*, but belongs to the content of the predicate, which is what makes it unsaturated. (Frege (1976) 1980, p. 101; my emphasis)

There is no denying that in his early writings Frege is not as accurate as we would wish him to be in distinguishing the level of linguistic expression, the level of sense, and the level of reference. Until 1891 he seems not to have quite made up his mind as to whether by 'predicate' he means the linguistic expression, its sense, or that which is predicated of an individual object. What is remarkable in the letter to Marty is the rejection of the traditional analysis in terms of subject, copula, and predicate. The notion of the copula belongs to grammar, and has no role to play in

⁶ In the Bartlett/Bynum translation the last sentence in the quoted passage is omitted, and, as a result, the reader cannot appreciate the connection between Frege (1882) and Frege (1918). Also the allusion to the Kantian *Anschauung* is lost.

logic. Predication and assertion part company very early in Frege's philosophical logic, and the role of the copula is redundant to both.

To sum up. The application of the part-whole picture to thoughts should be taken *cum grano salis*. It is only sentences that, strictly speaking, have parts, and even in the case of sentences what counts as a 'part' is anything but clear. We forget all too easily that the lexical classifications and the grammatical categories that we employ in describing language are of our own making. Indeed, Frege held a highly revisionary view of the traditional doctrine of what, in *logic*, should count as a semantically significant part of a sentence. Grammar—he held—can be a very misleading guide when it comes to logic proper. The logical structure displayed by sentences belonging to his conceptual notation is possibly the only type of structure to which the notion of an isomorphism between sentence structure and thought structure is applicable. What the metaphor emphasizes is that thoughts are *structured*, i.e. are not amorphous lumps, and under this respect the metaphor is an insightful one. However, the claim that the *structural constitution* of thoughts can be accounted for in terms of their having saturated and unsaturated parts is a questionable one for the reasons indicated above. It signals that the parts have to be different, but more has to be said on the difference between naming and predicating, if we have to gain a genuine understanding of matters which goes beyond a valuable syntactic insight. Moreover, not all that is relevant to the expression and identification of the thought expressed by a sentence on a given *occasion* can be recovered from the sentence's linguistic composition. Possibly, the contribution of the *context of utterance* adds in subtle ways to the identification of the thought which is being expressed on a given occasion by uttering a token of a sentence type, as Frege himself had pointed out in his 'First Logical Investigation'. In sum, matters become truly intricate when it comes to *situated* utterances of sentence types.

Frege's appeal to the Context Principle in his *Grundlagen der Arithmetik* of 1884 can be seen as an essential contribution to the semantics of sentence types, whereas his observations relating to the context of utterance and to indexical expressions pave the way to the semantics of sentence tokens. As I said, Frege appealed to the metaphor of saturation in order to elucidate the reasons why construing a concept (the referent of a predicate expression) as a function from objects to truth-values offers a solution to the traditional issue of predication (i.e. what it is for an object to fall under a concept). There is no denying that the functional conception is intimately connected with the conception of a predicate as that which remains from a sentence when we drop one or more occurrence of a proper name. A predicate, thus conceived, is a pattern that we *discern* in an array of sentences. Several decompositions of a sentence into saturated and unsaturated parts are possible. Whether such a pattern itself should be conceived in terms of functional application is a different and more intricate question. I have suggested that we keep apart the (interrelated) issues of what counts as a linguistic unit

(a declarative sentence of a given language, be it natural or artificial), what counts as a propositional unity (a thought, in Frege's terminology), and what it is for an object to fall under a concept (the problem of predication properly so called). Does the *functional* model apply at the level of the linguistic expression and at the level of the thought expressed? My tentative answer to both questions is negative. While I am inclined to think that it makes sense to speak of 'parts' in connection with thoughts, I very much doubt that the idea of an incomplete or unsaturated sense is useful to throw light on the semantic *function* that the sense expressed by a concept-word plays in the *context* of a given sentence. It is undeniable, however, that this is how Frege saw matters.

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Was Frege a Proto-Inferentialist?

During the last two decades there has been a Homeric struggle going on in many areas of philosophy between inferentialist and representationalist accounts of meaning and thought. The struggle is reminiscent of a controversy that raged in the early 1970s between communication-intention versus truth-conditional accounts of meaning—with Sir Peter Strawson championing the former and Donald Davidson the latter. Among the protagonists of today's animadversions are Robert Brandom, the champion of inferentialism, a near kin of Michael Dummett's justificationist theory of meaning, and Jerry Fodor, the proponent of a sophisticated form of representationalism. Even a cursory glance at their writings suffices to show how often Frege's views are appealed to and used to support various claims. Recently an attack on the inferentialist programme has been launched by Timothy Williamson, who has employed a whole battery of arguments to rebut a suggestion put forward by Paul Boghossian concerning the role played in the meaning constitution of logical connectives by inference rules.¹ In the context of his criticism of inferentialism Williamson opposes the suggestion put forward by Dummett and elaborated on by Brandom and Boghossian concerning the meaning of pejoratives.² Williamson's positive account draws substantively on Frege's remarks on tone and colouring, thus indirectly joining forces with Fodor in the anti-inferentialist camp.

But how come, one may wonder, that Frege's doctrines lend themselves to such widely different readings? Obviously the writings of a philosopher can be a source of insight for other philosophers who differ greatly from him. But it would be strange, alarming in fact, if it turned out that all these different accounts did equal justice to Frege's central concerns. In what follows I will use these different interpretations of Frege's doctrines as a foil to highlight one specific aspect of the controversy between inferentialist and representationalist accounts of concepts, i.e. the relationship between analyticity, synonymy, and definition.

¹ Boghossian (2003) and Williamson (2003).

² Dummett (1973a), pp. 453–7; Brandom (2002).

1. Inferentialism and Representationalism

As I said, both Brandom and Fodor incorporate bits and pieces of Frege's doctrines into their own work. Fodor takes Frege's principle of compositionality as the cornerstone of his theory of mental representations. In Fodor's construal the principle applies primarily to the language of thought. Considerations of normativity, rationality, and context dependence may play a role in accounting for communication in public languages, but have no bearing on the individuation of items in the language of thought.

Frege's insight that the meaning of a sentence is constructed out of the meanings of its component parts, with the attending clauses on productivity and systematicity, forms the cornerstone of semantic atomism. Fodor (1998) adopts a building-block version of the principle of compositionality while rejecting Frege's context principle, which gives pride of place to sentences put forward in acts of judgement. All conceptual role semantics offend against the principle of compositionality as Fodor and Lepore construe it, and Brandom's inferentialism is no exception to the rule. Needless to say, Quine's and Davidson's construal of Frege's principle(s) of compositionality in the framework of their theories of radical translation and interpretation is anathema to Fodor, for these theories are committed to semantic holism. While Fodor can allow for a moderate holism of belief fixation,³ semantic holism is incompatible both with atomistic theories of content and with molecularist theories of meaning and understanding like the one advocated by Michael Dummett. No wonder that many arguments put forward long ago by Dummett against Quine's indeterminacy theses have their counterpart in an atomistic scenario.⁴

The fundamental issue over which representationalists and inferentialists part company is how thinking should be conceived. Is thinking the ability to extract information from the environment at a conceptual and non-conceptual level or is it the ability to deploy reliably gained information in thought, action, and communication? Brandom opts for the latter characterization, Fodor for the former. He sees himself as carrying out a philosophical programme which has its roots in Kant's philosophy; further developments of Kant's legacy can be traced in the work of Frege, Wittgenstein, Sellars, and Dummett. Brandom's more specific aim is to work out a pragmatist theory of concept possession and linguistic competence.

Not all pragmatists are inferentialist: Quine, Rorty, and Davidson share a pragmatist heritage, which comes to the fore in their commitment to semantic and epistemological holism and in the way they connect belief and action in the framework of interpretationist theories of meaning and thought. However, none of them is committed to the strong inferentialism advocated by Brandom, who, on

³ Cf. e.g. Fodor (1998).

⁴ Dummett (1973b) and Dummett (1991c), ch. 10.

his part, contrasts his idea of expressive rationality with the notion of instrumental rationality characteristic of the pragmatist tradition (from Peirce to Davidson via Ramsey). As Fodor and Lepore (2001) point out in their review article of Brandom (2000), most semantic pragmatists hold a dual-aspect theory of content: grasping a content is viewed as a matter of knowing both how a word or concept behaves in inferences and how to apply the concept to objects. They go on to claim that in Brandom's approach the applying part drops out in favour of the inferential aspect. This criticism would be appropriate if Brandom endorsed a form of hyper-inferentialism, but he only subscribes to strong inferentialism. Strong inferentialism is the claim that the inferential articulation, *broadly construed*, is sufficient to account for conceptual content. The qualification 'broadly construed' alludes to the fact that the non-inferential circumstances of application and their consequences are taken into account. Concepts such as 'red' obviously have non-inferential circumstances of application and strong inferentialism must account for them (Brandom 2000, p. 28) without falling back on the Myth of the Given, forcefully criticized by Wilfrid Sellars.

Strong inferentialism, as I understand it, amounts to the claim that conceptual contents present themselves to thinkers in a form that makes them *implicitly* available for inferential deployment. Mere differential behaviour when presented with certain stimuli achieves the status of conceptualization only when it becomes inferentially elaborated 'as providing reasons for making other moves in the language game, and as themselves potentially standing in need of reasons that could be provided by making still other moves' (Brandom 2000, p. 17). Many creatures can spot red surfaces, but only few are *aware* of them *as* red; conceptualization, for all we know, is a prerogative of linguistic creatures. Expressing a content is *conceptualizing* it, 'addressing it in a form that can serve as and stand in need of reasons, making it *inferentially* significant' (Brandom 2000, p. 16). Brandom also wishes to emphasize the characteristically rational and social character of content. Concepts are norms determining the correctness of various moves in the game of giving and asking for reasons. The commitments one undertakes in using a word concern both what follows from its application and what is incompatible with it.

Whereas Frege and the young Wittgenstein concentrated on logical structure, Brandom, following Wilfrid Sellars,⁵ stresses the importance of *material inference*. Material inferences are our primitive way of articulating the bearing of a concept on another concept: the conditional is the linguistic construction which brings this bearing to the fore and thus opens the possibility of acceptance and critical scrutiny. The exclusive concentration on the logical machinery of deductive

⁵ Sellars (1956) 1997.

reasoning for accounting for the inferential articulation of our concepts has been in Brandom's opinion philosophically misleading.

According to Brandom, recognizing the priority of judging over other kinds of mental acts is what inferentialists such as Kant, Frege, and himself have in common. Undeniably, Frege emphasized the priority of sentences expressing judgements over the semantic function of proper names and concept-words. But this allotment of priority is fully compatible with representationalism. As Fodor and Lepore (2001) correctly stress, this does not make Kant or Frege inferentialists because, simply, 'thoughts are the units of inference' is entirely compatible with 'concepts are the constituents of thoughts'. However, the chief objection that Fodor and Lepore level against all brands of holistic conceptual role semantics is that such accounts must supply an answer to two key questions, i.e. Which inferences are meaning constitutive? and How is the compositionality of meaning to be explained? They doubt that any such answer can be supplied, unless one revives the analytic/synthetic distinction which has been shown to be untenable by Quine, and Brandom shows no inclination in this direction.

A natural answer to the first question raised by Fodor and Lepore that springs to mind to those who do not concur in Quine's judgement on analyticity, is that the content of a concept is constituted by its analytic entailments and, possibly, by its definition. The inferential role exploits the semantic structure of concepts, but is not constitutive of it. Apparently Lepore and Fodor believe that one can follow Quine in rejecting the analytic/synthetic distinction, while avoiding endorsing his semantic holism. I myself believe that an answer to the first question can be supplied and the construal of analyticity offered by Frege contains valuable insights. Since Fodor, Lepore, and Brandom agree with Quine on the issue of analyticity, I am bound to disagree with them. In fact, my chief criticism of Brandom's interpretation of Frege is that it overlooks Frege's insights on analyticity and that it misconstrues his contribution to semantics.

Fodor and Lepore view representational theories of mind as a species of the genus Semantic Cartesianism, which they characterize as follows:

Anybody is a Semantic Cartesian who holds that *having the concept F is being able to think about F-ness (as such)* and, correspondingly, that *having an expression that means F is being able to express one's thought about F-ness (as such)*. According to this sort of theory, the appeal to semantic notions (like 'representation of', 'thinking about') in accounts of linguistic and conceptual content is ineliminable. The order of exposition, according to all versions of Semantic Cartesianism, is *from the semantic to the intentional*, not the other way around.

(Fodor and Lepore (2001) 2002, p. 137)

Needless to say, they believe that the Cartesian is on the right track. Representationalists assume that thinking *about* (e.g. about *foolishness*) is what

is basic to theories of content; thinking *that* (e.g. thinking that *This man in front of me is a fool*) requires that we think *of* the man in question and *of* foolishness. According to representationalists *thinking about an object* has primacy over the propositional (*thinking that*); Brandom believes that *judging* has the conceptual priority, for assertoric contents are the smallest units that can enter into inferences.

My impression is that this alternative is too stark. Evans's Generality Constraint⁶—which encapsulates a (number of) genuine Fregean insights—intimates that thinking a thought requires the discernment of structure. We are never presented in thought with a *bare* object: an object is always thought, perceived, conceived in a certain way which need not be propositional in character but lends itself to assessment of correctness. A minimum requirement for understanding a concept is that we think of it as a feature that should be capable of being instantiated in many episodes of thinking that involve different objects. Evans (1982, p. 101) argues that thoughts are structured

...not in the sense that they are composed of several distinct *elements*, but in terms of their being a complex of the exercise of several distinct conceptual abilities. Thus someone who thinks that John is happy and that Harry is happy exercises on two occasions the conceptual ability which we can call 'possessing the concept of happiness'. And similarly someone who thinks that John is happy and that John is sad exercises on two occasions a single ability, the ability to think of, or think about, John.

Fodor's atomism runs foul of Evans's Generality Constraint, whereas Brandom's rational expressivism sets the standards of the conceptual too high, at the level of full-fledged contents capable of featuring as premises and consequences of an argument, thus neglecting a whole range of intermediate cases. For one may wonder whether, as a matter of fact, the material presented to us in judgements based on perception offers itself in a format ready for inferential processing. This is an empirical issue, not a (purely) philosophical one. Also, as far as linguistic activity is concerned, I have argued elsewhere (Picardi 2008) that Brandom sets the standard for what counts as exhibiting reasoning and concept possession too high.

2. Frege on Analyticity and Inference

As I said at the beginning, Brandom and Fodor incorporate bits and pieces of Frege's doctrines into their own work, but my impression is that they both do so at the cost of considerable mutilation and misrepresentation. They seem to be

⁶ Evans (1982), section 4.3.

dealing with two utterly different philosophers, and not with the work of a philosopher whose ideas underwent significant change while keeping a fundamental unity of inspiration. Some may be inclined to suggest that there is a simple way out of my puzzlement: the Frege which provides inspiration to Brandom is the author of the *Begriffsschrift* (*Conceptual Notation*), i.e. the Frege before the semantic turn of 1890. Let's call him FREGE ONE. The Frege dear to representationalists, and to Fodor in particular, is the author of *The Basic Laws of Arithmetic*, of *Sense and Meaning*, *Concept and Object*—let's call him FREGE TWO. But this is not a satisfactory answer. For one thing, Frege's masterpiece of 1884, *Die Grundlagen of Arithmetik*, squares neither with FREGE ONE nor with FREGE TWO.

The locus classicus on which Brandom rests his interpretation of Frege as a proto-inferentialist is §3 of *Begriffsschrift*:

A distinction between *subject* and *predicate* finds *no place* in my representation of judgement. To justify this, I note that the contents of two judgements can differ in two ways: either the conclusions that can be drawn from one when combined with certain others also always follow from the second when combined with the same judgements, or else this is not the case. The two propositions 'At Plataea the Greeks defeated the Persians' and 'At Plataea the Persians were defeated by the Greeks' differ in the first way. Even if a slight difference in sense can be discerned, the agreement predominates. Now I call that part of the content that is the *same* in both the *conceptual* content. Since *only this* has significance for the BS, no distinction is needed between propositions that have the same conceptual content. (Frege (1879) 1997, p. 53)

Here Brandom detects the seed of the idea that concepts are individuated by their inferential role. In 1879 Frege did not appeal to semantic notions. Luckily so, in Brandom's opinion, for he believes that notions such as truth and reference play only a subsidiary role in explaining the characteristic features of the conceptual. It is not as if Brandom held that such notions should be excised as irrelevant: on the contrary, they have a role to play, but only a subsidiary one. Brandom's logical expressivism wants to keep clear of the conception of logic as concerned with the laws of being true held by FREGE TWO. Logic has to do with the most general laws of judgement, as Frege tells us in his *Begriffsschrift*. Frege's semantic doctrines are seen by Brandom as backsliding into a representationalist, pre-Kantian conception of content. Indeed, Brandom goes as far as to suggest that since Frege deemed truth undefinable and opposed correspondence theories of truth, his conception of a thought is not 'representational'. To be sure, 'representational' is a tricky term, but I take it that neither Fodor nor Brandom mean 'pictorial' by it. As is well known, Frege held that the word 'true' should not be construed as expressing a relation between a thought and a fact, for this would trigger an

infinite regress. Whether the regress argument is sound as it stands has been the object of much controversy and need not detain us here.⁷ However, the claim that truth is undefinable has no tendency to show that there is no *representational* ingredient in Frege's concept of a thought. According to Frege the essence of the *concept* of truth is enshrined in assertoric force and not in the word 'true'—which he held (wrongly) unable to make a meaning contribution of its own to the sentence in which it occurs. Judging is characterized by Frege as a mental act in which thinkers engage when aiming at truth. However, assertion is not just the outer manifestation of such inner acts; it involves the social dimension of language, which Frege saw as the privileged vehicle both for the expression and for the communication of thoughts. Truth and assertion have, indeed, to be elucidated together, but this does not in the least indicate that Frege held that inferential connections are conceptually or metaphysically prior to semantic relations.

Not only does Frege's characterization of the notion of the sense of a proper name in his writings after 1890 as '*the way in which an object is given to us*'—i.e. as a way of thinking, perceiving, apprehending the object to which a predicate is ascribed in speech or thought—clearly suggests a representationalist reading. In fact, in all his writings after 1890, i.e. after the introduction of the distinction between *Sinn* and *Bedeutung*, Frege describes the connection between meaning and rules as flowing from the former to the latter: logical inferences are legitimate in so far as they are truth-preserving, i.e. in so far as they encode legitimate ways of extracting the consequences of (true) thoughts.⁸ It is precisely against this conceptual priority that Wittgenstein directed his criticism in his *Philosophical Investigations* (PI): rules of whatever kind do not 'flow' from meanings, nor are they answerable to semantic content spelled out in truth-conditional terms. Wittgenstein argues at length that it is misguided to ask whether the rules that govern a certain concept-word are in harmony with its meaning, conceived as determined, say, by its representational content. Logical rules, for example, *constitute* the meaning of logical connectives: a change of rules is a change of meaning, and then one might change the sign as well. In the context of his discussion of negation in §§547–57 of PI Wittgenstein writes:

- (a) 'The fact that three negatives yield a negative again must already be contained in the single negative that I am now using.' (The temptation to invent a myth of 'meaning'.)

It looks as if it followed from the nature of negation that a double negation is an affirmative. (And there is something right about this. What? *Our* nature is connected with both.)

⁷ Cf. Künne (2003).

⁸ Cf. Picardi (1998).

- (b) There cannot be a question whether these or other rules are the correct ones for the use of 'not' (I mean whether they accord with its meaning). For without these rules the word has as yet no meaning; and if we change the rules, it now has another meaning (or none), and in this case we may just as well change the word too.⁹

The whole controversy which Frege staged against formalist conceptions of arithmetic is predicated on the rejection of the conceptual priority of rules over meanings. Use ought to flow from meaning, and not the other way round, at least as far as the concept-script is concerned. To be sure, in a formalized language, owing to definitions and stipulations, we have the possibility of provisionally fixing the meaning (sense and reference) of certain words. But the newly introduced terms must prove their value by making possible the derivations of new and important theorems, thereby showing that analytic propositions can extend our (a priori) knowledge. And this is why Frege has a lot to say about analyticity, definition, and synonymy. Brandom's suggestion that we construe holistic conceptual content as consisting of inferential relations bears an obvious similarity to some of Wittgenstein's leading ideas, but it collides head on with Frege's way of approaching the issue of the harmony between meaning and rules. It is the rules that hold for a given sign which must be held responsible to content, spelled out in terms of truth-conditions, and not the other way round.¹⁰ If we follow Brandom's lead we will be deprived not only of many important insights, but also of excellent pieces of artillery that could be used to rebut Fodor's and Lepore's semantic atomism.

As we have seen, in his *Begriffsschrift* Frege states that two sentences differ in conceptual content if and only if they differ in inferential potential, i.e. if some consequences follow from the one but not from the other when they are combined with certain other sentences. As a sufficient condition for difference of content it is (perhaps) unobjectionable. However, the relevant question which exercised Frege after 1879 is whether a difference in inferential potential of two expressions (sentences or concept-words) is also a necessary condition for difference in conceptual content. And the answer is clearly negative. It is natural to maintain that the constitution of concepts is more fine-grained than their inferential potential. It suffices to think of various principles which were shown to be deductively equivalent, such as the Axiom of Choice and a Zermelo's well-ordering theorem: they are provably equivalent and hence knowable a priori, but neither are they synonymous nor are they analytically equivalent. The adoption of the extensional approach is perhaps responsible for many unsatisfactory formulations that we find in Frege's writings after 1890 concerning analyticity and

⁹ PI, p. 147, text printed under the line.

¹⁰ See for instance Picardi (1988).

synonymy. For example, Frege contends, against Husserl, that a definition should merely preserve reference. But, as Dummett points out, this criterion is patently inadequate:

The alternative definitions of a conic he [Frege] provides as an illustration indeed have no pretension to be equivalent in sense; but they are not merely coextensive: they are *provably* coextensive—provably so, that is, by mathematical means, and hence equivalent a priori even if not analytically so. This is, indeed, for Frege, a weaker notion than being synonymous, having the same sense, but is a great deal stronger than just being materially equivalent.¹¹

Perhaps it is no chance that Frege's masterpiece sits uncomfortably in Brandom's characterization. For, on the one hand, it is arguably the work in which the notion of content (*Inhalt*) is given the richest inferential treatment, but, on the other hand, it is undeniable that the process of making explicit the inferential potential enshrined in the concepts of arithmetic as a plant in a seed is conceived by Frege a step in a process of establishing his claim that numbers are objects—a distinctly objectivist claim. Since in his paper 'Frege's Technical Concepts' Brandom disagrees with Hans Sluga concerning a purely non-relational reading of the referential idiom employed by Frege, something needs to be said concerning the way in which *Grundlagen* fits in the overall inferentialist framework. One may confine FREGE ONE to the author of *Begriffsschrift*, but this seems an absurd mutilation, for, as I have said, *Grundlagen* offers the best examples of ways of extracting the inferential potential enshrined in certain concepts by way of a reconceptualization of the contents of sentences in which the relevant concept-words occur. Brandom believes that Frege's attempt to establish that numbers are (logical) objects is doomed to failure because of the high standards of individuation he sets on abstract objects. But this is a different issue altogether, which is nowadays intensely discussed thanks to the revival of logicism promoted by Crispin Wright and George Boolos.

I have argued elsewhere,¹² in agreement with Dummett,¹³ that if we want to do justice to Frege's conception of analyticity we have to offer an account of definition as close as possible to the one put forward by Frege himself in *Grundlagen*. It is true that while after 1890 Frege never tires of emphasizing that the discovery that proper names endowed with different senses have the same reference can add to our knowledge, he does not pay equal attention to the cognitive gain ensuing from the discovery of a sentence's analytic character or from finding out that two sentences are analytically equivalent. Frege makes a distinction between analyticity and cognitive synonymy. The latter should be immediately recognizable and

¹¹ Dummett (1991a), p. 29.

¹² Picardi (1992).

¹³ Cf. Dummett (1991a).

intuitively decidable, while the former cannot be so characterized. A salient feature of analytical sentences under Frege's construal of them is (a) that they should be provable by logic alone (second-order logic) within a formalized language; (b) that some of them at least make an essential appeal to definitions or are themselves couched in the form of a definition. It may prove useful to distinguish between logically equivalent sentences and analytically equivalent ones, the distinguishing mark of the latter being the appeal to explications and definitions. I suppose that Brandom would be ready to admit that a proof of the analyticity of arithmetic like the one envisaged by Frege may require explication in Carnap's sense and a survey of the concept's occurrence in many related sentences and the working out of its consequences. This is after all what making things explicit means. However, it seems to me that Brandom's holism forces him to a much stronger conclusion, i.e. that there is no way of distinguishing primary occurrences of a word or concept from secondary ones, and therefore, contrary to what molecularists hold, such a survey cannot be carried out. This is also why the quest for an improved understanding of analyticity is doomed to failure.

Frege's semantic theories are incompatible with holism. And I take it for granted that everybody agrees that Quine's semantic holism should be sharply distinguished from Frege's context principle: the latter applies to sentences, the former to whole theories or languages, as Quine himself emphasized in *Two Dogmas of Empiricism*. In many places in his writings Frege says that two sentences can express the same content, but in different ways. The carving up metaphor is appealed to by Frege in two different contexts: (a) when he describes the extraction of a predicate from a sentence by means of dropping one or more occurrences of a proper name; and (b) when he appeals to the context principle at crucial stages of *Grundlagen* in order to justify the introduction of logical objects. One may argue that the way Frege proceeds can be seen as a way of making explicit the ontological commitments to logical objects which are implicit in our pre-theoretical ways of talking. The introduction of the cardinality operators 'The number of Fs' and 'The number of Gs' as term-forming operators is grounded in the principles that these singular terms have the same referent if and only if there are just as many Fs as Gs. To borrow Dummett's terminology here, we are dealing with the process of reconceptualization; Frege can back the claim of the analyticity of arithmetic in so far as the definition of the cardinality operator is faithful to assumptions present in our pre-analytic linguistic practice.

Brandom may protest that there is nothing in his idea of logic, as a process of making explicit the inferential potential of arithmetical concepts, that is at odds with Frege's way of proceeding. A sentence, as Brandom, following Frege, never tires to stress, is capable of various logical analyses. Does this mean that there is no one thought that a sentence expresses, but only a variety of inferential potentials revealed by these different decompositions? Of course not: there is a constraint, and the constraint is precisely that all these carving ups (or decompositions)

should be seen as originating from the same conceptual content spelled out in terms of truth-conditions and reference. Frege clearly held that a necessary but obviously not a sufficient condition that these different ‘carvings up’ of the same content should satisfy is that they preserve truth. Contrary to what Brandom surmises, for Frege the notion of content and sameness of content must be secured by an appeal to the notion of truth and truth-preserving transformations. What is required is a notion of content which grounds inferential connections without being reducible to them. This may be a difficult goal to achieve, but I think it is well worth pursuing if we want to reach a satisfactory understanding of analyticity and priori knowledge.

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