

The notion of subjective probability in the work of Ramsey and de Finetti

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THE DECADE FROM the mid-twenties to the mid-thirties was undoubtedly the most crucial for the twentieth Century notion of subjective probability. It was in 1926 that Frank Ramsey wrote his essay "Truth and probability", presented at the Moral Science Club in Cambridge and published posthumously in 1931.¹ There he put forward for the first time a definition of probability as degree of belief, that had been anticipated only by E. Borel in 1924, in a review of J. M. Keynes' *Treatise on Probability*.² Ten years after Ramsey's paper, namely in 1935, Bruno de Finetti gave a series of lectures at the Institut Poincaré in Paris, published in 1937 under the title "La prévision: ses lois logiques, ses sources subjectives".³ In this paper subjective probability, defined in a way analogous to that adopted by Ramsey, was implemented with the notion of exchangeability, that de Finetti had already worked out in 1928–1930.⁴ Exchangeability confers applicability to the notion of subjective probability, and fills the gap between frequency and probability as degree of belief. It was only when these two were tied together that subjectivism could become a full-fledged interpretation of probability and gain credibility among probabilists and statisticians. One can then say that with the publication of "La prévision" the formation process of a subjective notion of probability was completed.⁵

¹ In Ramsey 1931, pp. 156–198.

² See Borel 1924.

³ See de Finetti 1937.

⁴ See de Finetti 1929 and 1930.

⁵ In a footnote added to the English edition of "La prévision" de Finetti says that he came to know Ramsey's work on probability in 1937, after the original version of "La prévision" was completed (in Kyburg and Smokler (eds.) 1964, p. 102).

Frank Ramsey and Bruno de Finetti are rightly considered the 'fathers' of subjective probability. However, their working almost at the same time on the same subject is probably responsible for the fact that in the literature their work is often referred to somewhat indistinctly, and their personal accomplishments confused and mixed together. It seems worthwhile to point out the similarities and differences existing between their views. This is precisely what will be attempted in the following pages. Reference to some of de Finetti's less known works, as well as Ramsey's recently published *Notes on Philosophy, Probability and Mathematics*⁶ will hopefully throw new light on such a comparison.

The conclusion to which this will lead can be anticipated by saying that in the work of each author there are elements that could provide an answer to the problems left open by the other. In particular, the notion of exchangeability, together with the clearcut difference traced by de Finetti between the definition of probability and its evaluation, are apt to solve the puzzlement that one encounters in Ramsey's last notes, as regards his own notion of probability. On the other hand, Ramsey's pragmatist philosophy, and especially his views on theories and universals, seem to provide a good ground to integrate de Finetti's notion of probability.

In the following pages, more space will be devoted to Ramsey than de Finetti. This seems appropriate not only in view of the fact that the present collection is centred on Ramsey, but also because de Finetti's philosophy of probability was the object of previous work of mine, to which the reader is referred.⁷

1. *The subjective theory of probability*

One or two notions of probability? An overall difference between the way of looking at probability adopted respectively by Ramsey and de Finetti should be noted at the outset. When developing his own view in "Truth and probability", Ramsey clearly meant to put forward a notion of probability intended for application to those

⁶ See Ramsey 1991a.

⁷ See Galavotti 1989 and 1991.

situations in which what matters are the decisions taken by single subjects. He did not seem to think of it as immediately applicable to science. Already in the "Foreword" to his 1926 essay he claims that probability has different meanings in logic and physics, and says that "we cannot be sure beforehand that the most useful interpretation of it in logic will be appropriate in physics also" (Ramsey 1931, p. 157). We know from Braithwaite's Introduction to *The Foundations of Mathematics* that Ramsey had planned to write a final section on probability in science, to be added to the paper. We also know from Ramsey's available manuscripts⁸ that he had planned a book "On truth and probability", of which he left a number of tables of contents. Of the projected book he only wrote the first part, dealing with the notion of truth.⁹ One can conjecture that he meant to include in the second part of the book the content of "Truth and probability", plus some more material on probability in science.

Ramsey's conviction that there are two notions of probability marks a sharp difference with de Finetti's position, since de Finetti always regarded the subjective notion of probability as the only admissible one, both in everyday life and in science. As a matter of fact, he used to consider science a continuation of everyday life. When reading Ramsey and de Finetti, this difference should be kept in mind, as it is the key to understanding the different kinds of subjectivism they put forward. This involves in the first place the admission of some notion of probability suited for physical theories, or 'chance', on the part of Ramsey, while de Finetti never admitted any such notion, which he simply regarded as meaningless. Furthermore, such a difference can be taken to account for the different roles played by the notion of utility in the work of Ramsey and de Finetti.

Probability and utility. The role of utility is an important issue that deserves some discussion. The core of the subjective interpretation

⁸ These form the Ramsey Collection, kept at the Hillman Library of the University of Pittsburgh, Special Collections Department.

⁹ This has been published as Ramsey 1991b.

amounts to the idea that probability is to be interpreted as degree of belief. It is shared by Ramsey and de Finetti, who regard the notion of probability as only explicable in psychological terms, and think that it can only acquire a precise meaning through an operational definition. In other words, in order to give probability some meaning, a way of measuring it has to be specified.

Here is where the scheme of bets and the related notion of utility, through the law of mathematical expectation, enter the picture. Ramsey refers to bets as “the old-established way of measuring a person’s belief”; a method he considers “fundamentally sound”, but “insufficiently general” (Ramsey 1931, p. 172). In view of the above, he chooses as a basis for his operational definition of probability as degree of belief a more general notion of preference. Degree of belief is then defined in terms of preferences through a set of axioms indicating how to represent its values by means of real numbers. Degrees of belief obeying such axioms are coherent—Ramsey uses the term ‘consistent’ in this connection—and coherent belief systems can be shown to satisfy the usual laws of finitely additive probability.

Ramsey’s axiomatization rests on the assumption that “behaviour is governed by what is called the mathematical expectation” which is introduced as “a law of psychology”. An example will show what this means: if event E occurs, you gain a dollars, if $\neg E$ occurs, you gain b dollars. What is the value of the situation for you? If the probability of E is q , the expectation of gain is $q \cdot a + (1 - q) \cdot b$. Ramsey’s assumption is that behaviour under uncertainty is governed by an intuitive assessment of expectations, which in turn can be led back to degrees of belief (q and $1 - q$ above). Degree of belief is then defined “in a way which presupposes the use of the mathematical expectation” (p. 174). Though not based on the traditional notion of bet where utility is measured in terms of money, Ramsey’s definition is strictly linked to the betting scheme. As he observes, his definition of degree of belief in a proposition p “amounts roughly to define the degree of belief in p by the odds at which the subject would bet on p , the bet being conducted in terms of differences of values as defined” (pp. 179–180), namely if the bet is conducted according to his way of defining preferences. Being

based on the principle of mathematical expectation, his definition of probability as degree of belief involves an appeal to some notion of utility.

Despite a widespread opinion, according to which Ramsey and de Finetti essentially agreed on the role played by the notion of utility and the betting scheme within the subjective interpretation of probability, their views on the matter show non-marginal divergences. For one thing, de Finetti does not take utility as an essential ingredient of his definition of probability. Probability and utility for him "have different cogent values" (de Finetti 1957, p. 6) and the appeal to utility is just an instrumental device to give probability an operational meaning.

Moreover, de Finetti looks at the betting scheme only as a "convenient way of talking about probability". In his essay "Sul significato soggettivo della probabilità" of 1931 he claims that the notion of probability can be "justified independently of the betting scheme" and, in addition to the well known quantitative definition in terms of fair betting quotients, puts forward an alternative definition of probability. This is qualitative in kind and is based on the primitive relation "at least as probable as" between two events. Of this qualitative, "more intuitive" notion of probability he shows that it is well defined, in the sense that a set of postulates formulated in terms of the above relation is given, and a demonstration is provided to the effect that satisfaction of such postulates implies coherence. The notion of 'coherence' in this context acquires a slightly different meaning than in the quantitative case. While in the latter saying that an individual behaves according to a coherent set of beliefs means that a book cannot be made against him, being coherent according to the qualitative notion of probability means that the individual in question does not entertain contradictory opinions regarding the probabilities to be assigned to a given set of events. De Finetti's conclusion is that it is in no way essential to adopt a quantitative notion of probability, expressible by means of a real number. This turns out to be the most convenient way of talking about probability, given its immediate operational meaning, but it is by no means the only one. It is worth noting that whereas the quantitative notion makes use of the betting scheme and is

based on mathematical expectation, the qualitative notion does not assume it.

The logic of uncertainty. In putting forward a subjective theory of probability both Ramsey and de Finetti regarded truth and probability as demanding two different kinds of logic. In this connection de Finetti talks of a 'logic of uncertainty' that starts where truth no longer applies but has to be substituted by the notion of 'practical certainty'. According to de Finetti between truth, or logical certainty, and practical certainty there is a gap that cannot be filled.

Instead of a logic of 'practical certainty' Ramsey refers to a 'larger logic', namely the 'logic of discovery, or inductive logic', as opposed to a 'lesser logic', namely the "logic of consistency, or formal logic". Also for Ramsey the two logics are separated by a gap, created by the fact that the lesser logic can be "interpreted as an objective science consisting of objectively necessary propositions". These are precisely those propositions that can be interpreted as 'tautologies' in Wittgenstein's sense (Ramsey 1931, p. 187). The larger logic can only have a psychological foundation. Again, this turns out to be in agreement with propositions 6.363 and 6.3631 of the *Tractatus*, where Wittgenstein says that "The process of induction is the process of assuming the simplest law that can be made to harmonize with our experience. This process, however, has no logical foundation but only a psychological one. It is clear that there are no grounds for believing that the simplest course of events will really happen". By quoting this passage, Ramsey opens a paper written in 1922 and called "Induction: Keynes and Wittgenstein", that he presented at the Cambridge Apostles Society.¹⁰

Before going back to this paper, which contains some very interesting remarks on Keynes' views on probability, it is worth mentioning that while distinguishing between two logics to deal with matters of certainty and uncertainty, both de Finetti and Ramsey find a

¹⁰ See Ramsey 1991a, pp. 296–301. On the first page of the manuscript of this paper one reads "Autumn 1922", while in an accompanying sheet to be found in file 007–06 of the Ramsey Collection, not written by Ramsey, the paper is dated October 20, 1923.

common ground for the two logics on a pragmatic basis. This is given by the fact that logic—in de Finetti's words—"teaches us the coherence of thought with itself" (de Finetti 1930a, see 1981 p. 261), or—in Ramsey's words—is "the science of rational thought" (Ramsey 1931, p. 191), the discipline that "enables us not to be inconsistent".¹¹ In this connection Ramsey often refers to Peirce's claim that "logic is self-control", paraphrased by him as the claim that "its value is to eliminate inconsistency in action".

2. *The criticism against Keynes*

The conviction that there is no continuity between the logic of certainty and that of uncertainty, but rather a pragmatic ground in which they are both rooted, marks a difference between Ramsey and de Finetti on the one side, and the upholders of a logical notion of probability—like J. M. Keynes and R. Carnap—on the other. Since the subjective theory of probability has sometimes been confused in the literature with the logical interpretation, it is worthwhile recollecting and comparing the criticisms urged by Ramsey and de Finetti against the logical view—mainly in Keynes' version. In the case of Ramsey, this will also illustrate the genesis of the subjectivism he advocates.

De Finetti's criticism. De Finetti's arguments against the logical interpretation of probability are exposed at length in the paper of 1938 "Probabilisti di Cambridge". De Finetti starts by expressing wide agreement—more apparent than substantial—with Keynes, concerning the idea that probability should be referred to propositions. He also welcomes Keynes' claim that a many-valued logic is not required in order to deal with probabilities. Later on in the same paper, however, he opposes the very central thesis of logicism, namely the claim that probability should be based on logical properties of propositions. This for him amounts to building the notion of probability on "the structure of nothing", or, in other

¹¹ See "Further Considerations" (1928), in Ramsey 1931, p. 201.

words, on the attribution of a realistic content to logical-mathematical expressions which are purely formal or symbolic".¹²

Here one can see de Finetti's main philosophical concern coming out, namely his urge to affirm an anti-realist perspective. I have argued elsewhere that this is the main trait of the Finetti's philosophy of probability,¹³ which is above all directed towards banning metaphysical claims from the foundations of probability. His anti-realism is reflected in the leit-motiv of his work, namely the claim that "probability does not exist",¹⁴ and is also at the root of his denial that there can be any sense in talking about 'objective probability'.

De Finetti's criticism of the logical interpretation of probability was resumed in 1967, in the course of a brief comment on Carnap's work.¹⁵ This is of special interest, because there he criticizes the notion of 'symmetry', which expresses in Carnap's terminology the same probabilistic property that the Finetti calls 'exchangeability'. The charge he makes against symmetry is that of being based on the idea that initial distributions express possible laws that reflect the existence of possible cases in nature. So, once again his criticism is directed towards the realistic import that in his eyes characterizes the logical point of view.

An important feature of de Finetti's philosophy of probability is worth recalling here. It amounts to his distinction between two components of probability judgments, namely the formal properties of probability—the probability calculus as derived from the notion of coherence—and the evaluation of probability. Now, while the probability calculus qualifies as a set of logical principles having univocal meaning, the evaluation of probability can be performed in various ways. In particular, he defines two different attitudes in this connection. First there is a 'rigid' attitude, which consists in choosing a particular probability function on the basis of reasons that are

¹² de Finetti 1938, English translation p. 86.

¹³ See Galavotti 1989.

¹⁴ This is to be found in many works by the Finetti. For instance, he wanted it to be printed in capital letters in the Preface of the English version of his *Theory of Probability* (1975).

¹⁵ See the "Replies" following de Finetti 1967.

judged 'objective' by those who take this attitude. Secondly, there is a more 'elastic' attitude, according to which coherent functions are all equally admissible, and the subject evaluating probabilities is apt to choose freely among them. While embracing this 'elastic' attitude, de Finetti charges as 'rigid' in their probability evaluations all interpretations of probability, with the sole exception of subjectivism. In other words, frequentists, logicians and upholders of the classical interpretation of probability are all sinners in this respect, as they all think there are objective reasons to privilege the choice of a particular probability function. In doing this, they confuse logical with extra-logical considerations and fail to see that the only requirement that can be placed on probabilities on logical grounds is coherence. Besides, one can only apply practical criteria to the choice of a probability function. Then, for de Finetti the frequentist, logical and classical views cannot be regarded as 'interpretations' or 'definitions' of probability at all, whereas it can be admitted that, under appropriate circumstances, such perspectives can provide useful hints for the evaluation of probabilities.

Ramsey's criticism. Ramsey's criticism is obviously addressed to Keynes, and is contained in a number of sources, in addition to "Truth and probability". Such sources include a review of Keynes' *Treatise*,¹⁶ the paper "On the hypothesis of limited variety"¹⁷ and the previously mentioned "Induction: Keynes and Wittgenstein", all written in 1922. The note "Criticism of Keynes", written sometime after 1926,¹⁸ is also relevant in this respect.

Unlike de Finetti's criticism, Ramsey's objections stem from the same background in which Keynes himself, and also Johnson were operating. This accounts for the fact that for Ramsey there is no need to claim, as de Finetti does, general agreement with the logical perspective adopted by Keynes. Moreover, the alternative Ramsey is confronted with is not between naive ontological realism and a logical way of looking at things—which is what de Finetti faces—but

¹⁶ See Ramsey 1922.

¹⁷ In Ramsey 1991, pp. 269–272.

¹⁸ In Ramsey 1991a, pp. 273–274. The paper is undated, but it contains a reference to an article by R. H. Nisbet appeared in 1926.

rather between Keynes and Wittgenstein. It amounts to the opposition between an attempt to ground probability on a logical basis comparable to that underlying deductive logic, and a psychologistic foundation of probability. As already pointed out, Ramsey took the second path. This move was essentially dictated by his dissatisfaction with the kind of relations on which Keynes wanted to ground probability. But let us follow Ramsey's arguments throughout his papers.

First of all, it is important to observe a point of analogy between our two authors. In his review of the *Treatise*, Ramsey addresses to Keynes a criticism closely reminiscent of de Finetti. In fact he opposes Keynes' view that "a probability may . . . be unknown to us through lack of skill in arguing from given evidence".¹⁹ De Finetti's previously mentioned claim that "there are no unknown probabilities"²⁰ comes to mind. The rejection of a notion of probability as a characteristic of reality, existing independently of the subject who evaluates it, seems to be a feature of Ramsey's perspective no less than de Finetti's.

A further criticism that Ramsey addresses to Keynes concerns the 'hypothesis of limited variety', which he regards as ill-founded, mainly because it is based on an implicit equiprobability assumption. Ramsey also observes that such a hypothesis is the opposite of the 'axiom of infinity', whose adoption within deductive logic he always opposed. It is well known that Ramsey's criticism led Keynes to doubt his own theory.

Instead of going into more detail in this connection, it seems appropriate to recall the passage which concludes the paper "On the hypothesis of limited variety". There Ramsey writes: "Induction would be far more satisfactorily justified by an inductive principle than by an inductive hypothesis. To say that those who use induction are reasonable because they know subconsciously some big fact about nature, necessarily very complicated, as is shown by the failure of all attempts to formulate it, is hardly good enough. It would be far better to make out that inductive procedure is reason-

¹⁹ See Ramsey 1922: p. 220 of the reprinted version (1989).

²⁰ See note 12. This point is also repeatedly stressed in de Finetti 1931a and 1931b.

able in itself, in virtue of some principle of probability, with the logical status of the Principle of Indifference. But how to state this unknown Inductive Principle is matter of great difficulty" (Ramsey 1991, p. 272). One can gather from this passage that in 1922 Ramsey, though dissatisfied with Keynes' version of the Principle of Indifference, had not yet come to its rejection so clearly expressed in "Truth and probability". In addition, he seems to be still searching for a logical justification of the inductive procedure.

A rather different attitude is to be found in "Induction: Keynes and Wittgenstein",²¹ which can be seen as a sort of bridge between the perspective characterizing the two papers discussed above and the position of "Truth and probability". In the paper in question Ramsey shows a different attitude that combines his dissatisfaction concerning Keynes' theory with the acceptance of Wittgenstein's point of view. The conviction that the 'hypothesis of limited variety' cannot be justified on logical grounds gives way to the search for a justification based on psychology rather than logic. "I see no logical reason for believing any such hypotheses;—he says—they are not the sort of things of which we could be supposed to have a priori knowledge, for they are complicated generalizations about the world which evidently may not be true" (Ramsey 1991a, p. 297). The conviction that no sound foundation can be given for the 'hypothesis of limited variety' goes hand in hand with more general doubts concerning the logical relations Keynes' theory is all about. "... the probability relations of which he ((Keynes)) speaks cannot be clearly identified" (*ibidem*, p. 298).

As regards the problem of finding a proper ground for induction, by the time he wrote "Induction: Keynes and Wittgenstein" Ramsey had clearly come to the opinion that, following Wittgenstein's suggestion, one should look for an answer to the problem in terms of psychology rather than logic. At the end of the paper, after referring to Hume's celebrated solution, Ramsey puts forward by way of a conjecture, of which he claims to be too tired "to see clearly if it is sensible or absurd", the idea that induction could be justified by saying that "a type of inference is reasonable or unrea-

²¹ See note 8.

sonable according to the relative frequencies with which it leads to truth and falsehood. Induction is reasonable because it produces predictions which are generally verified, not because of any logical relation between its premisses and conclusions. On this view we should establish by induction that induction was reasonable, and induction being reasonable this would be a reasonable argument" (Ramsey 1991a, p. 301). It has been observed²² that this claim contains the seed of the kind of inductive justification of induction that was to be developed by R. B. Braithwaite. Even more than that, Ramsey seems to envisage some sort of pragmatic justification of the inductive procedure, an idea that will reappear later in his writings.

The same kind of attitude is taken by de Finetti, who repeatedly discussed the problem of induction in his works and put forward a pragmatic justification of induction, essentially based on the notion of exchangeability. The core of his argument amounts to the claim that by means of the so-called 'representation theorem' inductive habits can be shown to lead to sensible conclusions. In other words, starting from a view of induction as governed by Bayes' rule combined with the condition of exchangeability, the representation theorem is apt to give us a reason why, in de Finetti's words, "we are also intuitively inclined to expect that frequency observed in the future will be close to the frequency observed in the past" (de Finetti 1972, p. 34). De Finetti regards this as perfectly in line with Hume's argument; indeed he claims to proceed one step further, as in his perspective the coherence of inductive behaviour does not just rest on habit and belief, but rather on a sound mathematical argument. The notion of exchangeability will be the object of further remarks in the following pages.

Going back to Ramsey, by 1926 the subjective turn of his philosophy of probability was completed. He had reached a fully developed subjective view, and was able to see in clear terms what was wrong with Keynes' theory. This led to the explicit assertion that "there are no such things" as the relations which are the object of Keynes' theory of probability. In the note "Criticism of Keynes",

²² See Sahlin 1990.

some reasons are given in favour of the alleged inexistence of those Keynesian relations. They include the following: “(a) Do we really perceive them [Keynes’ logical relations]? Least of all in the simplest cases when they should be clearest; can we really know them so little and yet be so certain of the laws which they testify? (What we do think of is what we would believe) . . . (c) They would stand in such a strange correspondence with degrees of belief” (Ramsey 1991a, pp. 273–274). Here Ramsey reaffirms the same empirical attitude pointed out before, except that now it does not concern only ‘unknown’ probabilities, but the logical relations Keynes is talking about. In this connection his position is close to de Finetti’s. Once the view is adopted that probability is to be identified with degree of belief, this is not to be reconciled with a conception of probability as a characteristic of reality, independent of the subject evaluating it.

On the same note, we find once again an expression of Ramsey’s pragmatism, when, commenting on Keynes’ proposed justification of probability relations, he says “This is nearer the truth it can only mean that they mostly lead from truth to truth otherwise poor consolation of being always justified in error”; and adds: “(f) But justification is not matter of logical relations only” (*Ibidem*).

Having adopted a notion of probability in terms of coherent degrees of belief, Ramsey does not need to ground his theory on a principle “not sound” (*Ibidem*), such as the Principle of Indifference. This is the main accomplishment he was able to achieve after having worked out the subjective notion of probability. The dismissal of this Principle is contained in “Truth and probability”, where he says: “The Principle of Indifference can now be altogether dispensed with; we do not regard it as belonging to formal logic to say what should be a man’s expectation of drawing a white or a black ball from an urn; his logical expectations may within the limits of consistency be any he likes; all we have to point out is that if he has certain expectations he is bound in consistency to have certain others” (Ramsey 1931, p. 189). Since the importance of this point has been stressed by others,²³ I will not comment on it further.

²³ See Zabell’s paper in this collection.

A remark contained in Ramsey's discussion of the Principle of indifference is nevertheless worth recalling: "anyone who tries to decide by Keynes' methods what are the proper alternatives to regard as equally probable in molecular mechanics, e.g. in Gibbs' phase-space, will soon be convinced that it is a matter of physics rather than pure logic" (*ibidem*). The passage suggests that scientific theories—particularly physical theories—can guide probability assignments; a view that de Finetti always strongly opposed. This relates to the comments made earlier in these pages, regarding Ramsey's admission of some notion of objective probability. Indeed, several remarks in his writings indicate that for him such a notion was presumably to be taken in connection with theoretically more developed sciences, like physics. For instance, in his note "Broad on probability and induction" he proffers agreement with Broad's claim that "those inductions which we regard as highly probable are so relative((ly)) to the belief that we really have got hold of the general ground plan of nature in the region of phenomena under investigation . . . Sciences where no such discovery has yet been made((,)) such((,)) e.g.((,)) as psychology and biology are almost at a prescientific level; then inductions carry no great conviction to anyone trained in the more advanced sciences" (Ramsey 1991a, pp. 267–268). The passage indicates that he regarded with sympathy the idea that induction is more warranted within highly theoretical sciences than within other sciences having not attained the same degree of development. This gives us a hint about the meaning ascribed by Ramsey to the notion of 'probability in physics', to which he refers in the "Foreword" to "Truth and probability". A further indication in this direction is given by some claims contained in Ramsey's note "Chance". There he first maintains that "Chances are degrees of belief within a certain system of beliefs and degrees of belief; not those of any actual person, but in a simplified system to which those of actual people, especially the speaker, in part approximate" (Ramsey 1931, p. 206). He goes on saying that "Probability in Physics means chance as here explained, with possibly some added complexity . . ." (*ibidem*, p. 209). What he favours here is a view according to which probability can receive an 'objective' meaning directly from scientific theories.

3. *Ramsey's discontents.*

We have seen how by 1926 the development of Ramsey's ideas on probability had evolved from the search for a satisfactory explication of probability on logical grounds to a subjectivist and pragmatist perspective. However, some notes from 1928 and 1929 offer evidence that by that time he was puzzled about some aspects of his own view.

We know from the manuscript published under the title *On Truth* that in the last years of his life Ramsey was writing on that subject. Still, some notes left from that period—including those published under the heading "Further considerations" (in Ramsey 1931) and additional material more recently published (in Ramsey 1991a)—testify that he was still reflecting on probability. More specifically, he was involved in a clarification of the notion of 'probability in physics', commented on earlier, and of some related notions, like 'chance', statistics, and the relationship linking frequency and degree of belief. This might be taken to suggest that he actually had in mind to write the second part of the planned book "On truth and probability", including the missing section on 'probability in physics'.

Evidence of the fact that Ramsey was not totally happy with the foundation of his own concept of probability is conveyed by some claims contained in a note bearing the title "Reconsiderations on causality, probability, etc.", dated September 1929—a few months before his death. Right at the beginning of the note Ramsey writes: "what is wrong with my probability theory is its externality" (Ramsey 1991a, p. 277). There follow a few quotations from Wittgenstein on the thesis that logic "is a consequence of analytic psychology" ("Es liegt im Begriff des Denkens dass man $p \cdot \neg p$ nicht denken kann"), and that it cannot be interpreted as a part of natural science ("Aber dieser Begriff des Denkens ist keiner naturwissenschaftlicher"). Going back to degree of belief he says: "All this is even clearer in my probability theory. Degree of belief is a useless scientific conception, and should not be introduced as one" (*ibidem*, pp. 277–278). Though the note is not completely clear, one might conjecture that Ramsey, unhappy with a justification of

induction based on the notion of 'reasonableness', when writing it was trying to think of belief "as part of our cognitive process". Belief, however, is not to be taken as such, being rather based on habit—whence the "externality" of the notion of probability in terms of degrees of belief.

In the note "Reasonable degree of belief"—published among "Further considerations"—Ramsey had already advanced doubts about the notion of 'reasonableness' of degree of belief. "To introduce the idea of 'reasonable'—he says there—is really a mistake; it is better to say 'usually', which makes clear the vagueness of the range" (Ramsey 1931, p. 203). Ramsey's discontent with the fuzziness of a notion based on habit could not be expressed more clearly. Another passage reads as follows: "Nor can the precise sense in which 'reasonable' or 'usually' can usefully be taken be laid down" (*ibidem*, p. 202).

Putting together the remarks appearing in these two notes from 1928 and 1929, one can see that Ramsey's puzzlement comes from the fact that the notion of degree of belief is in itself neither part of experience nor a genuine psychological notion. In view of this, it is exceedingly difficult to provide a good foundation for such a notion. Indeed, the problems he raises in the notes in question also seem to undermine the pragmatic attitude that we have seen expressed earlier in his writings, for this is no less based on some notion of 'reasonableness', or on the possibility of showing that probabilistic behaviour leads to truth more than to error.

This whole bunch of problems is absent from de Finetti's perspective. For him probability is a primitive notion, expressing the "psychological sensation of an individual" (de Finetti 1931a, p. 302), and there is no need to ground probability on some notion of rationality that transcends the individual who evaluates probability. The reasonableness of inductive procedure for de Finetti rests on exchangeability which—as we have already seen—represents the key notion of his justification of induction. Exchangeability, combined with Bayes' scheme—to which for de Finetti any valid induction must conform—indicates how frequencies modify initial evaluations of probability. In other words, it gives a method for updating such evaluations in response to the acquisition of new information. In so doing, exchangeability, according to de Finetti, gives a re-

sponse to Hume's problem, and proves the reasonableness of inductive behaviour.

The notion of exchangeability is the necessary step from a subjective view to a full-fledged subjective interpretation of probability, for it is exchangeability that guarantees the applicability of subjective probability to practical situations, especially to those encountered within experimental science. Exchangeability is the missing aspect in Ramsey's perspective, that which could have made him see the connection between degree of belief and frequency that—among other things—would have indicated how degree of belief can be justified on pragmatical grounds.

By way of a conjecture, one might say that at the end of his life Ramsey was not only looking for a notion like exchangeability, but indeed came close to it. Evidence that he was intrigued by the relationship between frequency and degree of belief is to be found in "Miscellaneous notes on probability" written in 1928 (in Ramsey 1991a). There he distinguishes four kinds of connections, namely: "(1) if degree of belief = γ , most prob((able)) frequency is γ (if instances independent). This is Bernoulli's theorem; (2) if freq((uency)) has been γ we tend to believe with degree γ ; (3) if freq((uency)) is γ , degree γ of belief is justified. This is Peirce's definition; (4) degree γ of belief means acting appropriately to a frequency γ ". And he adds: "It is this last which makes calculus of frequencies applicable to degrees of belief" (Ramsey 1991a, p. 275). Now, the result that tells us exactly how to treat the relation (4) is the representation theorem. It then seems that Ramsey would have found an answer to at least part of what he was looking for in the results that de Finetti was working out in the same years, but he did not have the chance to know.

It is nevertheless interesting to point out that Ramsey knew the property de Finetti calls 'exchangeability'. He probably took it from his teacher and colleague W. E. Johnson, whose 'permutation postulate' is equivalent to Carnap's symmetry and de Finetti's exchangeability. Another note by Ramsey, bearing the title "Rule of succession" (see Ramsey 1991a), contains a derivation of Laplace's rule of succession from the notion in question, named by him 'equiprobability of all permutations'.

4. *By way of a conclusion.*

The combination of the notion of exchangeability with the Bayesian scheme for updating probability evaluations that gives rise to the representation theorem marks the decisive step toward a fully developed subjective interpretation of probability. It was de Finetti's great merit to be able to make this step, that provided subjective probability with direct applicability to statistical inferences—at least to those of the inductive sort. One can say that it was through the notion of exchangeability that subjectivism gained respectability among statisticians, though this only took place in the Fifties, mainly through L. J. Savage's work.

The notion of exchangeability also plays a central role within de Finetti's philosophy of probability, as it provides the ground for his pragmatical justification of induction. A further element of great importance in de Finetti's perspective is his distinction between the formal properties of probability, like the probability calculus and the notion of coherence, and the complex procedure of the evaluation of probabilities, which is extra-logical and intrinsically context-dependent. This distinction, together with that combination of exchangeability and Bayesian conditioning known as the 'representation theorem', seem to convey at least a partial answer to Ramsey's puzzlements, for the first illustrates the relationship between logical and subjective components of probability as degree of belief, while the second fills the gap between degree of belief and frequency.

As we saw, within de Finetti's perspective these elements are combined with a view of probability as a primitive notion, corresponding to the feelings of those evaluating it. Probability can be seen as a qualitative notion, and defined accordingly, or it can be taken as quantitative, and become the object of an operational definition, which is apt to be built upon the scheme of betting.

As a counterpart to such a broad way of defining probability, de Finetti shows a dogmatic attitude with regard to the interpretation of probability, and does not admit of any sense in which probability can be given some sort of 'objective' meaning. To this he was led by his strongly anti-realistic and anti-metaphysical philosophy. It is the same attitude that prevented him from attaching any meaning to

such notions and methods as randomness and statistical tests, which gather wide consensus among researchers making use of statistical techniques.

Ramsey, on the other hand, was the proponent of a sophisticated philosophical position, which included a pragmatic view of theories, universals and laws. One can guess that de Finetti would have found it very congenial. Indeed, some of the problems that are left open by de Finetti's view of probability could find a solution within Ramsey's philosophy. Ramsey's claim that philosophy is "analysis of possibilities" (see Ramsey 1991a, p. 226) indicates a way of escaping metaphysics that does not imply the kind of extreme subjectivism to which de Finetti is led by his anti-metaphysical attitude. Moreover, Ramsey's view of knowledge as based on "a reliable process" and "leading to success"²⁴ fits in very well with a probabilistic approach, like the one taken also by de Finetti, and his redundancy theory of truth conveys within a sound philosophical position the dissatisfaction with stronger notions of truth that the Finetti expresses so strongly in his essay "Probabilismo" (de Finetti 1931b). The same holds for Ramsey's theory of universals and laws, which gives a direct answer to de Finetti's repeated claims to the effect that we only talk about facts, and that 'theories' (this is the term he uses in order to refer to generalizations, like statistical laws) have only an instrumental meaning (see de Finetti 1971). Last but not least, Ramsey's view of theories provides a proper ground on which the notion of 'objective probability' could be founded.

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²⁴ See Sahlin's article in this collection.

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