

Inductive Logic as Explication: The Evolution of Carnap's Notion of Logical Probability

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ABSTRACT

According to a popular interpretation, Carnap's interpretation of probability had evolved from a logical towards a subjective conception. However Carnap himself insisted that his basic philosophical view of probability was always the same. I address this apparent clash between Carnap's self-identification and the subsequent interpretations of his work. Following its original intentions, I reconstruct inductive logic as an explication. The emerging picture is of a versatile linguistic framework, whose main function is not the discovery of objective logical relations in the object language, but the stipulation of conceptual possibilities. Within this representation, I map out the changes that the project went through. Seen from such an explication-based perspective, inductive logic becomes quite hard to categorize using the standard labels.

1. INTRODUCTION

Throughout the second half of his philosophical career, Rudolf Carnap developed systems of inductive logic based on what he regarded as a logical concept of probability. From the first of his papers on probability (Carnap 1945b), all the way to the posthumously published accounts (Carnap 1971a, 1980), the successive formal systems of inductive logic continued to be expanded and refined. According to what seems to be the leading interpretation of the development of Carnapian inductive logic, both the formal features of the systems of inductive logic and their conceptual underpinning changed significantly between 1945 and 1970. As this view has it, the interpretation of probability on which Carnap's inductive logic was based evolved from a logical towards a subjective, or Bayesian, conception. One of the most explicit formulations of this interpretation can be found in the following quotation from Brian Skyrms:

By the end of his life, [Carnap] had moved from a logical conception of probability inspired by Keynes and a unique inductive rule based on a flat prior for an IID process to a subjective conception of probability and a class of inductive rules corresponding to the Dirichlet priors. (Skyrms 1996, 321)

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Skyrms is not alone here; similar statements have been put forward by Zabell (2005, 29–30), Galavotti (2007, 132–33), and Earman (1992, 113)—all of them leading philosophers of probability.

However, this view of the conceptual evolution of Carnap's inductive logic would appear, at first glance, to be at odds with his own view. In the accounts published in the 1960s, commonly referred to as the “late” Carnapian inductive logic, Carnap insists that his basic philosophical view on probability did not change. For instance, in the 1963 paper “Remarks on Probability,” he writes about his first monograph on probability (Carnap 1950b):

The major features of my theory, as explained in the book, are still maintained today. This holds for both the basic philosophical conception of the nature of logical probability, explained in the first half of the book, and the formal system constructed in the second half. (Carnap 1963b, 65)

Similar pronouncements can also be found in the “Replies and Systematic Expositions” of the 1963 Schilpp volume (Carnap 1963c, 971, 972). Moreover, in all of his publications on inductive logic, Carnap continued to insist on the logical character of his systems and his concept of probability—up to and including the last published programmatic account of inductive logic, “Inductive Logic and Rational Decisions” (Carnap 1971b).

The aim of this paper is to take a closer look at this apparent clash between Carnap's self-identification and the subsequent interpretations of his work. Are the modern accounts of Carnap's evolution misguided, or was Carnap himself delusional about the actual conceptual implications of the technical developments within his inductive logic? In order to answer this question, I will investigate how exactly Carnapian inductive logic changed over time and whether the changes that did occur constitute a break from a logical concept of probability and an acceptance of the subjective one. I will pay special attention to the interpretation of the inductive logic project against the background of Carnap's mature (post-tolerance) metaphilosophy.

This paper puts forward a conceptual analysis of the differences in the successive published versions of Carnap's inductive logic. My goal is not a precise reconstruction of the timeline in which the changes happened or the exact reasons for them. In the later works on inductive logic, Carnap offered very little motivation for his modeling and conceptual choices. For that reason, a proper analysis of the reasons for the changes within inductive logic would have to be primarily based on the unpublished materials; that will be a topic of future investigations. This paper is primarily based on the work that Carnap chose to publish.

Out of about twenty-five articles and books on inductive logic and related matters published by Carnap, a much smaller number can be identified as contributing significantly to the conceptual development of the whole program. The two earliest papers (1945a; 1945b), introducing the basic concepts and presuppositions, were later included in the main monograph of the early period, *Logical Foundations of Probability* (1950b, hereafter *Foundations*), which is the primary reference for that early stage of the development. Published only a little later, the small monograph *The Continuum*

of *Inductive Methods* (1952, hereafter *Continuum*) is considered an important step in the technical progress of inductive logic. In those early publications Carnap develops his understanding of inductive probability as a logical concept, and inductive logic as a project in explication.

In this paper, I will argue that this view of inductive logic as an explication of the concept of inductive probability should guide our understanding of the whole project. In explication, a sharp, new concept, the explicatum, is proposed as a replacement of an already used, but less clear concept, the explicandum. As I will further explain, Carnap saw the identification and specification (or “clarification”) of the explicandum as a significant part of this task, and devoted substantial effort to it. I will argue that the changes to Carnap’s conception of inductive logic are best understood by focusing on his changing view of the explicandum of the concept “probability,” not in changes to the formal system of inductive logic (the explicatum). These changes in the explicandum are laid out in the evolving lists of its informal specifications, appearing in the different publications.

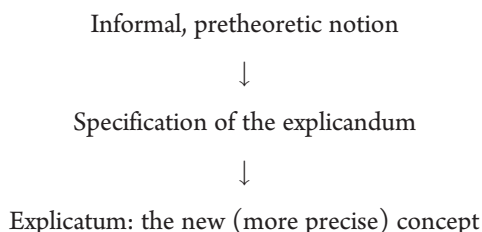
It is in the “Replies” section of the Schilpp volume (Carnap 1963c, largely written in 1957) where the modified characterization of the explicandum is first given (see section 2 below), so it is from that point that we can talk of the “late” Carnapian inductive logic. The notions introduced there were subsequently elaborated in “The Aim of Inductive Logic” (1962c), which was later reprinted, with slight modifications, as the introductory article in the posthumously published 1971 collection Carnap edited together with Jeffrey (Carnap 1971b). The second philosophically programmatic article is the “Inductive logic and inductive intuition” (1968). The latter two articles, together with the exposition included in the Schilpp volume, are the main sources on the conceptual side of Carnap’s mature inductive logic.¹

The paper proceeds as follows. In section 2 I introduce the interpretation of inductive logic as a project in explication and schematically sketch the structure of such projects. I then describe and analyze the early (section 3) and the late (section 4) inductive logic within this explication schema. Developed in this way, an understanding of what inductive logic exactly was allows me to precisely locate the changes that occurred in the development of the project, which is what I do in section 5. Finally, in section 6 I try to answer the motivating question of this paper, namely whether the development of Carnapian inductive logic is correctly characterized as an abandonment of the logical concept of probability and a move towards a subjective one. Section 7 considers some possible sources of such a reading in Carnap’s writing. I conclude in section 8 by pointing to some challenges and open questions, as well as sketching a further research agenda that stems from this work.

2. INDUCTIVE LOGIC AS AN EXPLICATION

Inductive logic was a project of explication; indeed, it is in the beginning of *Logical Foundations of Probability* that Carnap introduced this method in detail, as an introduction to the whole inductive logic enterprise. Hence it seems fitting—and it will prove useful—to take this declaration seriously and reconstruct inductive logic as an explication of the concept of probability. In basic terms, explicating a concept (the explicandum) consists of creating a new, precise concept (the explicatum), which

can then replace the explicandum in the relevant contexts. However, the picture of explication that emerges from the first sections of *Foundations*, is actually a three-level one. It can be summarized in the following schema:



At the top level lies the original, vague concept in all of its uses in either everyday language or within a particular disciplinary context. This initial concept is not yet the explicandum; the explicandum must be specified or characterized (“clarified”) in more precise terms than by just pointing to an ordinary-language or informal concept. Such a specification of the explicandum consists of choosing particular (possibly imprecise) uses and meanings of the concept on which the explication will focus. Only when an explicandum is narrowed down in this way can we construct the new, precise concept that captures the aspects of the pretheoretical concept highlighted in the explicandum. This three-tier picture provides the framework in which I will analyze Carnap’s inductive logic and its evolution. It will turn out that the differences between the early and the late publications belong to more than one level of the explication schema; moreover, the most important changes were not at the level of the explicatum itself.

To illustrate the explication process, suppose that my goal was to explicate the concept “warm” as my target informal, pretheoretic notion. First, I notice that this word is used in many different contexts; for instance, it is used to ascribe a particular property not only to physical objects (warm soup), but to other kinds of things (warm feelings, warm colors, warm personalities). The next step is the specification of the explicandum. I choose those uses and meanings of warm that I take to be exemplary for that aspect or use of this concept I am interested in explicating. In this case, I could say that I am interested in the concept “warm” as it is used to describe a particular quality of physical objects. At this point not everything has to be sharply defined; for instance, I can include “warm weather” as one of the uses I am interested in, in spite of it being not fully clear in what sense this use belongs to the category “property of physical objects.” Finally, I would formulate a precise explicatum within my general theoretical framework for temperature, maybe defining it by a sharp range of temperatures. The explicatum does not have to match the explicandum precisely; in this case, for instance, the explicated concept warm might not apply to all physical objects in the same way (warm sweater vs. warm soup).

In the case of inductive logic, the informal, pretheoretical notion Carnap focused on was the concept of probability. Before even proceeding to the clarification of the explicandum, Carnap points out that the word “probability” even within science is used in two fundamentally different kinds of ways, i.e., has two explicanda, which he

calls probability₁ and probability₂. Probability₁ is a semantic, purely logical concept; sentences conveying numerical values of this kind of probability are true or false in virtue of their meaning rather than any empirical facts. Sentences conveying values of probability₂ are true or false in virtue of empirical facts. Probability₁ is a property of sentences or propositions, while probability₂ is a property of events or things. It is probability₁ that Carnap wishes to explicate; and since the short characterization above is too vague and general to provide a basis for a rational reconstruction, it needs to be specified further in order to narrow down the desired explicandum.

Carnap gives us two versions of such a characterization, marking the two phases in the conceptual development of inductive logic. In the first version, in *Logical Foundations* (1950b, §41), there are three descriptions of probability₁ to characterize the explicandum. The first and most important one is the measure of evidential support, or the degree of confirmation, given by one sentence (a sentence representing an observation) to another (representing a hypothesis). Furthermore, probability₁ of a hypothesis h given an evidence statement e , is taken to be the fair betting quotient in particular kinds of bets on h , conditional on e . The third way of describing probability₁ is as an estimate of probability₂ values, that is as an estimate of relative frequency.

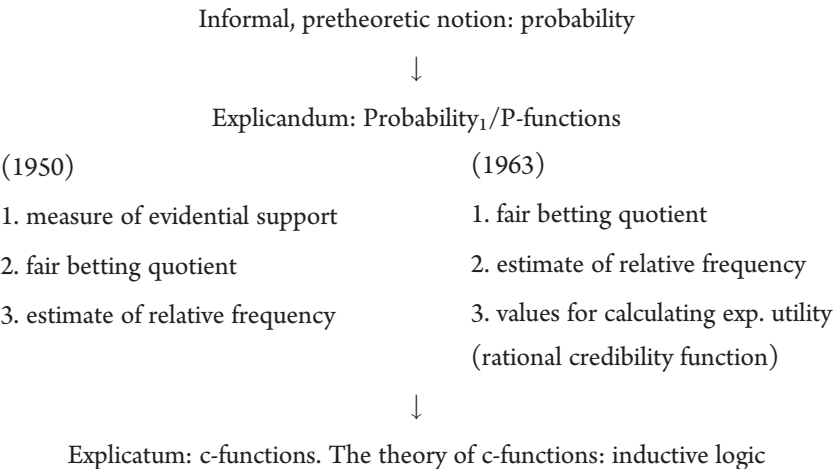
In the Schilpp volume (1963c, 967), Carnap still characterizes his project as one of explication, but provides a somewhat different list of ways to describe the explicandum. Probability₁ is still thought of as a fair betting quotient and an estimate of relative frequency. However, the most important meaning of probability₁ is now taken to be one that applies to decision making. The values of probability₁ for the relevant pairs of sentences are those numerical values that should be used as probabilities in the calculation of expected utility in a decision situation, in order to arrive at a rational choice.

Once the target concept is narrowed down to an explicandum, it is clear, or at least much easier to decide, which of its features should be captured in a newly designed explicatum. In the case of inductive logic (probability₁), the explicatum is the theory of confirmation functions, i.e., specific numerical functions defined on pairs of sentences (or propositions) of a particular formal language, like c^+ or another function from the λ -continuum. Such a formal theory of confirmation functions is the inductive logic proper, that is, a formal theory whose probability statements are analytic. This is the main sense in which inductive logic is logic: it is a formal theory of certain language forms, such that the determination of the truth values of (some of) its sentences is possible without recourse to empirical checks. That this is the sense in which Carnap understood the logical status of inductive logic has been acknowledged (Jeffrey 1973), but for some reason this awareness seems to have been lost later on (see section 6 below).

Through this brief exposition one can already locate some of the changes that the inductive logic project underwent between its early and late phases. While two out of three descriptions of the explicandum remained the same between 1950 and 1957, the description that Carnap at the respective time considered the most important did change. At the pretheoretical level nothing changed over the years; in the late works Carnap continued to talk about there being two probability concepts and

himself being interested in explicating the logical, or inductive, one. At the level of the explicatum, multiple changes occurred, pertaining to the kinds of formalism used in defining confirmation functions and the number of admissible functions—but Carnap’s overall view of the formal, logical status of his explicata did not change.

This general conceptual structure of the inductive logic project is schematically presented below.



In the next two sections, I will unpack this brief characterization of Carnap’s inductive logic, focusing on those features of the 1950 and the 1963 versions that will be the most relevant for answering the main question of the paper.

3. EARLY INDUCTIVE LOGIC: OBJECTIVITY, LOGICALITY, RATIONAL BELIEF

Both in the earliest papers on the topic and in *Foundations*, Carnap introduces the concept that he wants to explicate as the logical, semantic concept of probability. Such a characterization is necessarily imprecise, since, according to Carnap’s conception since *Logical Syntax*, semantic and logical concepts acquire their precise, correct meaning only within a clearly specified linguistic framework—and the task of explicating probability₁ is exactly the construction of such a precise framework. Hence, according to the schematic picture of explication introduced above, the explicandum needs to be made more precise by providing the description of its relevant uses and meanings.

The characterization of the explicandum “probability₁” considered most important in the early versions of inductive logic was as the degree of confirmation, or evidential support, between pairs of sentences. It was supposed to be a relation independent of anyone’s beliefs, established by the meanings of the sentences involved, as defined into the framework that stipulates their component vocabulary, formation rules, and transformation rules. The two other characterizations of the explicandum, as a fair betting quotient and as an estimate of relative frequency, operationalize the

concept further and specify its relation to the second explicandum of probability, that is probability₂.

The description of logical probability as a semantic concept does not mean that Carnap assumed that there was such a thing as an objective, formal-system-independent fact of the matter concerning the confirmation relations between sentences of, say, the language of science. Such a reading would mean that inductive logic was intended as a formalization of such semantic facts; that it was meant to capture the objective reality of confirmation, so to speak. However, such a reading is entirely at odds with Carnap's metaphilosophical views, which were fully developed by the time *Foundations* was published (the *Logical Syntax*, with its principle of tolerance, appeared in 1934, and "Empiricism, Semantics, and Ontology" [1950a] appeared in the same year as *Foundations*).

A system of inductive logic is essentially a linguistic framework like any other. An important feature of such frameworks is that they are constructed freely by their creators, and that any choices made during the construction cannot be classified as correct or incorrect; the frameworks only need to fulfill the goals of their maker. There is no notion of truth and theoretical correctness outside a specific linguistic framework (unless it be in a separate metaframework)—and the same applies to inductive logic.

Throughout the text of *Foundations*, Carnap repeatedly expressed the wish to narrow down as much as possible the range of admissible confirmation functions (i.e., the functions that qualify as rational within the system set up there)—even up to the point of there being only one such function. However, we should not read this as an aspiration to capture some objective, semantic reality. For Carnap, all talk of an objective, linguistic-framework-independent reality of semantic facts was meaningless. The reason for looking for stronger and stronger systems of inductive logic should rather be seen from the perspective of considering linguistic frameworks as tools. Inductive logic provides a tool—an explicated concept—to be used by an inductive reasoner who wants to make their inductive practice more precise and consistent (see especially [1952, §18]). And the stronger the tool, the easier it will be to apply it.

That this is the interpretation of inductive logic that Carnap was after does not only follow from evoking his metaphilosophy; this attitude is visible in various places throughout the *Foundations*, and is sometimes expressed in rather striking ways. Section 62 of *Foundations* contains a discussion of formal systems of inductive probability developed by other authors such as Keynes, Hosiasson, and Jeffreys. Carnap concludes that once we subtract the unrestricted applications of the principle of indifference (which lead to inconsistency), all of the previous systems for inductive probability are equivalent to what he calls the theory of regular c-functions. Such a theory is quite weak: outside of some trivial cases involving tautologies and contradictions, it does not give any specific numerical values of $c(h, e)$. That is, any assignment of confirmation-function value for (practically) every pair of sentences is consistent with the theory. Any such theory, Carnap proceeds, is too weak to be a useful tool for an inductive reasoner. Moreover, these theories are unnecessarily weak: making them stronger is both a practical requirement and a real conceptual

possibility. This is contrasted with the situation in the theory of probability₂, that is relative frequency. There, the specific probability values cannot be predetermined by the theory: they have to answer to the empirical facts. The counterparts of the sentences of the form $c(h,e)$ are factual in the theory of probability₂: they are considered true only if certain things are the case in the world. A theory of probability₂ should not analytically exclude anything that is empirically possible. The situation is different, however, for the theory of inductive probability:

On the other hand, in the case of a theory of probability₁, there is no reason for this restriction. A sentence of the form ' $c(h,e) = r$ ' is not factual but L-determinate. Therefore, a logicomathematical theory of probability₁, in other words, a system of inductive logic, can state sentences of this form. (Carnap 1950b, 344)

That is to say, the formal systems of inductive logic are defined in such a way that the truth value of sentences stating the c -values is determined solely by the semantic rules of the system—it is independent of any empirical facts and no empirical checks are required to establish the truth of these sentences. And since as creators of this formal system, we can choose its semantic and syntactic rules as we please (according to the principle of tolerance), we are therefore free to construct systems of inductive logic that are as strong as we want and need—as long as they in the end do the job we want them to do, such as helping us in our inductive reasoning.

Naturally, one should still have some principled reasons for the choice of particular features of the theory of confirmation functions. Spelled out in the explication framework introduced above, this means that the formal features of the explicatum should correspond in significant ways to the features of the explicandum that were highlighted at the middle level. In *Foundations*, Carnap does not explicitly state the particular reasons for the adoption of specific axioms; such focus will appear at the later stages of the project, as we will see shortly. However, some of such reasons are implicit in the text.

In *Foundations*, a lot of attention is given to the parallels between deductive and inductive logic. The first of these parallels is the one explored above in connection with Carnap's overall approach to linguistic frameworks and the freedom for choosing them. But there is also a more specific connection. Inductive logic is repeatedly characterized as a generalization of deductive logic: just as deductive logic is the logic of inclusion of semantic ranges of sentences, so is inductive logic the logic of their partial inclusion—this view immediately leads to the adoption of the standard axioms for probability. Also, from this perspective it is natural to focus on certain invariance and symmetry considerations as a basis for the axioms—just as in deductive logic. Hence the axiom of symmetry, which (roughly) requires atomic sentences to have the same prior probability, is introduced into the system as the one that says that certain features of the confirmation function should be invariant under permutations of individuals, just as in deductive logic (§90).

Finally, at the level of the explicatum, inductive logic evolved even within its early period. The focus of *Foundations* is on defining a wide class of confirmation

functions (constructing the theory of regular, symmetric c-functions), and zooming in on one of them, dubbed c^* . In the *Continuum*, Carnap finds a way to parameterize the original wide class of functions, and hence offers the whole class as the explicandum. Whoever wishes to apply the tool of inductive logic to their inductive investigations is free to choose one of these functions according to her purposes and preferences.

To sum up, the early inductive logic was concentrated on making precise the concept of inductive, or logical, probability, understood chiefly as the degree of confirmation, but used also as an estimate of relative frequency and a fair betting quotient. Carnap's goal was to construct a formal system defining a class of confirmation functions which would correctly capture the clear cases of the use of inductive probability, while at the same time being as strong as possible, in order to facilitate the application of the system in actual inductive reasoning. The axioms of the system were chosen mainly according to certain considerations of invariance and independence of particular interpretations, in parallel with deductive logic.

4. MATURE INDUCTIVE LOGIC: CREDIBILITY AND DECISION MAKING

While the notes containing the updated systems of inductive logic had been circulating from the mid-1950s (see Carnap 1973) and some indication of the new developments had been given in a book published in German (Carnap and Stegmüller 1959), the first full presentation of what we can today consider the mature inductive logic appeared in print in the Schilpp volume (1963c). It is there that the new understanding of the explicandum is introduced and the subsequent publications can be seen as a gradual elaboration of the ideas that originate in that first presentation.

The target concept of the explication is the same as in the early system: the logical, or inductive concept of probability—probability₁ as opposed to probability₂. Once again, the explicandum needs to be described in more detail before the explicandum can be constructed. At this point, Carnap dismisses the characterizations of probability₁ as degree of confirmation and degree of rational belief as too vague and ambiguous (1963c, 967). The first dismissal constitutes a clear break from the earlier approach; the second does not, since in *Foundations* (§12) he had already criticized the idea that the explicandum of inductive probability should be understood in terms of degrees of belief.² The reasons for the dismissal of degree of confirmation as the main description of inductive logic's explicandum are discussed in detail in the preface to the second edition of *Foundations* (1962b). In short, that concept turned out to be ambiguous between a number of possible readings.

On the positive side, Carnap retains the interpretation of logical probability as a fair betting quotient (in specific cases) and as an estimate of relative frequency. However, this time the main work is done by a new description, according to which the value of logical probability is a number to be plugged in as the probability value used to calculate the expected utility in decision situations, in order to arrive at a rational choice. In Carnap's own words this becomes:

Suppose that the following offer is made to X: the benefit B will be bestowed upon him in case the event h occurs. If h does not occur, he will receive nothing. Then $P(h,e)$ [the logical probability of h given e] is the factor by which the utility of B for X must be multiplied in order to calculate the rational subjective value for X of the offer. (Carnap 1963c, 967)

(The above formulation is somewhat sloppy in not explaining the role of e; from the text that follows it can be deduced that here e is assumed to represent the entire observational knowledge of X.) The connection of logical probability to expected utility calculation was stressed in *Foundations* as well. However, now some additional decision-theoretic machinery is introduced, as well as the concept of credibility.

The standard calculation of expected utility of an act for an agent X requires a numerical representation of X's preferences (in the form of a utility function) and beliefs (in the form of a credence function). Carnap postulates that there should also be something like a rational utility function (see Carnap 1962c, 306) and a rational credence, which, combined via expected utility maximization, would lead to the rational choice or decision in the given situation. He tried to develop the fundamentals of the former, in the form of rational utility theory, in a note on value concepts which was recently transcribed from an unpublished manuscript (Carnap 2017). Inductive logic was meant to supply the latter, that is a theory of functions that would correspond to rational belief functions of a special kind.

The task of inductive logic, then, is to construct c-functions that would correspond to rational belief functions; what would these look like? First of all, Carnap assumes that in the target scenario the agent has to be rational in the sense of allowing only the evidential statements themselves to affect her credence (as opposed to factors such as emotions). This means that there should be a function, call it Cred (credibility), underlying any possible credence of that agent, specifying that for any body of observational knowledge e, the agent's credence for a hypothesis h after learning e—and nothing else—equals $\text{Cred}(h|e)$. Only when such a credibility function exists is the agent considered rational in the relevant sense. However, a credibility function is not automatically rational just because it underlies an agent's credence.

Some intuitive criteria of rationality for credibility functions can be formulated, and logical probability is understood as assigning values for degrees of rational credibility. The explicandum for probability₁ can now, against this background, be given a new description using the new concept of credibility: logical probability is to be understood as a rational credibility function (1963c, 971). Hence, the c-functions of the inductive logic system are to be constructed in such a way that their features will correspond to the properties of such "informal" rational credibility functions. A credibility function can also be thought of as credence at time zero, that is credence of an agent in an idealized situation of not yet having acquired any observational knowledge. In such a situation, the agent does not know yet of any nontautological sentences whether they are true or not. Probabilities have to be assigned to sentences based not on data, but only on the structure of the sentence, e.g., the number of atomic predicates in it.

Carnap stressed in multiple places that he was not interested in formulating rationality requirements for regular credence, but for credibility instead—and he was aware that this was a significant difference between his approach and the standard Bayesian one. He also admitted that this concept is a highly idealized one. It is not easy to reconstruct the exact reasons for Carnap's choice of this particular concept. On the one hand, he draws a parallel with other sciences where theoretical progress had often been achieved by the construction of more and more abstract concepts (1971b, sec. 5). On the other hand, there is the insistence on looking for stronger systems, combined with the awareness that there does not seem to be any reasonable rationality requirements for credence that go beyond probabilistic coherence (1962c, 316). At the same time, it seems to me that we should not underestimate here the strongly semantic character and ambition of the inductive logic project, which seems not to have disappeared in the later phases. Probability₁ is for Carnap a semantic concept, and so even at the level of the explicandum, probability statements should follow only from the stipulated meanings of the sentences involved. Seen from this perspective, the construction of the concept of credibility can be understood as aiming at distilling the pure import of stipulated linguistic meaning on rational belief, i.e., the precise degree to which rational belief depends on the framework in which it is articulated.

Carnap maintained the position that the description of the explicatum in terms of rational decision making rather than confirmation facilitates the construction of precise, numerical functions at the level of the explicatum, thanks to the developments in decision theory (1963c, 972). This assertion shows once again how deep the commitment to the explicatum being a useful, workable concept runs—the need for a precise explicatum in this case drives the search for appropriate characterizations of the explicandum (note also that there appears to be no corresponding insistence on the explications being correct, in line with Carnap's general attitude of tolerance). It remains something of an open question whether these late systems of inductive logic have proved themselves to be as useful as intended. On the one hand, Carnap's symmetrical functions—this time under the guise of exchangeable hypotheses—are one of the main types of models used in Bayesian statistics. On the other hand, the connection that he drew between the symmetry axiom and rationality requirements, even if applied only to a very specific and abstract situation, has been widely criticized (see, e.g., Zabell [2011, sec. 13.2]).

But it must be kept in mind that the discussion above pertains to the “informal” description of the explicandum, and not to the explicated concept itself. That is, notions such as utility, credence, and credibility appear only at the middle level of our explication schema. They are there to facilitate the understanding of how the target concept of the explication may appropriately be used in reasoning. Still, even the operationalization of the concept of logical probability in terms of decision making is constructed in a way that ultimately focuses on the semantic nature of the concept: the values of rational credibility functions depend only on the meanings of the terms occurring in them.

The actual explicatum continues to be a purely formal, mathematical object. Here, just as in the earlier versions of the system, the explicatum is constructed by stipulating a set of axioms which the confirmation functions must obey. Yet the focus on the parallels with deductive logic and invariance considerations is pushed into the

background; the additional, new interpretation of inductive probability as rational credibility function opens up new possibilities for drawing principled reasons for particular axioms of inductive logic. Now the axioms can be informed by the intuitions concerning rational decision making.

This picture of the axioms of inductive logic as corresponding to rationality constraints on credibility functions appears in the Schilpp volume and is subsequently developed in more detail, all the way into Carnap's last works on the topic (see especially Carnap [1971b]). Still, the question of whether and how inductive logic in this picture can actually be construed as normative for belief has not been investigated so far; some pointers towards a possible answer can be found in works by Steinberger (2017) and Carus (2017). First, the question of the normative import of any linguistic framework becomes problematic at best, given Carnap's commitment to the principle of tolerance. Second, in the case of inductive logic, norms for belief and action do enter the picture even at the level of characterizing the explicandum; hence, they cannot be seen as directly imposed by the linguistic framework of inductive logic itself, which is constructed at the explicatum stage. This issue deserves to be given more attention.

Focusing the explication on rational decision making means that some of the axioms for the confirmation functions are easily grounded in by-now standard arguments for probabilistic coherence in terms of betting behavior, which also corresponds nicely to one of the other descriptions of the explicandum, that of a fair betting quotient. The axiom of symmetry, however, receives a slightly different treatment, as it continues to be motivated by a logical invariance argument (1971b, 23). This is precisely the point where Carnap's inductive logic has received the most criticism from the proponents of probabilism as the only rationality constraint on rational belief. Here is not the place to weigh in on that debate. What I can say, however, is that in Carnap's inductive logic-as-explication view, nothing forces the rationality considerations that enter the picture at the level of the explicandum to be restricted to the practical rationality of betting; in principle, invariance considerations could be playing a similar role.

As we have seen, the further elaboration of the explicandum introduced in the mature version of inductive logic brought with it a host of notions which could now be used to inform the construction of explicata. The resulting system, however, retained its basic structure: the target concept on the top, described in a number of particular ways, and explicated via a set of formalized confirmation functions. We are now ready to approach the main question of this paper, that is: to what extent the changes in inductive logic between Carnap's early and mature versions constitute a development from a logical towards a subjectivist approach to probability.

5. WHAT DID AND DID NOT CHANGE

Now that the two phases of the inductive logic project have been reconstructed in a schematic way which facilitates comparisons, we can more easily spot the changes that the project went through, and locate these changes in particular places in the explication structure. First, I will recall Carnap's own interpretation of the evolution of his project up to 1962, reported in the preface to the second edition of *Foundations*

(written in early 1962). Then, based on the discussion of the previous sections, I will list the changes to the project that were not so explicitly acknowledged—as well as the relevant features of the system which did stay the same from the beginning all the way until the last accounts. Finally, I will assess the extent of these changes, before turning to an evaluation of whether the target notion of inductive probability underwent a significant modification in the process.

Carnap starts the preface to the second edition of *Foundations* with a statement that he still maintains the basic tenets of his theory, both on the philosophical and on the formal level. He then proceeds to list the modifications that were introduced to the system. On the technical side, the requirements of logical independence of atomic sentences and of completeness of primitive predicates are dropped, due to the developments of techniques allowing for dealing with previously forbidden cases. The most attention is given to the changes within the concept of confirmation: abandoning it as a description of the explicandum, and the ambiguous ways it was used in the original version of the book.

In that preface Carnap insists that he does not intend this change to be an abandonment of the logical concept of probability, of which inductive logic is still intended as an explication. We can see here the difficulties that focusing on a particular aspect of the explicandum (degree of confirmation) caused to the development of a workable explicatum. Hence, to be able to develop the project further, one had to find more fruitful ways to describe the explicandum—for instance, by focusing on the way that the target concept is used in decision making, rather than on trying to capture its meaning in use-independent ways.

And it is this change—the introduction of a more strongly use-related description of the explicandum—that is the most prominent one, which also drives a lot of the other developments. Replacing degree of confirmation with rational credibility as the most prominent characterization of the explicandum made extra resources available to supply reasons for the proposed axioms of the explicated system. In the mature phase, these reasons were mainly the rationality constraints on decision making; note however that the latter were not part of the inductive logic proper (i.e., the explicatum). In the early phases, as already noted, the stress was put primarily on certain invariance considerations meant to correspond to their counterparts in deductive logic. This perspective does not vanish entirely from the mature systems, since the symmetry axiom retains its original justification. Moreover, we have to keep in mind that even in the late phase the focus is on rationality criteria for beliefs in a very special situation: a situation where the beliefs are supposed to be influenced solely by the semantic content of the sentences involved.

While the intention that the c-functions be tools offered to the inductive reasoner is maintained throughout the whole development of the project, the intended scope of application of these tools does change. In the early phase the choice of a particular inductive method implies a commitment to use it in all inductive endeavors of the agent (see Carnap 1952, §18). In the mature versions of the system this commitment is significantly weakened and there is much more talk of choosing an inductive method to suit a particular inductive investigation rather than fixing the whole “inductive character” of the agent (for discussion, see Groves [2015, sec. 2.6.5]). But

Carnap himself would probably have disagreed with interpreting this development as a move towards a subjective interpretation of the concept of probability₁—it was merely a widening of the possibilities for applying the proposed tool, not a change in the interpretation of the concept that the tool was modeled to reflect.

It is equally important for my current purposes to note what did not change in the transition from the early to the mature inductive logic. The immediate thing to mention here, of course, is that the picture of the whole enterprise as a project in explication was never abandoned. Carnap's conception of explication applies equally to the late and the early inductive logics: their aim is to construct a formal concept as an improvement and replacement of an existing, informal one. Systems of inductive logic were tools to be used by reasoners who want to improve their inductive methods, i.e., their ability to draw the best and most useful conclusions from data.

Then there is the issue of logicity. Carnap considered inductive logic to be logic, regardless of what kinds of concepts were used in the characterization of the explicandum. He maintained this position even in his last paper, "A Basic System of Inductive Logic" (1971a; 1980), where the object language on which c-functions are defined was assumed to be partially interpreted via the inclusion of so-called attribute spaces (for discussion, see Sznajder [2016]). This conception of inductive logic as a form of logic is consistent with Carnap's late (semantic) conception of what a logic is: it is a formalized language whose sentences are to be evaluated solely by the formal rules of the system and not by empirical or factual checks. Moreover, Carnap continued to call the target concept of his explication, that is, the concept of inductive probability, a logical or semantic concept. That is to say, even in informal, pre-explicated uses, sentences containing the notion of inductive probability were taken to be judged true or false by their users solely on the basis of the stipulated meanings of the terms used in them. This interpretation is not of course inconsistent with there being more than one admissible confirmation function once the target concept is explicated.

I have now identified the modifications within inductive logic that developed over the years 1942–70 and have located them in a coherent representation of the whole project. Knowing what we know now, can we say that these changes were significant enough as to transform inductive logic into something else than what it was in the beginning? How important were the revisions?

The focus on the use of the notion of inductive probability rather than its semantic meaning as degree of confirmation did not really change the system fundamentally; it was only a change in the salience given to different aspects of the target concept. The awareness of the use of inductive probability for rational decisions was present from the early stages of the project. In one of the early papers Carnap recognizes, for instance, that one of the main functions of the concept of inductive probability is to provide the base for rational decision making:

Our discussions have shown that statements of probability in the logical sense, that is, degree of confirmation, may be used as a guide for practical decisions. To use them in this way does not imply any apriorism, because the purely logical probability statements are not meant to be used in isolation but rather in

application to the concrete knowledge situation at the time of the decision.
(Carnap 1947, 147)

After all, the concept of probability is explicated within inductive logic in order to provide a rational reconstruction of the everyday practice of inductive reasoning (1945a, 95). In *Foundations* (§49–51), moreover, the specific use of inductive probability in determining rational choice through expected utility maximization is a prominent feature of the narrative.

At the same time, the strong semantic focus of the explicandum does remain in place in the later inductive logic, even if the reader might be distracted from it by the talk of utilities and rational beliefs. The rationality of belief that Carnap is after goes beyond thinking of rational degrees of belief as coherent belief states of some agent. He is looking for belief states that are influenced only by the meaning of sentences involved; wants to strip a belief state of any reference to nonsemantic, factual knowledge—hence the creation of the concept of credibility and the insistence on the semantic character of the target concept of the explication.

Overall, the general goal and the structure of the inductive logic project do not change significantly. While the formalism did undergo some changes,³ Carnap's interpretation of the status and the role of the explicatum level of the picture did not change at all. What did change is some of the characterizations of the explicandum; this change allowed for a more principled way of constructing the explicatum, due to a close connection between intuitions about rationality of decisions and the axioms for *c*-functions. It is clear that the revisions from the early to the late inductive logic were not significant enough to make the late one unrecognizable from the viewpoint of the early one. What remains to be answered is whether the changes were significant enough to warrant the interpretation that Carnap had moved from a logical to a subjective notion of probability.

6. LOGICAL OR SUBJECTIVE?

Assessing the degree to which Carnap moved away from a logical and toward a subjective interpretation of probability requires knowing what we mean by these two terms. Somewhat unsurprisingly, there appears to be no consensus on the exact features of the different interpretations, especially when it comes to the logical one. Therefore, my method in what follows will consist in focusing on some of the features of the two interpretations and assessing Carnap's development of his inductive logics against that background—building on the understanding of the project developed over the previous sections. I will focus especially on the interpretations that other writers have proposed of the development of Carnapian inductive logic or offered as arguments for the claim that Carnap's underlying notion of probability had changed.

In practically every philosophical introduction to probability, (the early) Carnap is listed as the champion of the logical interpretation, considered as one of the three dominant approaches to probability: the logical, the Bayesian or subjective, and the empirical. This is not surprising, since Carnap himself, as mentioned repeatedly above, takes the target concept of his explication to be exactly that: the logical

concept of probability. The question remains whether the other authors agree with Carnap on what that means precisely.

One of the most popular versions of what the logical interpretation of probability actually amounts is that its proponents believe that there is only one true, correct probability function for a given language, in which the objectivity and logicity of logical probabilities is grounded. This way of thinking immediately leads to the a view of Carnap as having moved away from such a "logical" conception on the grounds that he gradually extended the range of admissible conformation functions. For instance, Suppes (2002, 198) explicitly quotes this as the reason that Carnap is to be seen as moving towards a subjective notion of probability, or at least as not being able to sustain the logical one. The same perspective is also adopted by Skyrms, as evidenced by the following quote, as well as the one in the opening section of this paper:

At the beginning of his investigations in inductive logic Carnap hoped that all of scientific inference could be based on one inductive rule. That rule would have a necessary status. The logical character of inductive logic would derive from the logical status of this rule and of the prior which led to it. He soon started down the road that leads from Keynes to de Finetti. Ever larger classes of inductive rules were seen as part of inductive logic. In the end he saw a need for further expansion, in particular to deal with the three problems discussed here [i.e., analogy, value continuum], and in general to arrive at an adequate treatment of scientific inference.

In posthumous "Basic System" (1971, 1980), Carnap realizes somewhat reluctantly that he has become a subjective Bayesian. The logic in inductive logic is now the logic of coherence. (Skyrms 1996, 334)

A similar interpretation is alluded to by Kyburg (1970, 71), who first identifies the difference between the logical and subjective interpretation as pertaining only to a different number of rationality constraints on graded belief, and goes on to proclaim: "but Carnap is no true subjectivist; in the back of his mind he still entertains the belief that there is only one rational credibility function." (The rationale for this latter assertion is not given in Kyburg's text and, in light of the above discussion, is clearly false.)

The same reading can be found in Gillies:

In the logical interpretation, the probability of h given e is identified with the rational degree of belief which someone who had evidence e would accord to h . This rational degree of belief is considered to be the same for all rational individuals. The subjective interpretation of probability abandons the assumption of rationality leading to consensus. (Gillies 2000, 53)

However, as we have seen, Carnap was never committed to there being one "objective" (in the sense of being "correct") logical probability function. Even in *Foundations* the proposal of c^* as the main explicatum for logical probability is not made in the spirit of proposing the one, ultimate, correct function.⁴ In a letter to de

Finetti from 1963 [Carnap \(1963a\)](#) addresses this very point, saying that while at times he thought that it could be possible to find enough rationality requirements to constrain the range of admissible confirmation functions to a single one, he does not think that he had “published this thought as an assertion.” Even more importantly, such a reading would be entirely at odds with his general metaphilosophical views: an explication is a proposal for a linguistic framework (or for a concept within such a framework) and has nothing to do with any objective correctness. The idea of a single true or “correct” probability function amounts to a kind of probabilistic realism of just the kind that Carnap rejected throughout his career, and most especially in his later years (see 1950a). So if we take the logical interpretation of probability to be the assumption that there is only one correct or rational probability function for a given language, Carnap never accepted a logical interpretation of probability.

According to commentators on logical probability, one critical feature of logical probabilities is that they are supposed to be objective. And this objectivity has to be different from the objectivity of statistical probability, which is grounded in empirical facts. If objectivity is to be a defining feature of logical probability, how can we think of probabilities being objective in spite of there being a plurality of admissible probability functions?

In all phases of the project—including the late articles—Carnap stressed that his notion of probability is objective and not personalistic or subjective. What he meant was that a sentence of the form “the logical probability of h given e ” is fully formed and meaningful without referring to a particular agent. What makes a concept subjective, then, is precisely that its meaning requires a reference to a particular person. According to Carnap, there are concepts that are intrinsically subjective in this sense, for instance the concept of being preferable (preferable for whom?)—but logical probability is precisely not that kind of concept (see 1950b, §12). If it was, then, in a way, that would be a truly subjective conception of probability, one that requires all probability statements to involve an actual subject, that is an agent. A denial of that perspective constitutes a truly objective notion of probability: one whose values by definition depend only on the meanings of the sentences involved and not on characteristics of any particular agents. And if this was what is meant by the logical interpretation of probability, then Carnap did not abandon it at any point in the development of his inductive logic, as we have seen above. Additionally, it would be interesting to ask whether what is usually considered subjectivism about probability is actually subjective in this current sense.

One could try to argue—and some of the commentary to the effect that widening the range of admissible confirmation functions over time meant an abandonment of the objective concept of probability can be read in this way—that once there is more than one rational confirmation function, logical probability statements contain an implicit reference to the choice of the confirmation function (say, to a particular value of the λ parameter). And this is assumed to introduce a subjective element into the system, one that effectively leads to adopting the subjective, or personalistic, notion of probability. However, this would not be the kind of subjectivity I have described above; it is rather subjectivity understood as a certain amount of freedom granted to someone who applies the explicated concept as a tool. But within the system itself,

the explicit reference is to a value of a parameter, not to the person whose characteristics might nicely correspond to that parameter. And when it comes to subjectivity understood as a freedom of choice afforded to the user of any linguistic framework, Carnap was the leading champion of this view since formulating his principle of tolerance in the *Logical Syntax* (Carnap 1934, §17).

What about references to rational agents and rational beliefs, though? After all, the main characterization of the explicandum in the late period is of logical probability functions as rational credibility functions—does that formulation presuppose an implicit reference to an agent, even if it is an idealized one? Carnap himself continued to insist—both in the early and the later periods—that such references are only a manner of speaking that facilitates understanding of the underlying objective concept, in the same way as considering the application of deductive consequence to rational reasoning does not render deductive logic subjectivistic. There probably are people who do not agree with him on that. What matters for my main question is that even if this reference to rational belief sneaks in an implicit reference to an agent that would threaten Carnap's understanding of objectivity, this is already the case in the early period of inductive logic's development. This means that if we want to count this interpretation as an instance of subjectivism about probability, we would have to classify both the early and the late inductive Carnap as a subjectivist—not just the late one.

We can conclude, then, that if “logical” and “objective” probability are understood in the above sense of not containing an implicit reference to an agent in the sentences involving that concept, then Carnap's explicandum was either logical and objective throughout the two periods (if you agree with him on whether references to rational belief do not spoil this picture), or he was a subjectivist from the start (if you think that a plurality of admissible probability functions necessarily implies a subjective component). Whichever of these two interpretation we prefer, there is no change in either respect between the early and the late period.

Another argument for Carnap's move towards subjectivism about probability points to his acceptance of the fair betting odds interpretation and the connection of probability to decision making via expected utility calculation. As we have seen, however, both of these features are present already in *Foundations*—the change in the later period was rather an increase in the relative weight of such applications, rather than a drastic change in substance. Hence, under this interpretation of what subjectivism amounts to, we would also be forced to classify Carnap as a subjectivist from the start.

There is also an interpretation according to which Carnap should be classified as belonging to what has become known as objective Bayesianism. Objective Bayesians subscribe to the interpretation of probability as rational degree of belief, while also postulating rationality constraints that go beyond the basic axioms of probability. If we were to grant this interpretation, then Carnap's early inductive logic would have to be classified as Bayesian in spirit as well. But this raises the prior question of the difference between objective Bayesian and the logical conception of probability; was Carnap both of these in the beginning? Or should we retire the logical label entirely, to a large extent against his own identification? Additionally, such a picture is a little

too simplistic, for it ignores some important features of the project and its motivation. In particular, as I have tried to show in section 4, the focus on credibility instead of credence in mature inductive logic was motivated by the search for a certain semantic stability of logical probability. This sort of perspective is quite unique to Carnap's approach and, in my opinion, it makes his inductive logic as a whole sufficiently different from the other, credence-based approaches for us to reconsider the labels such as objective Bayesianism or the logical interpretation of probability.

Taking all of the above into account it hardly seems possible to assign categorical labels to the early and the late inductive logic. As explained in the first part of this paper, neither the widening of the range of admissible confirmation functions nor the inclusion of decision-theoretic notions in the characterization of the explicandum changed Carnap's convictions about the nature of the concept he was explicating or of the logicity of the resulting systems. So it is all but impossible to claim that he had abandoned the logical concept of probability or become a subjectivist. If our criterion for being a subjectivist is rather weak and includes only the acceptance of the betting-quotient interpretation of epistemic probabilities and of the role of probability in expected utility calculations, then we would have to classify Carnap as a subjective Bayesian right from the beginning. Such an interpretation, however, would ignore some of the most important features of the Carnap's intended interpretation of probability as specified in the characterizations of his explicandum.

Finally, if the objectivity and logicity of probability₁ is to be understood in the strict Carnapian sense of implying no implicit references to any subjects—that is, a conceptual rejection of a psychologistic interpretation of the concept—then Carnap's explicandum was always meant to be objective and hence logical. Overall, Carnap's own conception, as reconstructed above, was to a large extent *sui generis*, diverging significantly from both the early logical interpretation of probability and from mainstream Bayesian subjectivism. Which is exactly why we can find it difficult to categorize Carnap as the one or the other—in some significant ways it simply cannot be done.

7. THE ORIGINS OF CERTAIN MISCONCEPTIONS

In this section I will speculate on some possible motivations for the opinions on the development of inductive logic that led to the initial question of this paper. It would of course be impossible to deduce the exact motivation of the authors whose claims have provided the starting point of my discussion. Still, one can point to some formulations in Carnap's own text which, when taken out of the context of his entire project, can encourage the kinds of readings discussed in the previous section.

This holds especially for the popular reading of the early period of inductive logic as striving for the definition of the one correct confirmation function. Carnap's own writing often invites this sort of reading, as can be seen in the following quote from *Foundations*: "Our task will be to construct the rest of inductive logic by narrowing the class of c-functions and finally selecting one of them. This will be done in later chapters" (Carnap 1950b, 337). It is quite natural to interpret "our task" here as referring to the entire goal of the inductive logic enterprise. But this sentence can also, more straightforwardly, be understood as simply sketching what is about to happen

in the current book—a summary of sorts rather than a manifesto regarding ultimate conceptual commitments.

Still, some other assertions in *Foundations* are harder to interpret in the same spirit, such as the following: “We have to lay down further requirements in addition to regularity, and thereby finally come to the selection of one particular c-function” (1950b, 345). Putting this sentence in context gives us a better idea of why Carnap is writing about the need to narrow down the theory to a single c-function—but it does not change the fact that indeed, having a single c-function appears to be the final goal of the project. The context here is the need for a theory strong enough to be useful for an actual reasoner wishing to apply it (see section 3 above)—the assumption being that what is ultimately needed is to be able to compute a single value of the confirmation function for any pair of sentences. The reason for seeking such a strong, single-function theory is not the need for objectivity or logicity; as we have seen, those features of the theory would not be compromised by making the theory weaker. But it does seem that Carnap hardly succeeds in presenting his goals clearly enough, thus inviting the reading of the inductive logic project as striving for the definition of the one true, correct confirmation function.

And what is the evidence for Carnap's move towards Bayesian subjectivism? The previous section already discussed whether the inclusion of decision-theoretic concepts in the explanation of the explicandum amounts to a deep conceptual shift in the underlying interpretation of the concept of probability. As for other textual evidence, the following quotation from the late “Basic System” has been proposed as such (see Zabell 2005, 2011):

I think there need not be a controversy between the objectivist point of view and a subjectivist or personalist point of view. Both have a legitimate place in the context of our work, that is, the construction of a system of rules for determining probability values with respect to possible evidence. At each step in the construction, a choice is to be made; the choice is not completely free but is restricted by certain boundaries. Basically, there is merely a difference in attitude or emphasis between the subjectivist tendency to emphasize the existing freedom of choice, and the objectivist tendency to stress the existence of limitations. (Carnap 1980, 119)

At the first glance, it can look like Carnap here is indeed making some concessions to the subjectivist view. Upon further examination, however, things no longer look so straightforward. First, we should look at what immediately succeeds the above quotation: “I give more attention to the latter, because in my work I am mainly interested in discovering new rationality requirements which lead to narrower boundaries.” That is, Carnap identifies himself explicitly as someone who belongs on the objectivist side of this scale.

What we have here, then, is Carnap saying that he recognizes that there can be different points of view and different modeling choices in constructing a conceptual framework for probability as degree of confirmation. This is not a very surprising

statement coming from someone who had been championing this sort of peaceful coexistence of different approaches to the same problems for many years. His own work, still, he classifies as belonging to the objective side—in what sense is this text evidence for Carnap's "substantial retreat from the hard-core credibilism of the *Logical Foundations of Probability*" (Zabell 2005, 30)? The interest in narrowing down the range of admissible confirmation functions via stronger rationality requirements was, as argued above, already an important part of the motivation behind *Foundations*. This quote from the later work shows, rather, a certain continuity of intentions, a continuity extending over the possible changes in some of the methods used in the construction of the system.

Another possible source of certain interpretations of inductive logic can be Carnap's explicitly mentioned references. In the early period especially, he spoke of himself as being inspired by, or building on the work of, people such as Keynes, Wittgenstein, or Waismann—the main figures in what is widely considered the "logical" approach to probability. Later on, he would mention people like Savage or de Finetti and visibly draw inspiration from their work. Still, all of this does not have to mean that Carnap was taking in the tools developed by others together with every conceptual aspect of the theories that accompanied these tools.

8. CONCLUSIONS

This paper began with the question of whether Carnap had abandoned the logical interpretation of probability in the 1960s in favor of a more subjectivist one. Somewhat unsurprisingly for the philosophy of such a protean concept as probability, the answer to the main question of the paper turns out to be: it depends. In particular, it depends on what you mean by the logical and the subjective concepts of probability. In looking for the answer, though, a better understanding of Carnap's inductive logic project has been gained, which focuses on its structure as a project in explication, and makes it possible to see that the basic motivation and structure of the project developed in an incremental and continuous way, rather than in two radically discontinuous stages.

I want to stress, however, that in highlighting the conceptual coherence and continuity within the evolution of Carnap's inductive logic, I do not mean to deny that inductive logic underwent any changes at all. It obviously developed significantly between the 1940s and the 1960s. Nor do I deny that Carnap was influenced by the developments he saw on the Bayesian side—the changes in the explicandum highlighted throughout this paper are evidently the fruit of this influence. What this paper does argue is that in spite of these changes there is a fundamental conceptual continuity within the project, and that it is not accurate to characterize the changes as a transition from a logical to a subjectivist conception of probability. Rather, facing the example of inductive logic as offering a very specific understanding of probability, we may have to rethink what we mean by the logical and subjective or Bayesian concept of probability and what the true differences between them are.

A similar question to the one raised in this paper has recently been addressed by Groves (2016), who also goes back to the explicative roots of Carnap's inductive

logic program. Groves analyses Lakatos's attack on Carnapian inductive logic and argues, contrary to Lakatos, that inductive logic did not change over time in ways that would render it a degenerate research program (according to the methodology of research programs proposed by Lakatos). My work here can be seen as a continuation and expansion of this theme, but dispensing with the Lakatosian filter. In his dissertation, Groves (2015) offers a much more detailed presentation of the changes in the formal systems of inductive logic, that is, the changes in the explicatum. This perspective was not needed for answering my question, but it might be a useful supplement to the interpretation presented here.

Carnap's inductive logic has rarely been analyzed in the wider context of his philosophy and his metaphilosophical perspective. Commentators on it and those who have developed it further have focused almost exclusively on the formal side of the proposal; its critics have repeatedly pronounced the logical conception of probability unsustainable and simply dead without much attention to the larger philosophical context. I hope that the present paper belongs to what will soon be a popular approach to Carnap's project: looking at inductive logic in its larger context, and taking into account its conceptual roots in the more general perspective of Carnap's philosophy. It might quite possibly turn out that the main value of Carnap's inductive logic lies somewhere other than in its technical continuity with subsequent developments in Bayesian statistics. But a comprehensive demonstration of that more general hypothesis remains to be achieved.⁵

NOTES

1. It has to be said, however, that these publication dates can be misleading when it comes to tracing the development of inductive logic in time. *Foundations*, though published in 1950, was mostly finished by 1947; and the inductive-logic part of the "Replies" in the Schilpp volume was written in 1957 (Carus 2017). Still, the exact timeline here does not have a bearing on the argument of this paper.
2. It comes as a bit of a surprise, then, that in correspondence Carnap did use the characterization of inductive probability as degree of rational belief (see for instance letters to Kyburg [Carnap 1962a] and to de Finetti [Carnap 1963a]). This does not need to be inconsistent with the position expressed in the Schilpp volume. The characterization of inductive probability as a rational degree of belief is not deemed incorrect in the Schilpp volume; it is simply deemed too vague to provide a useful explicandum. The characterization of inductive probability as rational credibility can be seen as a way of making the original vague characterization more precise. At the same time, this vague formulation might be fine when talking about the project in general terms—also in *Foundations* (44) the talk of rational degrees of belief was considered pedagogically useful.
3. In *Foundations*, Carnap defined *c*-functions as symmetric, regular probability functions, and focused on one of them, the *c**. Later on, the symmetric regular functions were arranged in a single-parameter continuum, the λ -continuum of inductive methods. In the late inductive logic, more functions were added, resulting in the λ - γ -continuum, which allows for nonequal prior probabilities for atomic predicates. Another difference between the early and the late period lies in the switch from defining *c*-functions on sentences to defining them on the corresponding sets of models for the object language. For further details on the development of the explicata in inductive logic, see Groves (2015).
4. "It will not be claimed that *c** is a perfectly adequate explicatum for probability, let alone that it is the only adequate one" (1950, 563).
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REFERENCES

- Carnap, R. 1934. *Logische Syntax der Sprache*, Wien: Verlag von Julius Springer.
- . 1945a. "On Inductive Logic," *Philosophy of Science* 12(2): 72–97.
- . 1945b. "The Two Concepts of Probability," *Philosophy and Phenomenological Research* 5(4): 513–32.
- . 1947. "Probability as a Guide in Life," *Journal of Philosophy* 44(6): 141–48.
- . 1950a. "Empiricism, Semantics, and Ontology," *Revue internationale de philosophie* 4(11): 20–40.
- . 1950b. *Logical Foundations of Probability*, 1st ed., Chicago: Univ. of Chicago Press.
- . 1952. *The Continuum of Inductive Methods*, Chicago: Univ. of Chicago Press.
- . 1962a. Letter to Henry Kyburg, 16.03.1962, in Richard C. Jeffrey Papers, 1934–2002, ASP.2003.02, Archives of Scientific Philosophy, Archives and Special Collections, University of Pittsburgh Library System, Box 9 Folder 9.
- . 1962b. *Logical Foundations of Probability*, 2nd ed., Chicago: Univ. of Chicago Press.
- . 1962c. "The Aim of Inductive Logic," in E. Nagel, P. Suppes, and A. Tarski, eds., *Logic, Methodology and Philosophy of Science: Proceedings of the 1960 International Congress*, Stanford, CA: Stanford Univ. Press, 303–18.
- . 1963a. Letter to Bruno de Finetti, 30.07.1963, item RC 084-16-01 in Rudolf Carnap Papers, 1905–1970, ASP.1974.01, Archives of Scientific Philosophy, Archives and Special Collections, University of Pittsburgh Library System.
- . 1963b. "Remarks on Probability," *Philosophical Studies* 14(5): 65–75.
- . 1963c. "Replies and Systematic Expositions," in P.A. Schilpp, ed., *The Philosophy of Rudolf Carnap*, La Salle, IL: Open Court, 859–1012.
- . 1968. "Inductive Logic and Inductive Intuition," in I. Lakatos, ed., *The Problem of Inductive Logic*, Amsterdam: North-Holland Publishing Company, 258–67.
- . 1971a. "A Basic System of Inductive Logic, Part I," in R. Carnap and R.C. Jeffrey, eds., *Studies in Inductive Logic and Probability*, vol. I, Los Angeles: University of California Press, 33–165.
- . 1971b. "Inductive Logic and Rational Decisions," in R. Carnap and R.C. Jeffrey, eds., *Studies in Inductive Logic and Probability*, vol. I, Los Angeles: University of California Press, 5–31.
- . 1973. "Notes on Probability and Induction," *Synthese* 25(3/4): 269–98.
- . 1980. "A Basic System of Inductive Logic, Part II," in R. Carnap and R.C. Jeffrey, eds., *Studies in Inductive Logic and Probability*, vol. II, Los Angeles: University of California Press, 7–155.
- . 2017. "Value concepts (1958)," *Synthese* 194: 185–94.
- Carnap, R. and W. Stegmüller 1959. *Induktive Logik und Wahrscheinlichkeit*, Wien: Springer-Verlag.
- Carus, A.W. 2017. "Carnapian Rationality," *Synthese* 194: 163–84.
- Earman, J. 1992. *Bayes or Bust? A Critical Examination of Bayesian Confirmation Theory*, Cambridge and London: MIT Press.
- Galavotti, M.C. 2007. "Confirmation, Probability, and Logical Empiricism," in A.W. Richardson, and T. Uebel, eds., *The Cambridge Companion to Logical Empiricism*, Cambridge: Cambridge University Press, 117–35.
- Gillies, D. 2000. *Philosophical Theories of Probability*, London and New York: Routledge.
- Groves, T. 2015. *Let's Reappraise Carnapian Inductive Logic!*, Ph.D. thesis, University of Kent.
- . 2016. "Lakatos's Criticism of Carnapian Inductive Logic Was Mistaken," *Journal of Applied Logic* 14: 3–21.
- Jeffrey, R. 1973. "Carnap's Inductive Logic," *Synthese* 25(3/4): 299–306.
- Kyburg, H. 1970. *Probability and Inductive Logic*, London: The Macmillan Company.
- Skyrms, B. 1996. "Carnapian Inductive Logic and Bayesian Statistics," in *Statistics, Probability and Game Theory*, volume 30 of *IMS Lecture Notes—Monograph Series*, 321–36.
- Steinberger, F. 2017. "Frege and Carnap on the Normativity of Logic," *Synthese* 194(1): 143–62.
- Suppes, P. 2002. *Representation and Invariance of Scientific Structures*, Stanford, CA: CSLI Publications.

- Sznajder, M. 2016. "What Conceptual Spaces Can Do for Carnap's Late Inductive Logic," *Studies in History and Philosophy of Science* 56: 62–71.
- Zabell, S. 2005. "Symmetry and Its Discontents," in *Symmetry and Its Discontents. Essays on the History of Inductive Philosophy*, Cambridge: Cambridge University Press, 3–37.
- . 2011. "Carnap and the Logic of Inductive Inference," in D.M. Gabbay, S. Hartmann, and J. Woods, eds., *Handbook of the History of Logic*, vol. 10: *Inductive Logic*, Elsevier, 265–309.